Improved Dynamic Response of Piezoelectric Composite Shells using Multiobjective Optimization and Closed Loop Control

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Abstract
A multiobjective optimization procedure is developed for improving the vibratory response of composite shells with distributed piezoelectric patches under a variety of loading conditions. The objective is to minimize the overall maximum deflection associated with the first five modes of vibration and the static deflection as the shell is subjected to electrical, mechanical and combined loading. Constraints are imposed on the natural frequencies and stresses. The stacking sequence is used as a design variable to investigate the most efficient stiffness distribution. A closed loop control system is designed using Linear Quadratic Gaussian (LQG) controller. The multiple objective function problem is formulated using the Kreisselmeier-Steinhauser (K-S) function approach. Using this approach, the multiobjective constrained optimization problem is reduced to the unconstrained optimization of a single envelope function (the K-S function). A Simulated Annealing algorithm is used as the search algorithm. Numerical results are presented for a variety of loading conditions to show the significant improvements in vibratory response from optimizing the stacking sequence. The influence of the orientation of the piezoelectric patch is also investigated.

Nomenclature

\[ \sigma_{ij} = \text{stress} \]
\[ \varepsilon_{ij} = \text{strain} \]
\[ E_k = \text{electric field} \]
\[ e_{ijk} = \text{piezoelectric constants} \]
\[ D_i = \text{electric displacement} \]
\[ c_{ijkl} = \text{elastic constants} \]
\[ b_{ij} = \text{dielectric permittivity} \]
\[ \phi = \text{electric potential} \]
\[ \tilde{u}_i = \text{deformation on displacement boundary} \]
\[ \tilde{t}_i = \text{traction on displacement boundary} \]
\[ q = \text{charge accumulated (sensing signal)} \]
\[ \gamma = \text{structural damping ratio} \]
\[ f_i = \text{components of force per unit volume} \]
\[ S_D = \text{charge boundary} \]
\[ S_o = \text{stress boundary} \]
\[ F_i(\phi) = i^{\text{th}} \text{objective function} \]

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† Professor, Mechanical and Aerospace Engineering Dept, Arizona State University, Mail Stop 6106, Tempe, AZ, 85287.
$g_{\text{max}}$ = maximum constraint value  
$f_i^*$ = $i^{th}$ reduced objective function  
$f_{\text{max}}$ = largest constraint value of the reduced objective function  
$\rho$ = pull down factor  
$L(t)$ = Kalman filter gain matrix  
$K(t)$ = optimal feedback gain matrix  
$K$ = gain matrix  
$J$ = performance index  
$R$ = input energy weight matrix  
$Q$ = intermediate state weight matrix  
$U_i^k$ = in-plane displacements ($i=\alpha, \beta$) $k$ denotes the k-th layer of the laminate  
$u_i, \phi_i$ = laminate unknowns for ($i=\alpha, \beta$)  
$w$ = laminate unknowns  
$\theta_i^k, \psi_i^k$ = layerwise unknowns for ($i=\alpha, \beta$)  
$f_i(z), g_i(z)$ = through-laminate-thickness functions of higher order odd and even distributions ($i=\alpha, \beta$)  
$\phi_j$ = rotational degrees of freedom for ($j=\alpha, \beta$)  

I. Introduction

Aerospace vehicles are susceptible to high vibratory loads due to the high unsteady load and complex aerodynamic environment in which aircraft components must operate. Over the last decade, a significant amount of research has been conducted by using induced strain actuation for improved vibration and noise control in aircraft components\(^1\)-\(^4\). The use of these transducers allows the imposed changes to be tailored according to the conditions sensed by a particular structural component. Most importantly, by using distributed sensors and actuators, control can be achieved over a much larger bandwidth than by using conventional mechanisms. Piezoelectric materials can be used as active vibration dampers for structures where they act as both sensors and actuators. When placed in discrete locations on a structure, these materials can sense movement and control that motion via localized strains. Although these materials generate very low strain, they cover a wide range of actuation frequency and, therefore, have more practical applications. Composites have an advantage over metals due to their excellent toughness and strength while maintaining low strength-to-weight ratio. Therefore, research has also been reported in using embedded/surface bonded actuators and sensors to improve the vibratory characteristics of composite structural elements\(^5\)-\(^8\). A multitude of aerospace components, ranging from fuselage to engine containment systems, can be modeled as composite shells. Therefore, improved vibratory performance and dynamic response of anisotropic shell structures is an important research area and is the topic of the present work.

A significant amount of work has been reported in using formal optimization techniques to improve the performance of composite plates and shells using passive techniques. Yuan\(^9\) investigated the effect of stacking sequence on the distribution of thermal stresses. Hwang\(^10\) used the Tsai Wu failure criterion as the objective function and investigated optimum stacking sequence using a state-space approach. Walker\(^11\) examined the multiobjective design of laminated composite shells with respect to buckling and torsional loads. They formulated the performance index as the weighted sum of individual objectives in order to obtain Pareto optimal solutions of the design problem. Haftka et al.\(^12\) used Genetic Algorithms (GA) with built-in repair strategies for the stiffness tailoring of composites against buckling. A GA with elitist selection for best-fit single individual was also used by Haftka et al.\(^13\) in optimizing the stacking sequence of a composite as the objective function. Optimization of piezoelectric composite plates for improved vibratory performance was addressed by Thornburgh and Chattopadhyay\(^7\). They used a higher order plate theory with passive control to improve the vibratory response of laminated plate structures with distributed PZTs. More recently, Kim et al. addressed the modeling and closed loop control of free vibration of piezoelectric shell structures\(^8\). The goal of the present paper is to further extend this work by improving the dynamic response of composite shells under a variety of loading conditions.

The efficient design of smart structural systems with distributed sensors and actuators requires the integration of high fidelity analysis tools, robust controllers, and optimization techniques. In the analysis of composite laminates, classical laminate theory (CLT) and first-order shear deformation theory (FSDT) have traditionally been used. These theories are limited by the fact that CLT ignores transverse shear stresses and FSDT uses ad hoc correction
factors to compute them. Also, valuable information regarding local stress distributions is neglected in both theories. Although three-dimensional approaches are more accurate than two-dimensional theories, their implementation can be very expensive in practical applications. Layerwise approaches are alternatives since they are capable of modeling laminated stress distributions. However, computational effort increases with ply number, which makes them computationally prohibitive. A computationally efficient new layerwise laminate theory has been developed by Zhou et al\textsuperscript{14} addressing transverse shear stress continuity and ensuring accuracy at local levels. In this paper, the refined layerwise theory is used in the analysis of composite shells with distributed piezoelectric patches under a variety of mechanical and electrical loadings. In the analysis of integrated piezoelectric structures, it is important to accurately model the electro-mechanical field interactions. A completely coupled theory, integrating thermal, mechanical and electrical fields was developed by Gu et al\textsuperscript{15} and further refined by Zhou et al\textsuperscript{16}. A simplified two-way electro-mechanical model was later developed by Thornburgh and Chattopadhyay\textsuperscript{7}.

In this paper, the completely coupled model from Zhou et al\textsuperscript{16} is used in conjunction with the refined layerwise theory\textsuperscript{5} in the dynamic response analysis of composite shells with distributed PZTs. A multidisciplinary procedure is developed, integrating the refined analysis with a multiobjective hybrid optimization algorithm and closed loop control to investigate the design tradeoffs associated with stiffness tailoring and changes in piezoelectric patch orientation angle on the vibration control.

II. Problem Statement

The objective is to minimize the vibratory response of the shell structure under mechanical, electrical and combined electro-mechanical loading conditions. Therefore, the maximum deflections associated with the first five modes of vibration as well as the static deflection are used as objective functions. The stacking sequence is used as design variable and constraints are imposed on the natural frequencies and stresses. In addition, geometric constraints are also imposed on the design variables to avoid unrealistic design. Since the problem involves multiple objective functions, the Kreisselmeier-Steinhauser multiobjective formulation is used\textsuperscript{17}. In the K-S function approach the original objective functions are transformed into reduced objective function, which take the form of constraints. The problem reduces to the unconstrained minimization of the K-S function, which significantly simplifies the search procedure. A Simulated Annealing algorithm\textsuperscript{18} is used in conjunction with the K-S function approach to address the closed loop control of composite shells. The influence of the orientation of the piezoelectric patches in vibration reduction is also investigated.

III. Analysis of Piezoelectric Composite Shells

Composite shells with distributed piezoelectric actuators and sensors and simply supported boundary conditions are modeled using a combination of the refined layerwise theory and the two-way coupled electro-mechanical field formulation. Consider a shell element with \( N \) plies described in orthogonal coordinates, \( \alpha \) and \( \beta \), along the shell mid-surface as shown in Fig 1.
Figure 1. Shell geometry

The area of an infinitesimal rectangle is denoted $dS$ and the volume of an infinitesimal parallelepiped is denoted $dv$.

\begin{align}
    dS & = A_\alpha (1 + z/R_\alpha) A_\beta (1 + z/R_\beta) \, d\alpha \, d\beta \\
    dv & = A_\alpha (1 + z/R_\alpha) A_\beta (1 + z/R_\beta) \, d\alpha \, d\beta \, dz
\end{align}

Where $A$ represents the coordinate shell coefficient and $R$ is the geometric coordinate radius. The thickness coordinate, $z$, is measured from the mid-surface and $R_\alpha$ and $R_\beta$ are the radii of curvature of the $\alpha$ and $\beta$-curves.

The orthogonal coordinate system can express the volume and surface area of the shell in a differential form. The implementation of the initial curvature is important in the development of an accurate thick shell theory. It must be noted that the terms $(1 + z/R_\alpha)$ and $(1 + z/R_\beta)$, which address the variation of surface curvature for different values of thicknesses, are included to take into account the initial curvature effects in thick laminates.

The layerwise zigzag displacement field accounts for the interlaminar continuity of transverse stresses at each interface of lamina as well as the traction free boundary conditions on the top and bottom surfaces of the laminates. The displacements of a point with the coordinates $(\alpha, \beta, z)$ are described using the superposition of first-order shear deformation and layerwise functions. The layerwise functions describe the zigzag in-plane deformation through the laminate thickness. When the PZT patches are actuated, the stresses are located at the interface between actuator and composite shell. The high stress concentration is described with one more rotational degree of freedom for the displacement field of the PZT laminate. Then, the equilibrium of the composite shell with piezoelectric actuators is described with the layerwise functions of.
\[ U^k_{\alpha}(\alpha, \beta, z) = (1 + \frac{z}{R_{\alpha}}) u_{\alpha}(\alpha, \beta) + z \phi_{\alpha}(\alpha, \beta) + f_{\alpha}(z) \theta_{\alpha}^k(\alpha, \beta) + g_{\alpha}(z) \psi_{\alpha}^k(\alpha, \beta) + z \varphi'_{\alpha}(\alpha, \beta) \delta_{ik} \] 
\[ U^k_{\beta}(\alpha, \beta, z) = (1 + \frac{z}{R_{\beta}}) u_{\beta}(\alpha, \beta) + z \phi_{\beta}(\alpha, \beta) + f_{\beta}(z) \theta_{\beta}^k(\alpha, \beta) + g_{\beta}(z) \psi_{\beta}^k(\alpha, \beta) + z \varphi'_{\beta}(\alpha, \beta) \delta_{ik} \] 
\[ U^k_z(\alpha, \beta, z) = w(\alpha, \beta) \]

Where:

\[ \delta_{ik} = \begin{cases} 
0 & \text{for } i \neq k \\
1 & \text{for } i = k 
\end{cases} \]

In these equations, a superscript \( k \) denotes the k-th layer of the laminate and a superscript \( i \) denotes the i-th PZT layer. Laminate unknowns (\( u_{\alpha}, u_{\beta}, w, \phi_{\alpha} \) and \( \phi_{\beta} \)) and layerwise unknowns (\( \theta_{\alpha}^k, \theta_{\beta}^k, \psi_{\alpha}^k \) and \( \psi_{\beta}^k \)) are used to address in-plane deformation. The rotational degrees of freedom (\( \varphi_{\alpha}^i \) and \( \varphi_{\beta}^j \)) are introduced in the PZT layer to describe electrical actuation and stress concentration. Next, the Kronecker Delta (\( \delta_{ik} \)) is introduced to describe additional rotational degree of freedom in the PZT layer. The through-laminate-thickness functions \( f_{\alpha}(z) \), \( f_{\beta}(z) \), \( g_{\alpha}(z) \), and \( g_{\beta}(z) \) describe the global deformation of the composite shell.

\[ f_{\alpha}(z) = f_{\beta}(z) = \sinh(z) \]
\[ g_{\alpha}(z) = g_{\beta}(z) = \cosh(z) \]

Where the functions \( f_{\alpha}(z) \), \( f_{\beta}(z) \), \( g_{\alpha}(z) \), and \( g_{\beta}(z) \) are higher order odd and even distributions. Note that Eq. (2) results in \( 4N + 5 + 2N_p \) number of structural unknowns in the displacement field, where \( N \) is the total number of layers and \( N_p \) is the number of PZT layers.

Conventional sensor modeling neglects the nonzero electric field induced by mechanical deformation. This leads to a violation of the conservative charge law. It also mispredicts the charge accumulated on electrodes significantly. In this paper, a higher order field description of electrical field is used to accurately describe the nonuniform distribution of electric field through the thickness of piezoelectric layers. Two-way coupling between the mechanical and the electrical fields is modeled to extract accurate sensing information generated by piezoelectric sensors\(^{15-16,19}\). The constitutive relations governing stress, strain, charge and electric field can be written as follows:

\[ \sigma_{ij} = c_{ijkl} e_{kl} - e_{ik} E_k \]
\[ D_j = e_{ij} E_k + b_{ij} E_j \]

Where the quantities \( e_{ij} \) and \( \sigma_{ij} \) denote the components of strain tensor and stress tensor, respectively and \( E_i \) and \( D_j \) denote the components of electric field and electric displacement, respectively. The quantities \( c_{ijkl} \), \( e_{ij} \) and \( b_{ij} \) represent elastic constants, piezoelectric constants and dielectric permittivity, respectively. The relationship between charge variations generated by the sensors and delamination parameters will be quantified using variational principle as follows\(^{15-16,19}\):
\[
\int_{t_0}^{t} \left( \int_{V} \left( \rho \ddot{u}_i + \gamma \dot{u}_i + \sigma_{ij} \ddot{e}_{ij} \right) dV - \int_{S_\sigma} \dot{t}_i \delta u_i dS - \int_{S_\theta} D_{ij} \delta \phi_{ij} dV + \int_{D_S} q \delta dS \right) dt = 0
\]  \tag{6}

Where the quantity \( \Phi \) denotes electric potential applied in the piezoelectric device. The quantities \( u_i, t_i, \Phi \) and \( q \) denote the deformation on the displacement boundary \( S_\sigma \), traction on the stress boundary \( S_\theta \), voltage on the potential boundary \( S_\phi \), charge accumulated (sensing signal) on the charge boundary \( D_S \), respectively, and \( \rho, \gamma \) and \( t_i \) denote mass density, structural damping ratio and components of the force per unit volume, respectively. The procedure will provide means to identify structural characteristics using the charge accumulation, \( q \). The above formulation represents the piezoelectric and converse piezoelectric effects simultaneously, as opposed to the conventional sequential approach. Thus, energy transformation from the electrical to the mechanical field and vice versa is accounted for. This theory provides an accurate description of the piezoelectric on the composite shell.

A piezoelectric patch may possess anisotropic characteristics if the mechanical properties are unique in each direction. Likewise, the configuration of the three mutually orthogonal crystal axes affects the polar axis that induces strain on the host media out of the plane\(^{20} \). Depending on how the cubic crystal axis or element face aligns to the polarization axis, it can elevate or degrade the mechanical properties of the piezoelectric material. The piezoelectric patch rotates about the rhombohedral crystal axis, which can elevate or degrade its mechanical properties. Generally, the user orients the material in the axis of polarization (best mechanical fit) direction perpendicular to the surface of the electric potential direction. Because actuator properties change based on the polarization angle, it is necessary to optimize the sensor configuration and location under a specific bending load. Additionally, it is pertinent to search for the best configuration of the piezoelectric patch to that provides the best damping performance. To investigate the effect of physical orientation of the PZTs on vibration control, the optimization process is repeated over a range of orientation angle, \( \Phi (-90^\circ < \Phi < 90^\circ) \).

### IV. Optimization Problem Formulation

The maximum deflections of the first five modes and the static deflection are used as objective functions and the ply stacking sequence is the design variable. Three different loading conditions are studied. These include the electrical load due to applied voltage, the radially applied mechanical load and combined electro-mechanical load. Constraints are imposed such that the stresses in the shell do not exceed the ultimate failure or maximum fiber stress in either compression or tension. That is,

\[ -\sigma_{m-compression} < \sigma_{m} < \sigma_{m-tension} \]

To avoid changes in frequency, with reference to the host structure, constraints are imposed on the first five natural frequencies. Geometric constraints are also imposed on the design variables; where \( \Psi \) represents the stacking sequence angle in the range of \( -90^\circ < \Psi < 90^\circ \).

The K-S function is used to handle the multiple objective functions. First, the original objective functions, \( F_{io}(\Phi) \) (where \( \Phi \) is the design variable vector) are converted to reduced objective function, which take the form of constraints, as shown in Eq.(5).

\[
f_i^* = \frac{F_i(\Phi)}{F_{io}(\Phi)} - 1 - g_{max} \leq 0
\]  \tag{7}

Where \( g_{max} \) is the maximum possible physical constraint. The K-S envelope function is then expressed as follows.

\[
F_{ks}(\Phi) = f_{max} + \frac{1}{\rho} \ln \sum_{m=1}^{M} \rho(f_{io}(\Phi) - f_{max})
\]  \tag{8}
Where \( F_{ks}(\phi) \) is a single composite function that combines all of the objective functions and \( f_{max} \) is the largest constraint corresponding to the new reduced constraint function \( f_m(\phi) \). It must be noted that \( f_{max} \) is not equal to \( g_{max} \). The quantity \( \rho \) is a pull-down factor. A high value of \( \rho \) ensures that the K-S function curve lies close to the maximum reduced objective constraint surface and a low value of \( \rho \) yields an envelope that represents all the constraints.

The Simulated Annealing algorithm is used as the search algorithm. The procedure mimics the probability of simulated annealing of metals by generating a random number in Boltzmann’s probability distribution function, a step size, a temperature reduction factor, and a step reduction factor. The step size controls the increment for generating random numbers, and the temperature reduction factor changes accordingly for convergence issues. Finally, the step reduction factor controls the magnitude of the step size. The general strategy of this technique warrants that a higher value of an objective function (K-S composite function in this case) is acceptable under a set of conditions because a probability exists that an absolute minimum may still be present if more runs are generated. This operation leads to a robust method for finding the global minimum by reducing the chances of neglecting the absolute minimum of the system if the initial conditions are not favorable mentioned by Seeley.

V. Control System Design

In the design of control system, the use of Linear Quadratic Regulator (LQR) controller is not always feasible because in most practical applications not all states are measurable. Unlike the standard LQR, which requires full state feedback, a LQG controller allows the user to specify the measurable states resulting in a much more practical approach in terms of autonomous applications. Therefore, the LQG controller system is used in this research. The equations of motion in state-space can be expressed as follows.

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) + w(t) \\
y(t) &= C x(t) + v(t) \\
u(t) &= -K x(t) \\
\dot{x}(t) &= A x(t) + B u(t) + L[y(t) - C \dot{x}(t)]
\end{align*}
\]

(9)

Where \( \dot{x}(t) \) is the state space, \( y(t) \) is the output vector, \( u(t) \) is the input vector and \( L \) defines the Kalman filter gain matrix for the LQG controller. The Kalman filter \( L \) substantially improves the autonomous systems by estimating the past, present, and future states even when the exact condition of the system is unknown. It is an iterative solution to discrete linear data filtering which averages the limits of data. At the root of the controller is a recursive algorithm, which has \( n \) discrete-time Riccati equations. The goal is to minimize all the points of the steady state deflections as the states are approaching zero. The performance measure index is a matrix providing insight on the strength of the controller.

\[
J = E \sum_{t=1}^{\infty} [x^T(t)Qx(t) + u^T(t)Ru(t)]
\]

(10)

\( J(i) \) is a hybrid sum of “everything which is desired to be small”, with weighting matrices provided for the tolerances in these dynamics. \( R \) is the input weighted energy matrix, and \( P \) is the prior estimated error covariance.

VI. Results

Table 1 shows the fixed geometric parameters of the composite shell for the case studies in this work. The composite shell is simply supported with nine piezoelectric patch actuators mounted on top and bottom surfaces. The stress limits used in the optimization are 2.1GPa as the ultimate tensile strength and 1.72 GPa as the ultimate compressive strength. Table 2 shows the mechanical and electrical properties of the composite structure and piezoelectric patches. Because the stiffness of the composite shell changes as a result from the stacking sequence, the natural frequencies may shift during optimization. To avoid drastic changes in natural frequencies, relative to the host structure, the first four modes are constrained to remain within 15 percent of the original values. The fifth mode had a higher frequency fluctuation, so the constraint was set at 20 percent. For the electrical loading case, a potential of 200 V is applied to each of the nine PZT patches. For the mechanical loading case, a uniform load of magnitude 1000 Pa is applied radially on the composite shell. A stacking sequence of \([90/0/90/0/90/0/0/90/0/90/0] \) is used.
as the reference design. The optimum stacking sequence obtained for the three loading cases, electrical, mechanical, and combined, are presented in Tables 3 and 4 for L/h=100 and 200, respectively. Note that each stacking sequence is unique for a predefined piezoelectric orientation. Figures 2-8 illustrate the influence of the PZT orientation on the objective function. The reduction in modal deflection, associated with the first five modes, for the plate with L/h=100 are shown in Figs. 2 – 4 (for all three loading conditions). Similar results are presented in Figs. 5 – 7 for the plate with L/h = 200. In these figures, a negative percentage indicates an increase in deflection. Figure 8 shows an example of the first five mode shapes of the optimized shell structure subject to electrical loading. The differences in static deflections, before and after optimization are illustrated in Figs. 9 – 11 for L/h=100.

### Table 1. Shell dimensions.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>.3m</td>
</tr>
<tr>
<td>Arc Angle</td>
<td>60°</td>
</tr>
<tr>
<td>Length</td>
<td>.6m</td>
</tr>
<tr>
<td>Length/thickness ratio</td>
<td>100,200</td>
</tr>
</tbody>
</table>

### Table 2. Mechanical and electric properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Composite</th>
<th>Piezoelectric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus 1-dir (GPa)</td>
<td>144.23</td>
<td>63</td>
</tr>
<tr>
<td>Elastic Modulus 2-dir (GPa)</td>
<td>9.65</td>
<td>58</td>
</tr>
<tr>
<td>Elastic Modulus 3-dir (GPa)</td>
<td>9.65</td>
<td>58</td>
</tr>
<tr>
<td>Shear Modulus 1-dir (GPa)</td>
<td>4.14</td>
<td>24.6</td>
</tr>
<tr>
<td>Shear Modulus 2-dir (GPa)</td>
<td>4.14</td>
<td>24.6</td>
</tr>
<tr>
<td>Shear Modulus 3-dir (GPa)</td>
<td>3.45</td>
<td>24.6</td>
</tr>
<tr>
<td>Poisson Ratio (1-2)-dir</td>
<td>.3</td>
<td>.28</td>
</tr>
<tr>
<td>Tensile Yield Strength (MPa)</td>
<td>2100</td>
<td>41 (Dyn. Tensile)</td>
</tr>
<tr>
<td>Compressive Yield Strength (MPa)</td>
<td>1720</td>
<td>76 (Static loading)</td>
</tr>
<tr>
<td>Density (kg/m3)</td>
<td>1389</td>
<td>7600</td>
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<tr>
<td>Dielectric perm. 1-dir (nFarad/m)</td>
<td>N/A</td>
<td>15.3</td>
</tr>
<tr>
<td>Dielectric perm. 2-dir (nFarad/m)</td>
<td>N/A</td>
<td>15.3</td>
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<tr>
<td>Dielectric perm. 3-dir (nFarad/m)</td>
<td>N/A</td>
<td>15</td>
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<tr>
<td>Piezoelectric charge constant 1-dir (um/V)</td>
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<tr>
<td>Piezoelectric charge constant 2-dir (um/V)</td>
<td>N/A</td>
<td>230e-6</td>
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<tr>
<td>Piezoelectric charge constant 3-dir (um/V)</td>
<td>N/A</td>
<td>230e-6</td>
</tr>
</tbody>
</table>
### Table 3. Optimum Stacking Sequence; L/h=100.

<table>
<thead>
<tr>
<th>Ply</th>
<th>15/15 30/30 45/45 60/60 75/75 90/90 15/-15 30/-30 45/-45 60/-60 75/-75 90/-90 15/15 30/30 45/45 60/60 75/75 90/90 15/-15 30/-30 45/-45 60/-60 75/-75 90/-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ply 2</td>
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<tr>
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<td>15 30 60 30 0 45 30 15 -60 -30 45 30</td>
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<td>Ply 6</td>
<td>30 15 0 -15 30 15 30 30 0 15 15 15</td>
</tr>
<tr>
<td>Ply 8</td>
<td>15 30 60 30 0 45 30 15 -60 -30 45 30</td>
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<tr>
<td>Ply 9</td>
<td>30 15 0 -15 30 15 30 30 0 15 15 15</td>
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<tr>
<td>Ply 10</td>
<td>15 15 15 -30 -30 15 15 -30 -30 0 15 0</td>
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<tr>
<td>Ply 11</td>
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<tr>
<td>Ply 12</td>
<td>-15 45 -30 0 -60 -45 -30 75 75 15 15 0</td>
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<td>Ply 13</td>
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<td>Ply 19</td>
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<td>15 15 15 -30 -30 15 15 -30 -30 0 15 0</td>
</tr>
<tr>
<td>Ply 23</td>
<td>15 30 60 30 0 45 30 15 60 0 30 45</td>
</tr>
<tr>
<td>Ply 24</td>
<td>30 15 0 -15 30 15 30 30 0 15 15 15</td>
</tr>
</tbody>
</table>

### Table 4. Optimum Stacking Sequence; L/h=200.

<table>
<thead>
<tr>
<th>Ply</th>
<th>15/15 30/30 45/45 60/60 75/75 90/90 15/-15 30/-30 45/-45 60/-60 75/-75 90/-90 15/15 30/30 45/45 60/60 75/75 90/90 15/-15 30/-30 45/-45 60/-60 75/-75 90/-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ply 1</td>
<td>15 -15 -15 0 15 -15 -15 0 15 0 15 0</td>
</tr>
<tr>
<td>Ply 2</td>
<td>-15 15 -15 -30 0 0 15 -45 0 -15 -30 30</td>
</tr>
<tr>
<td>Ply 3</td>
<td>0 -45 -15 -15 75 60 -30 -45 45 75 45 0</td>
</tr>
<tr>
<td>Ply 4</td>
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<tr>
<td>Ply 18</td>
<td>30 15 0 -15 30 15 30 30 0 15 15 15</td>
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</table>

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Examining the first, second, and fifth modes of vibration in Fig. 2, for the electrical loading case, the improvements in reduction range from 2-5 percent for all PZT orientations considered. A significant benefit reduction of nearly 14 percent is achieved for the fourth mode. The optimization procedure is ineffective in suppressing the maximum deflection associated with the third mode (12.5 percent increase, compared to reference configuration). Similar observations are made on all other loading cases (Figs. 2 – 4, L/h = 100). This phenomenon is not due to the bending-torsion coupling, which can cause the maximum deflection of a single dynamic mode to increase. As seen from Fig. 8, for the case with 45/45 PZT orientation and electrical loading, such coupling is observed only in the fourth and fifth mode shapes. Therefore, the increase is perhaps due to the fact that while the K-S function approach minimizes the envelope function, a uniform improvement in all of the individual objectives may not be possible. In such cases, it may be of interest to apply weight factors to the individual objective functions\textsuperscript{23}. In all three loading cases, the PZT orientation has a strong effect on the modal response. The parametric studies conducted illustrate that larger reductions in the modal deflections are achieved at certain angles.

![Figure 2. Deflection reduction under electrical loading; L/h=100.](image-url)
Figure 3. Deflection reduction under mechanical loading; L/h=100.

Figure 4. Deflection reduction under electro-mechanical combined loading; L/h=100.
Examining Fig. 5, when the dimensions of the shell is set to $L/h=200$, the same trends are observed for the first, second, and fourth mode as Fig. 3. However, the reduction in the deflection associated with the fifth mode is more significant. Similar observations can be made for the other loading cases (Figs. 6 and 7). For example, the first and second modes of deflection were reduced by about 2 - 4 percent and the deflection of the fifth mode undergoes a reduction of 4-10 percent, compared to the reference design. Also, comparing the electrical to mechanical loading of Figs. 5 and 6, it is apparent that stiffness tailoring of a composite depends upon also the type of deterministic load encountered. Once again, it is observed that the PZT orientation plays an important role in the dynamic response and close loop control.

![Figure 5. Deflection reduction under electrical loading; L/h=200.](image-url)
Figure 6. Deflection reduction under mechanical loading; L/h=200.

Figure 7. Deflection reduction under electro-mechanical combined loading; L/h=200.
Figure 8. Optimized mode shapes under electrical load; 45°/45° PZT orientation.

The static deflection also shows significant reductions, on the order of 48 – 50 percent, as seen from Figs. 9-11, for the shell with L/h=100. The results obtained illustrate the significance of stacking sequence in improving the dynamic and static response of shell structures with closed loop control. This demonstrates how the stacking sequence is very crucial to the stiffness tailoring of the composite shell in the static as well as dynamic loading cases. The parametric studies conducted shows that the PZT orientation angles affect the dynamic response and closed loop control of composite shells. Further studies are necessary to examine the observed trends.
Figure 9. Comparison of static deflection under electrical load; L/h=100.

Figure 10. Comparison of static deflection under mechanical load; L/h=100.
VII. Concluding Remarks

A procedure has been developed to improve the vibratory response of composite shells with distributed piezoelectric patches under a variety of loading conditions. The maximum deflection associated with the first five modes of vibration and the static response is minimized using stacking sequence as design variable. A refined layerwise theory is used in conjunction with a two-way coupled electro-mechanical field formulation to determine the response of composite shells subjected to electrical, mechanical and combined electro-mechanical loading conditions. Constraints are imposed on the natural frequencies and stresses. A closed loop control system is designed using Linear Quadratic Gaussian controller. The Kreisselmeier-Steinhauser (K-S) function approach is used to formulate the multiple objective function problem. A Simulated Annealing algorithm is used as the search algorithm. Parametric studies are conducted to investigate the influence of the orientation of the piezoelectric patch on the dynamic response. The following observations can be made from the present study.

1) The developed framework is efficient and applicable for the design of composite structures with improved vibratory characteristics.
2) The K-S function approach in conjunction with the LQG controller is efficient in improving the overall dynamic modal response under all loading conditions.
3) Significant improvements are observed in the reductions of the individual objective functions associated with the first, second, fourth, and fifth modes of vibration.
4) Significant reductions are also observed in the static deflection.
5) The laminate stacking sequence and the orientation of the piezoelectric transducers play an important role in improving the dynamic response.
Acknowledgments

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References