ABSTRACT

Target class measurements, if available from automatic target recognition systems, can be incorporated into multiple target tracking algorithms to improve measurement-to-track association accuracy. In this work, the performance of the classifier is modeled as a confusion matrix, whose entries are target class likelihood functions that are used to modify the update equations of the recently derived multiple models CPHD (MMCPHD) filter. The result is the new classification aided CPHD (CACPHD) filter. Simulations on multistatic sonar datasets with and without target class measurements show the advantage of including available target class information into the data association step of the CPHD filter.

Keywords: CACPHD, Multiple models CPHD (MMCPHD), Classification

1. INTRODUCTION

Traditionally, only kinematic measurements are used to solve the difficult data association problem in multitarget tracking. However, incorporating target class measurements into the tracking algorithm can have significant benefits in terms of improved track accuracy and track purity [1]. In this paper, we present a new multitarget tracker, the CACPHD, that integrates target class information into the data association step of the Cardinalized Probability Hypothesis Density (CPHD) filter.

Feature aided (or classification aided) tracking has been previously proposed for most of the major multitarget trackers. In [2], Gaussian feature likelihood functions were used to help in the data association of the Joint Probabilistic Data Association (JPDA) filter. In [3], an extension to the Probabilistic Multihypothesis Tracker (PMHT) which accounts for noisy target classification measurements when performing data association is derived. In [4], the incorporation of target classification output into the likelihood function for multidimensional (S-D, S > 2) or multiframe assignment is presented. And in [5], Gerard et al. put forth a method, based on a feature aided Multi Hypothesis Tracker (MHT), that detects and classifies odontocete echolocation clicks as well as estimates the number of animals that are vocalizing.

Here, we propose the feature aided CPHD filter, which we refer to as the CACPHD filter. Its derivation relies heavily on the equations of the multiple model CPHD filter, hence we first turn our attention on summarizing the operation of the MMCPHD filter.

2. MULTIPLE MODELS CARDINALIZED PROBABILITY HYPOTHESIS DENSITY FILTER

2.1 CPHD Filter

Mahler introduced a new approach to multitarget tracking based on finite set statistics (FISST) [6], in which target states and measurements are modeled as random finite sets (RFS). Mahler proposed that the probability hypothesis density (PHD) be propagated as a first moment approximation to the multitarget Bayesian posterior distribution instead of the full posterior distribution [6]. The probability hypothesis density PHD is the intensity function of an RFS and represents the density of the expected number of targets per unit volume [7], [8]. In the bin model [9] approach to the PHD filter employed here, the PHD surface is discretized into infinitesimally small bins and the probability of each bin containing a target is predicted and updated.
The Cardinalized PHD (CPHD) filter [10] offers a nice cure to the premature “target death” problem [9] that plagues the PHD filter through adding memory to the target-number process, and although it introduces an artificial linkage between all targets’ existence probabilities (a significant effect observed as “spooky action at a distance” in [11]), it seems to work very effectively in most applications. The CPHD filter is a PHD filter with a hierarchically “supervising” Hidden Markov Model (HMM) that describes the number of targets in the scene; that is, the cardinality of the Poisson RFS is not restricted to be Poisson distributed, and can in fact be arbitrary. The only restriction for this distribution is that its first moment be equal to the expected number of targets. In other words, the CPHD filter approximates the Bayesian multitarget tracking problem with an i.i.d. cluster process. In the HMM used by the CPHD filter, the target number is external to the PHD surface itself and moreover it affects the PHD surface (as seen in Figure 1).

2.2 Multiple Models CPHD (MMCPHD) Filter

Multiple models (MM) approaches assume that a system operates according to one of a finite number of models; these models can have different noise, structure, state dimensions and/or unknown inputs. The dynamic MM estimator, used for systems that, with time, transition from one model to another, employs a Bayesian framework: based on prior probabilities of each model being currently in force, posterior probabilities are calculated [12]. By assumption, the model switching, also known as model jumping, is a Markov chain with known model transition probabilities.

While a nonmaneuvering target can usually be described well by a single model, a bank of different hypothetical target motion models characterizing possible maneuvers may be needed to operate in parallel in order satisfactorily to track a maneuvering target. MM approaches are attractive because, due to their exploration of the set of viable target motion models, they achieve better performance over the corresponding single-model filter; with some justification the MM filter is sometimes referred to as having adaptive bandwidth.

Multiple model extensions for well established tracking algorithms exist, such as the IMMJPDA (Interacting MM version of the Joint Probabilistic Data Association filter) [13] and IMMMHT (IMM version of the Multiple Hypothesis Tracker) [14]. As for the (recently-developed) PHD filter, Pasha et al. [15] first proposed augmented PHD filtering for linear jump Markov models and later extended their MMPHD to nonlinear jump Markov models [15]. Additionally, Punithakumar et al. [16] have developed a MMPHD using a particle filter implementation.

The multiple model version of the considerably more complex CPHD filter has been recently derived in [17], with the challenge lying in describing the interrelation between the models. Figure 2 shows the HMM used by a MMCPHD filter with two models, in which there is “crosstalk” between the two PHD surfaces and the target number affects them both.
Here, the MMCPHD filter is recast as the Classification Aided CPHD filter that is able to ingest and provide target class information. In the following, we summarize our previous work on the MMPCHD filter and next, in Section 3, we introduce the modifications to the prediction and update equations of the MMCPHD filter that turn it into the CACPHD filter. In Section 4, we show results of the application of the CACPHD filter to simulated multistatic sonar data. We conclude in Section 5.

### 2.2.1 MMCPHD Prediction Equations

In [17], the MMCPHD filter for two target motion models has been derived using a bin model approach. The MMCPHD prediction equation for the bin probabilities obtained was:

\[
P\{U_k(i)=1, r_k=q|Z_1^{k-1}\} = b(i, r_k = q) + \sum_{\bar{q}=1}^{\bar{q}=2} t_{\bar{q}q} \sum_j f(i|x_j, r_{k-1}=\bar{q})P_s(x_j, r_{k-1}=\bar{q})P\{U_{k-1}(j)=1, r_{k-1}=\bar{q}\} \tag{1}
\]

where \(U_k(i)\) is the probability that at time \(k\) bin \(i\) contains a target, \(r_k\) is the model at time \(k\), \(Z_1^{k-1}\) are the measurements received up to time \(k-1\), \(b(i, r_k = q)\) is the birth probability for a target to appear in bin \(i\) under model \(r_k = q\), \(P_s(x_j, r_{k-1}=\bar{q})\) is the probability of survival from \(k-1\) to \(k\) for the target in bin \(j\) with model \(r_{k-1}=\bar{q}\), \(f(i|x_j, r_{k-1}=\bar{q})\) is the probability that a target located at \(x_j\) in model \(r_{k-1}=\bar{q}\) propagates to bin \(i\) and \(t_{\bar{q}q}\) is the Markov transition matrix probability that a target switched from model \(\bar{q}\) to model \(q\), where \(q \in \{1, 2\}\).

The MMCPHD prediction step for the probability mass function (pmf) of the number of targets in model \(r_k = q\) is given below:

\[
p_k|k-1(n, r_k = q) = \sum_{\sum_{\bar{q}=1}^{\bar{q}=2} i_{\bar{q}} = 0} \psi\left(n - \sum_{\bar{q}=1}^{\bar{q}=2} i_{\bar{q}}, r_k = q\right) \times \prod_{\bar{q}=1}^{\bar{q}=2} \sum_{i_{\bar{q}}=1}^{N_{\text{max}}}(i_{\bar{q}}) t_{i_{\bar{q}}q} (1-t_{\bar{q}q})^{i_{\bar{q}}} \sum_{n'=1}^{N_{\text{max}}}(n') P_{k-1}(n', r_{k-1}=\bar{q})(1-P_s)^{n'-i_{\bar{q}}} P_s^{i_{\bar{q}}} \tag{2}
\]

\(\psi(\cdot, r_k = q)\) is the number of new-born targets at time \(k\) with model \(r_k = q\), \(\times\) stands for multiplication, \(N_{\text{max}}\) is the ceiling on the number of targets, and \(P_s\) is the average probability that a target survives, i.e. does not “die” between scans.

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**Figure 2.** The underlying hidden Markov model (HMM) of the inference process of the MMCPHD filter. Observe the “crosstalk” between the two PHD surfaces \(U_s\) and \(W_s\) and that the target number \(N_k\) affects them both.
2.2.2 MMCPHD Update Equations

The update step for bin probabilities is written for the case of missed detections and also for the case of successful target detections:

\[
p(U_i, r_k = q | Z) = \frac{L(Z | U_i, r_k = q)}{L(Z)} \cdot p(U_i, r_k = q) = (1 - P_d) \frac{L(Z | U_i, r_k = q, V_i = 0)}{L(Z)} \cdot p(U_i, r_k = q) + P_d \frac{L(Z | U_i, r_k = q, V_i = 1)}{L(Z)} \cdot p(U_i, r_k = q)
\]  

(3)

where we define the “visibility” indicator \(V_i\) at time \(k\) as a Bernoulli random variable with probability \(P_d(x_i)\) which is unity if the target in bin \(i\) is detected at time \(k\), \(U_i\) is shorthand for \(U_k(i) = 1\) and \(L(\cdot)\) are likelihood ratios with respect to the null hypothesis of all targets being clutter.

The update step for the target cardinality pmf is given by:

\[
p(n, r_k = q | Z) = \frac{L(Z | n, r_k = q)}{L(Z)} \cdot p(n, r_k = q) = \frac{\sum_{c = \max(|Z| - n, 0)}^{\left|Z\right|} L(Z | n, r_k = q, c) p(c)}{\sum_{q = 1}^{2} \sum_{c = 0}^{\left|Z\right|} L(Z | n, r_k = q, c) p(n, r_k = q) p(c)} \cdot p(n, r_k = q)
\]

(4)

where \(p(c)\) is the clutter pmf, \(p(r_k)\) is the prior model pdf and the explicit expressions for the likelihood ratios used in the above equations, i.e., \(L(Z | U_i, r_k = q, V_i = 1)\), \(L(Z | U_i, r_k = q, V_i = 0)\), \(L(Z | n, r_k = q)\) and \(L(Z)\) can be found in [17].

2.2.3 Track Management (TM)

In the Gaussian Mixture (GM) implementation of the CPHD filter, the posterior PHD surface is approximated by a GM and hence, the propagation of the whole surface can be replaced by the propagation of the weight, mean and covariance of each mode in the mixture. At each time scan, mode means and covariances are propagated by an Extended Kalman Filter (EKF) while mode weights are calculated using the prediction and update equations.

A multitarget tracker is composed of a filter followed by track management policies. In its original form the GMCMPHD filter is not able to provide scoreable tracks, so a track display/management scheme had to be devised [18] [19] in order to make the transition from filter into tracker. This is a set of policies dealing with track initiation, update and deletion, spawning, mode pruning and merging [17]. Note that track management is separate from the operation of the filter.
In our GMCPHD tracker, modes propagate through their offspring and it is this connectivity over time that allows for track management. Each Gaussian Mixture mode maintains a record of its “father” mode, i.e. the mode at the previous scan that was updated and became the current GM mode. It also has the same track identification number (ID) as its father.

In the case of our MMCPHD filter, the track management scheme described above is applied to an extra “display” layer, obtained by fusing corresponding Gaussian modes in the two models through moment matching. Hence, track management policies do not interfere with the operation of the two CPHD filters. For example, only the Gaussian modes that compose the extra (third) layer are assigned track IDs; the process is illustrated in Figure 3. The classification aided CPHD (CACPHD) filter inherits this track management scheme from the MMCPHD filter.

3. CLASSIFICATION AIDED CARDINALIZED PROBABILITY HYPOTHESIS DENSITY FILTER

In most conventional tracking systems, e.g. radar and sonar systems, only the target kinematic measurements are used to perform measurement-to-track association while the available target class information is typically relegated to postprocessing. On the other hand, in feature- (or classification-) aided tracking, target class information is integrated into the data association step of the filter, and generally promotes improved tracking accuracy, purer tracks and better track continuity [1].

In the following, we modify the way data association is done in the MMCPHD filter to include the simultaneous use of both target classification information and target kinematic information. The benefits of this approach are revealed, for example, when tracking two targets based on multistatic sonar data in which using the target kinematic measurement likelihoods alone can not satisfactorily differentiate between the targets. We refer to the new filter as the classification aided CPHD (CACPHD) filter.

Let \( N \) be the number of possible classes for a target and \( \zeta \) be the classifier output. The output of the classifier is a target class, specifically it is the predicted class to which the target under consideration belongs according to the classifier. Thus,

\[
\zeta \in \{1, 2, \cdots, N\}
\]

We can model the classifier accuracy according to a confusion matrix of the form \( C_{matrix} = [c_{ij}] \) where

\[
c_{ij} = Pr\{\zeta = j|\kappa = i\}
\]

and \( i, j = 1, 2, \cdots, N \). Note that the entries in the confusion matrix, e.g. \( c_{ij} \), represent the likelihood of the true class being \( i \) when the observed class (aka classifier output) is \( j \). Hence, the target class likelihood function for a measurement of target class \( j \) is the \( j \)-th column in the confusion matrix[1].

Here, we consider two possible target classes and assume that the target class likelihood functions above are independent of additional variables such as target state.

The CACPHD filter consists, like the MMCPHD filter, of two CPHD filters that closely interact. Incorporating the target class likelihood functions into the CACPHD filter requires only the update equations for bin probabilities in the MMCPHD filter to be modified. In particular, in the MMCPHD filter, the likelihood of a measurement originated from a target with motion model \( r_k = q \) is:

\[
L_z = \sum_j f(z|x_j, r_k = q) \frac{p(U_j|r_k = q)}{\sum_l p(U_l|r_k = q)}
\]

Similarly, for the CACPHD filter*, the likelihood that a measurement with class \( \bar{q} \) originated from a target in class \( c_k = q \) is given by:

\[
L_z = \sum_j f(z|x_j) c_{\bar{q}q} \frac{p(U_j|c_k = q)}{\sum_l p(U_l|c_k = q)}
\]
In order to convert the MMCPHD filter into the CACPHD filter, Eq. 8 is plugged into the expressions for the likelihood ratios that appear in Eqs. 3 and 4. For example, in the MMCPHD filter, the likelihood ratio in the case of a successful detection is:

\[
L(Z|U_i, r_k = q, V_i = 1) = \sum_{j=1}^{m} p(c = m - j) \frac{(m-j)!}{m!} \sum_{s=1}^{m} \frac{f(z_s|x_i, r_k = q)}{c(z_s)} \times \\
\sum_{n=j}^{\infty} n(n-1) \cdots (n-(j-1)) p(n|r_k = q)(1-P_d)^{n-j} \frac{1}{\sum_{l} p(U_l|r_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)}
\]

Likewise, in the CACPHD filter, Eq. 10 is matched by:

\[
L(Z|U_i, c_k = q, V_i = 1) = \sum_{j=1}^{m} p(c = m - j) \frac{(m-j)!}{m!} \sum_{s=1}^{m} \frac{f(z_s|x_i, c_k = q)}{c(z_s)} \times \\
\sum_{n=j}^{\infty} n(n-1) \cdots (n-(j-1)) p(n|c_k = q)(1-P_d)^{n-j} \frac{1}{\sum_{l} p(U_l|c_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)} \frac{1}{\sum_{l} p(U_l|c_k = q)}
\]

with the \( L_z \) terms as given in Eq. 8. Note that the likelihood functions for clutter appearing in Eq. 11 consist of a product between a likelihood function for the kinematic component and a likelihood function for the class.

4. RESULTS

4.1 Setup

We compared the classification aided CPHD (CACPHD) filter and the original CPHD filter on two simulated datasets. In the first scenario, two targets cross paths. A monostatic sensor at (0m,-3000m) with \( \sigma_{tdoa} = 0.1 \) sec and \( \sigma_{bearing} = 5^\circ \) collected measurements at a ping period of 60 sec for a total scenario time of 3600 sec from the two targets in Figure 4.a. Target 1 moves from southwest to northeast and target 2 moves from northwest to southeast. Both targets have \( \text{SNR}_{1km} = 40dB \) (SNR that would be observed from a specular target contact 1 km away from a monostatic sonar) and are in Rayleigh clutter; the detection threshold was 7dB. In the second scenario, two targets move on parallel paths from left to right (target 1 is the lower target and target 2 is the
upper target). The monostatic sensor collected measurements of the same quality from the targets moving from left to right in Figure 4.b. Note the large uncertainty in bearing the received measurements display in Figure 4.

In the CACPHD filter, the Markov transition matrix was considered to be $t_{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ as it was assumed that targets do not switch between class types. In both scenarios, class measurements were generated from Gaussian distributions $\mathcal{N}(\mu, \sigma^2)$, where $\mu_1 = 6$ for the first target, $\mu_2 = 3$ for the second target, $\mu_0 = 0$ for the clutter and $\sigma_1 = \sigma_2 = \sigma_0 = 1$. Therefore, the confusion matrix was set to:

$$C_{matrix} = \begin{bmatrix} 1 - \text{normcdf}(\tau, \mu_1, \sigma_1) & \text{normcdf}(\tau, \mu_1, \sigma_1) \\ 1 - \text{normcdf}(\tau, \mu_2, \sigma_2) & \text{normcdf}(\tau, \mu_2, \sigma_2) \end{bmatrix}$$

where $\tau = \frac{\mu_1 - \mu_2}{2}$ and $\text{normcdf}(\tau, \mu, \sigma) = \int_{-\infty}^{\tau} \mathcal{N}(x, \mu, \sigma^2) dx$. Note that setting the confusion matrix to $C_{matrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$ turns off the use of the classification information and reduces the CACPHD filter equations to the original CPHD equations. To decide if a class measurement came from target 1 or target 2, a likelihood ratio test was performed.

### 4.2 Tracks

Figure 5 shows tracking results for the scenario with crossing targets. When the classification information is turned off, i.e., not used as in the original CPHD filter, a false track that could belong to either target appears soon after the targets cross paths. With the classification information turned on, i.e., used as in the new CACPHD filter, this track (although still appearing soon after the trajectories cross) becomes a true track. However, visually, it is hard to clearly see the benefits of using target class information and hence, the calculation of appropriate metrics of performance is needed.

Figure 6 shows tracking results for the scenario with parallel targets. In this scenario, the effect of using target class information is obvious. Without class measurements, the track following target 2 is lost prematurely and the track following target 1 crosses over to follow target 2 towards the end of the scenario. Neither of these problems is encountered when taking into account class information as for the CACPHD results.

### 4.3 Metrics of Performance

As metrics of performance (MOPs), we report: track detection probability ($P_D$), i.e., the ratio of the total duration of all true tracks and the total scenario duration; track fragmentation ($FRAG$); number of false tracks ($FT$); and root mean square error ($RMSE$) evaluated only where tracks exist. Further details on calculation of these metrics can be found in [20].

Table 1 shows the MOPs calculated for the scenario with crossing targets. A noticeable improvement brought about by the addition of class information is the significant reduction of the RMS error for target 1. Similarly, table 2 shows the MOPs calculated for the scenario with parallel targets. Here, the striking improvement is in the track probability of detection for target 1. Additionally, RMSE is much lower for both targets when using class measurements.

<table>
<thead>
<tr>
<th></th>
<th>$P_D1$</th>
<th>$P_D2$</th>
<th>FRAG1</th>
<th>FRAG2</th>
<th>$FTs$</th>
<th>$RMSE1$</th>
<th>$RMSE2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPHD</td>
<td>0.98</td>
<td>0.98</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>467.47</td>
<td>313.73</td>
</tr>
<tr>
<td>CACPHD</td>
<td>0.98</td>
<td>0.98</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>415.53</td>
<td>325.99</td>
</tr>
</tbody>
</table>

Table 1. Metrics of performance for scenario with crossing targets.

<table>
<thead>
<tr>
<th></th>
<th>$P_D1$</th>
<th>$P_D2$</th>
<th>FRAG1</th>
<th>FRAG2</th>
<th>$FTs$</th>
<th>$RMSE1$</th>
<th>$RMSE2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPHD</td>
<td>0.98</td>
<td>0.39</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>191.09</td>
<td>209.41</td>
</tr>
<tr>
<td>CACPHD</td>
<td>0.98</td>
<td>0.98</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>174.09</td>
<td>160.20</td>
</tr>
</tbody>
</table>

Table 2. Metrics of performance for scenario with parallel targets.
5. CONCLUSION

In this work, we have proposed the integration of available target class information into the data association of the CPHD filter. The update equations of the recently introduced MMCPHD filter were modified to include the target class likelihood functions given by the confusion matrix that models the accuracy of the target classifier. The result is the classification aided CPHD (CACPHD) filter. Its track management scheme is also inherited from MMCPHD filter. Note that in the case of a single target class and in the case in which the classifier is turned off (i.e., a target class measurement has a 50%-50% chance of belonging to each of the two possible target classes), CACPHD equations reduce to the equations in the MMCPHD filter. Furthermore, if there is no transitioning between the models (as reflected in the Markov transition matrix being the identity matrix), the update equations reduce to the corresponding equations in the original CPHD filter.
Performance comparison with and without the use of target class information in the data association step of the filter was done on multistatic sonar scenarios. Metrics of performance showed that incorporating target class measurements into the filtering improved measurement-to-track association results (and therefore track RMSE) in scenarios with significant ambiguity in using kinematic information alone. Future avenues of research include the online estimation of the target-feature parameters.

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