1. **Preliminaries.** We say \( f(n) = O(g(n)) \) if \( f(n) \leq cg(n) \) for all \( n \geq n_0 \) for some constants \( c \) and \( n_0 \). We say \( f(n) = \Omega(g(n)) \) if and only if \( g(n) = O(f(n)) \). Also, \( f(n) = \Theta(g(n)) \) if \( f(n) = O(g(n)) \) and \( f(n) = \Omega(g(n)) \).

A partial list of functions in increasing order is: \( O(1), (\log n)^{\epsilon}, \log n, (\log n)^{1+\mu}, n^{\epsilon}, n, n^{1+\mu}, 2^{n^{\epsilon}}, 2^n, 2^{n+\mu} \) where \( 0 < \epsilon < 1 \) and \( \mu > 0 \) are constants.

Stirling’s approximation: \( n! \approx (n/e)^n \sqrt{2\pi n} \).

\[
\sum_{i=1}^{n} i = n(n+1)/2, \quad \sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6, \quad \sum_{i=1}^{n} i^3 = n^2(n+1)^2/4.
\]

2. **Master theorem.** Consider the recurrence relation: \( T(n) = aT(n/b) + f(n) \), where \( a \geq 1 \) and \( b > 1 \) are constants. **Case 1:** If \( f(n) = O(n^{\log_b a - \epsilon}) \) for some constant \( \epsilon > 0 \), then \( T(n) = \Theta(n^{\log_b a}) \). **Case 2:** If \( n^{\log_b a} = \Theta(f(n)) \), then \( T(n) = \Theta(f(n) \log n) \). **Case 3:** If \( f(n) = \Omega(n^{\log_b a + \epsilon}) \) for some constant \( \epsilon > 0 \) and \( af(n/b) \leq cf(n) \) for some constant \( c < 1 \), then \( T(n) = \Theta(f(n)) \).

3. **Randomized algorithms.** A Monte Carlo algorithm runs for a prespecified amount of time and its output is correct with high probability. By high probability we mean a probability of \( \geq (1 - n^{-\alpha}) \), for any constant \( \alpha \). A Las Vegas algorithm always outputs the correct answer and its run time is a random variable. We say the run time of a Las Vegas algorithm is \( \tilde{O}(f(n)) \) if the run time is \( \leq c\alpha f(n) \) for all \( n \geq n_0 \) with probability \( \geq (1 - n^{-\alpha}) \), for some constants \( c \) and \( n_0 \).

4. **Data Structures.** A dictionary supports three operations: INSERT, DELETE, and SEARCH. A (min) priority queue supports: INSERT, FindMin and DeleteMin. If one uses a 2-3 tree, each of these operations takes \( O(\log n) \) time, \( n \) being the number of elements in the data structure. In the Union-Find paradigm, there are \( n \) singleton sets to begin with: \( \{1\}, \{2\}, \ldots, \{n\} \).

We are interested in performing a sequence of union and find operations. If one uses WeightedUnion and CollapsingFind, a sequence of \( m \) operations can be completed in \( O(m\alpha(m)) \) time where \( \alpha(.) \) is the inverse Ackermanns function.

5. **Sorting.** Given a sequence of \( n \) numbers (or keys), the problem of sorting is to rearrange this sequence in either nondecreasing order or nonincreasing order. The mergesort algorithm has a worst case run time of \( O(n \log n) \). The run time of quicksort is \( O(n^2) \) in the worst case and \( O(n \log n) \) on the average.

A sorting problem is called general sorting if the only information known about the keys is that the keys are from a linear order. Any general sorting algorithm makes use of comparison as the basic operation. It can be shown that any general sorting algorithm needs \( \log(n!) = \Omega(n \log n) \) comparisons in the worst case.

We can sort \( n \) keys in \( O(n) \) time if the keys are integers in the range \([1, n^{\epsilon}] \) (for any constant \( c \)).

6. **Selection.** Given a sequence of \( n \) keys and an integer \( i (1 \leq i \leq n) \), the problem of selection is to identify the \( i \)-th smallest key from out of the \( n \) keys. Quickselect algorithm takes \( O(n^2) \) time in the worst case and \( O(n) \) time on the average. BFPRT algorithm takes \( O(n) \) time in the worst case.

7. **Matrix Multiplication.** Strassen’s algorithm takes \( O(n^{\log_2 7}) \) time.

8. **Convex Hull.** The convex hull of \( n \) points in the plane can be computed in \( O(n \log n) \) time.