Acoustic Channel Tracking with the Cardinalized Probability Hypothesis Density Filter and the Multiple Hypothesis Tracker

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Abstract—Two datasets, one simplistic that assumes direct observation of paths and the other based on observations derived from compressed sensing and an assumed OFDM communications underpinning, simulate underwater acoustic channels. The Cardinalized Probability Hypothesis Density filter and the Multiple Hypothesis Tracker are applied to these wireless channels. The performances of the two trackers are evaluated and compared using the Optimal Subpattern Assignment metric.

Index Terms—Cardinalized Probability Hypothesis Density (CPHD) filter, Multiple Hypothesis Tracker (MHT), Optimal Subpattern Assignment (OSPA) metric, Compressed Sensing (CS)

I. INTRODUCTION

Accurate estimation of fading wireless channels is important for receiver design. A straightforward way to achieve this is to periodically transmit pilot symbols, but this method decreases data rates and power efficiency. Therefore, it is more desirable to do adaptive channel tracking, in which a pilot preamble is transmitted for the receiver to have an initial estimate and then the receiver tracks the channel with decision feedbacks [24].

The algorithms of choice for adaptive wireless channel tracking are the Least Mean Squares (LMS) and the Recursive Least Mean Squares (RLS) filters, which have become very popular due to their simplicity. Many modifications based on these two filters have been put forward [13]. Sequential Monte Carlo filtering was also applied to this problem [14].

In this work, we take a different approach and look at the performance of two high complexity trackers. CPHD is a recently developed tracker that can provide better performance due to its ability to propagate the estimated number of targets present at the scene and its ability to perform well without requiring measurement to track association. The PHD and its variants are inherently "track managed", meaning that the number of targets is estimated along with their locations; the MHT, generally accepted at present as the tracker of choice in data association problems, also performs track management but does so as an overlay. Note that here the "tracks" (acoustic paths) may indeed appear and disappear, and any tracker must account for that.

Several challenges arise such as measurement to track association uncertainty, sensor detection uncertainty, false alarms and noise.

A. CPHD

Mahler introduced a new approach to tracking in which target states and measurements are modeled as random finite sets (RFS). After a prediction step, the multi-target posterior distribution is updated in time using Bayes formula. This is however impractical in most applications and therefore the probability hypothesis density (PHD) filter was developed. The PHD is the first-moment approximation to the multi-target Bayesian posterior distribution [9].

Under linear Gaussian dynamics and the assumption of state independence for the probability of detection and the probability of survival, closed form filter equations are given in [18]. In that work, the posterior PHD surface is approximated by a Gaussian Mixture and is shown to remain a Gaussian Mixture after the update step. Hence, the propagation of the whole surface can be replaced by the propagation of the weight, mean and covariance of each mode in the mixture. Mode means and covariances are propagated by an Extended Kalman Filter while mode weights are calculated using the PHD equations. In common with other similar trackers such as the MHT, the number of Gaussian modes could increase exponentially with the number of scans, thus pruning, merging, etc. are necessary to make the approach practical. A Sequential Monte Carlo implementation of the PHD also exists [19].

The cardinalized PHD (CPHD) filter was proposed by Mahler as a fix for the “target death” problem of the PHD filter [6]. The Cardinalized Probability Hypothesis filter is a recursive filter that propagates both the posterior likelihood of (an unlabeled) target state and the posterior cardinality density, i.e. the probability mass function of the number of targets [12].

In our analysis [8], we divide the state space into infinitesimal bins and for each bin, we ask the question: “Is there a target at this location?” The filter contains equations for the prediction and the Bayesian update of the probability of each bin containing a target. Integration of the PHD surface over a volume gives the expected number of targets therein. In the limiting case in which bins’ volumes go to zero, the filter’s equations converge to the PHD/CPHD filter of Mahler [12] and to the Intensity Filter of Streit [17].
In its original form the GM-CPHD filter is not able to provide scoreable tracks, so a track management scheme was devised [15] [7]. This is a set of policies dealing with events such as track initiation, update, merging, spawning and deletion.

B. MHT

Blackman describes the MHT as a deferred decision logic in which alternative data association hypotheses are formed whenever observation-to-track conflict situations occur. Then, rather than choosing the best hypothesis or, in effect, combining the hypotheses as in the Joint Probabilistic Data Association (JPDA) filter [1], the hypotheses are propagated into the future in anticipation that subsequent data will resolve the uncertainty [15]. Tracks are updated by the Kalman Filter.

In the hypothesis oriented MHT implementation, hypotheses are carried over from the previous scan. Then, on the receipt of new data, each hypothesis is expanded into a set of new hypotheses by considering all observation-to-track assignments for the tracks within the hypothesis. A log likelihood ratio (LLR), also known as a track score, is all that needs to be computed (and maintained) in order to assess the validity of a track. In order to avoid a potential combination explosion in the number of hypotheses (and tracks within those hypotheses) that an MHT system can generate, a number of techniques have been developed to keep this potential growth in check, such as clustering, hypothesis and track pruning (deletion), and track merging [5].

In the track oriented MHT implementation, rather than maintaining, and expanding, hypotheses from scan to scan the hypotheses formed at scan \( k - 1 \) are discarded. The tracks that survive pruning are predicted to the next scan \( k \) where new tracks are formed, using the new observations, and reformed into hypotheses. Typically there are many more hypotheses formed than tracks and therefore the track oriented MHT is the preferred approach [3], [14]. In this work, a track oriented MHT was used.

C. OSPA

The metric chosen to evaluate performance is the Optimal Subpattern Assignment (OSPA) metric [18]. OSPA is defined as:

\[
d^c_p (X, Y) = \left[ \frac{1}{n} \left( \min_{\pi} \prod_{i=1}^{m} d^c(x_i, y_{\pi(i)})^p + c^p(n-m) \right) \right]^{\frac{1}{p}}
\]

where \( m \) and \( n \) are the cardinalities of \( X \) and \( Y \), respectively, \( \prod_{i=1}^{m} \) is the set of permutations on \( \{1, 2...n\} \) and \( d^c(x, y) = \min(c, d(x, y)) \) is the distance between \( x \) and \( y \) cut off at no more than a distance \( c \).

The first term in Equation 3 represents the penalty for localization errors while the second represents the penalty for cardinality errors. The auction algorithm [3] is used to solve the assignment problem.

Parameters \( p \) and \( c \) need to be tuned for a particular problem. It should be mentioned that as \( p \) increases, the metric becomes more unforgiving of outliers and that \( c \) represents a penalty for cardinality errors with respect to localization errors.

Prior to the calculation of the OSPA metric, the Gaussian Mixture modes are thresholded and then combined, i.e. when two modes are within a certain distance, only the mode with higher weight is kept. This is a necessary peak-extraction step, with the benefit of indicating which modes should go into assignment along with the ground truth.

III. DATASETS

The trackers were applied to two datasets: the first dataset was simulated to have a low level of difficulty while the second dataset was generated using compressed sensing and displays a higher level of difficulty. In future work, the trackers will be applied to real data obtained in an underwater experiment.

A. Simulated Data

The scenario contains one stationary source and one moving receiver, both located \( d_1 = 2m \) (meters) underwater. The sea depth was set to be \( d = 15m \) to mimic shallow water experiments. The receiver starts at a distance \( d_{SR} = 200m \) from the source and travels at a speed \( v = 2.5m/sec \) away from the source and parallel to the sea floor.

We looked at \( N = 11 \) paths for 25 snapshots. Reflections occur off both the sea bottom and the surface. Ray tracing was performed to calculate the delay of each path, i.e. the time difference of arrival (TDOA) with respect to the direct path. Figure 3 shows a path that undergoes five reflections at the first time step when the receiver is at a distance \( d_{SR} = 200m \) from the source. The Doppler of each path was calculated as \( v \cos \alpha \), with \( v \) being the speed of the receiver and \( \alpha \) being the angle between the direction of motion of the receiver and the direction of propagation of a path when reaching the receiver. Once the ground truth was created, Gaussian noise of different covariances was added to both the delay and the Doppler values. The resulting measurements are fed to the tracker.

The direct path is referred to as path 1. Five paths (labeled 2-6) were simulated to have their first reflection off the sea floor and five paths (labeled 7-11) were simulated to have
their first reflection off the sea surface. The paths are ordered by increasing number of reflections, not by increasing delay, e.g. path 2 has one reflection off the sea floor, path 3 has one reflection off the sea floor followed by a reflection off the sea surface. Figure 1 displays path 6. Due to symmetry, this resulted in path 3 and path 8 traveling the same total distance and arriving at the receiver at the same angle. Thus, path 3 has the same delay and same Doppler truth as path 8; path 5 and path 10 also coincide (see Figure 2). This setup challenges the trackers to recognize when two targets share the same location.

B. Compressed Sensing Data

The measurements of compressed sensing data are obtained through channel estimation for a zero-padded (ZP) OFDM system. There are $K = 1024$ subcarriers, 256 of which are pilot subcarriers, 96 null subcarriers, and 576 data subcarriers. Given a bandwidth of $B = 9.77$ kHz, the symbol duration is $T = 104.86$ msec; the guard time was set to as $T_g = 24.6$ msec and the simulation SNR was set to 30 dB.

We assume a path-based underwater acoustic channel model as in [2]. The channel parameters can be modeled as:

$$ c(\tau, t) = \sum_{p=1}^{N_p} A_p \delta(\tau - \tau_p(t)) \tau_p(t) = \tau_p - a_p t $$

(2)

where $A_p$, $\tau_p$, and $a_p$ are the amplitude, delay, and Doppler scale of the $p$th path. Hence, the channel can be determined completely by the $N_p$ triplets of $(A_p, \tau_p, a_p)$.

The simulated channel is generated as follows.

The amplitudes are Rayleigh distributed with the average power decreasing exponentially with delay. We assume the path amplitude $A_p$ will change from block to block, i.e. block-fading channel.

Delay values are based on the ground truth. As there is only a slow relative motion between transmitter and receiver, the introduced delay change will be small. Therefore, the $\tau_p$ are predictable.

Doppler values were generated with a standard autoregressive (AR) model

$$ a_p(k+1) = \rho * a_p(k) + (1 - \rho^2) * v(k), $$

(3)

where $\rho = 10^{-5}$, $v(k)$ is a unit normal Gaussian noise, and the Doppler rates for each path at the first time scan were drawn from a zero-mean uniform distribution. As the relative moving speed is limited into one small range, the $a_p$’s are small and also predictable. With the standard velocity standard deviation $\sigma_v$, the maximum possible Doppler is $\sqrt{3}\sigma_v f_c / c$ (the sound speed is set to $c = 1500$ m/s).

Since UWA channels are sparse in nature, we can use the compressed sensing method to estimate the channel parameters $(A_p, \tau_p, a_p)$. We use the basis pursuit (BP) algorithms developed in [2], [11]. The resulting compressed sensing estimates $(\hat{A}_p, \hat{\tau}_p, \hat{a}_p)$ are fed into the tracker as measurements.

Two channel tracking systems for underwater acoustic OFDM systems are possible:

- Channel delay tracking system (DT): only the delay space is tracked while all the paths have similar Doppler scales. An inter carrier interference (ICI)-ignorant receiver is used.
- Delay-Doppler tracking system (DDT): both the delay and the Doppler profile are tracked. We explicitly consider the possible Doppler scales for each discrete path.
and an ICI-aware receiver is used as in [4], [10].

We report results on the delay-Doppler tracking system.

IV. RESULTS

A. CPHD Results on Simulated Data

Representative results of CPHD tracking this particular channel are shown in Figures 3 and 4. The Gaussian modes of CPHD are 4-dimensional, consisting of delay, rate of change of delay, Doppler and rate of change of Doppler. Figures 3 and 4 show the same modes, but Figure 3 shows their delay component while Figure 4 shows their Doppler component. The ground truth is shown in blue, the measurements in green and the filter estimates in red.

The delay values are plotted in Figure 3 on the x-axis and so are the Doppler values in Figure 4. Path 1 and path 7 are very close in both delay and Doppler and thus hard to distinguish in the figures. The height of the filter estimates represents the weight of the mode placed at that particular location, i.e. how confident the CPHD is that a target is present at that particular location. The height of the ground truth and the height of the measurements have been fixed arbitrarily as only their x-axis locations are meaningful.

It should be noted that during the 25 time scans the CPHD overestimates the number of targets by at most one target. CPHD correctly identifies when two targets share the same location (as described in Section III.A) and assigns weights of 2 to modes generated by coinciding paths (see Figure 3).

Figure 5 presents the computed OSPA metric at each time scan. The localization component (e-loc) and the cardinality component (e-card) of the OSPA metric are plotted along with the total OSPA metric (e-ospa) as defined in Equation 1. We used $p = 2$ in our calculation of OSPA and therefore the localization error and the cardinality error do not directly add up to the total error but they follow Equation 1. It can be noted that there is a trend for the OSPA to decrease over time, i.e. CPHD becomes better at filtering as more scans of data arrive.

To verify that the tracker performs as expected, OSPA was also calculated using the raw measurements instead of the filter estimates along with the ground truth. In this case, there is no error in cardinality because we always have as many measurements as ground truth entries. Therefore, the localization component and the total OSPA error are equal (see Figure 6). By comparing Figures 5 and 6 it can be seen that the OSPA is smaller when the filter estimates are used than when the raw measurements are used which proves that applying the CPHD filter is advantageous.
1) Spooky Action at a Distance: In a side analysis, the probability of detection of a target (i.e. path) was lowered from $P_D = 1$ to $P_D = 0.9$ while the false alarm rate was still zero. In Figure 7, at scan 17 all paths are detected and all modes have similar weights close to 1. In the next scan (see Figure 8), the seventh path in order of increasing TDOA was not detected. The weight of its mode was strongly reduced while the weights of all the other modes increased. This is the phenomenon of spooky action at a distance [9] in which a missed detection of a target has a significant effect on targets arbitrarily far from the missed detection target.

B. MHT Results on Simulated Data

The results of MHT tracking the targets in the simulated dataset are shown in Figure 9. The delay values are on the x-axis while the Doppler values are on the y-axis. The TDOA with respect to the direct path decreases for all paths due to the motion of the receiver away from the source and the Doppler increases for all paths due to the decrease in the angle between the direction of arrival at the receiver of each path and the direction of the receiver motion. Therefore, tracks appear to move northwest in this display. It can be seen that tracks are initialized at scan 3.

Unlike the CPHD, our implementation of the MHT was not able to deal with the case of two targets sharing the same location and therefore paths 8 and 10 were removed from the dataset prior to the application of the tracker. The gating step in the MHT could be modified, at the expense of added complexity, to allow the tracker to deal with the case of closely spaced targets. On the other hand, CPHD doesn’t require gating.

Figure 10 presents the computed OSPA metric with the MHT estimates at each time scan. The removal of paths 8 and 10 prompted a recalculation of the OSPA metric with the raw measurements (instead of the tracker estimates) along with the ground truth. As expected, it was found that the OSPA is smaller for the MHT estimates than for the raw measurements. The difference was on the order of $10^{-9}$.

C. CPHD Results on Compressed Sensing Data

Representative results of CPHD tracking this channel are shown in Figures 11 and 12. The compressed sensing step described in Section III.B gives Doppler measurements that are of significantly worse quality than the delay measurements and therefore harder to track. This is also reflected in the error in localization calculated for the CPHD estimates on the compressed sensing dataset (see Figure 13) being much larger than for the MHT estimates.
larger than the error in localization calculated for the CPHD estimates on the simulated dataset (see Figure 5).

Moreover, the compressed sensing step introduces false alarms in the set of measurements to be fed to the tracker thus providing a more challenging scenario. However, the filter is able to correctly estimate the number of targets as the cardinality component of the OSPA metrics reaches and stays at zero (see Figure 13). By varying the SNR from 30dB to 5dB in 5dB decrements, it became evident that the number of false alarms increases with decreasing SNR. This is also visible when comparing Figures 3 and 14.

D. MHT Results on Compressed Sensing Data

The results of MHT tracking the targets in the compressed sensing dataset are shown in Figure 15. Once again, the low quality of the Doppler measurements can be noticed as the measurements for each of the targets (shown in green) display a considerably larger variance over the y-axis (i.e. Doppler-axis) than the ground truth values (shown in blue). The tracks created by the MHT are shown in red.

The error in localization calculated for the MHT estimates on the compressed sensing dataset (see Figure 16) is also much larger than the error in localization calculated for the MHT estimates on the simulated dataset (see Figure 10) but is in agreement with the error in localization calculated for the CPHD estimates on the compressed sensing dataset (see Figure 13). Like the CPHD, MHT is able to provide acceptable performance and to correctly identify the number of targets present at the scene.

OSPA was also calculated using the raw measurements instead of filter estimates along with the ground truth. When comparing the OSPA for the raw measurements (see Figure 17) with the OSPA for the CPHD (see Figure 13) and MHT (see Figure 16) estimates, it can be seen that both trackers are effective in handling the cardinality error and maintaining a low localization error.
V. CONCLUSIONS

Underwater acoustic channels are naturally sparse, meaning that most channel energy is concentrated on a few delay and/or Doppler values. Recently, advances in the new field of compressive sensing have led to numerous applications on sparse channel estimation, often furnishing better quality estimates than conventional least squares methods [2]. Combining filtering and a compressed sensing step can potentially lead to lowering bit error rates (BER) at the receiver.

UWA channels have large delay spread and significant Doppler effects, and hence fall into the category of doubly (time- and frequency-) spread channels. This is a difficult environment for tracking.

While most efforts to track wireless channels concentrate on applying simple and quick filters, we propose to take a different route and look at the performance of high complexity trackers such as CPHD and MHT. Such advanced trackers should be able to provide improved results in challenging scenarios. Performance is evaluated based on the OSPA metric, an attractive metric due to its tuning flexibility and ability to penalize errors in both cardinality and localization.

Our two scenarios have been designed to mimic (but at reduced difficulty) the real data recorded in the SPACE08 experiment off the coast of Marthas Vineyard, MA (Oct. 14 to Nov. 1, 2008). Running the trackers on this experimental dataset is the next step and the final goal of this work.

In both scenarios, the CPHD and the MHT perform well, successfully dealing with false alarms and correctly estimating the cardinality of the targets while maintaining low, acceptable localization errors. When comparing Figure 16 and Figure 17, MHT seems to have a small advantage over the GM-CPHD: MHT has a slightly smaller localization error and it is faster at achieving a cardinality error of zero.

A potential issue to watch for in the transition to real data is that in an underwater environment, the SNR will be lower than the SNR used in generating the compressed sensing estimates which would increase the number of false alarms significantly.

On a last note, it was interesting to observe the spooky action at a distance behavior of the CPHD. However, this is not of concern when working with real data for which the false alarm rate would be quite high as this behavior only appears when the false alarm rate is zero.

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