CE 240
Soil Mechanics & Foundations
Lecture 8.3

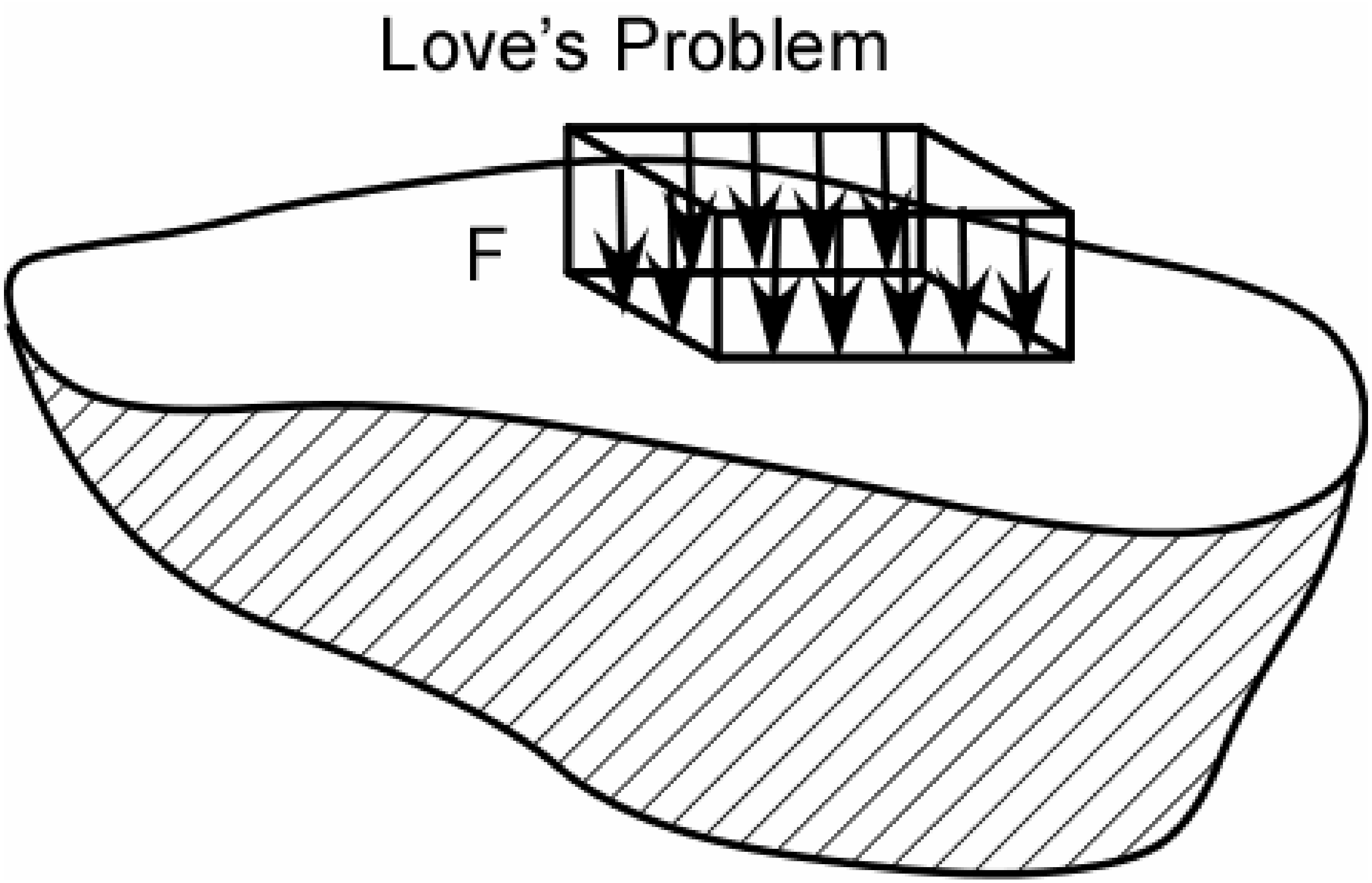
Stresses in a soil mass III

(Das, Ch. 9)

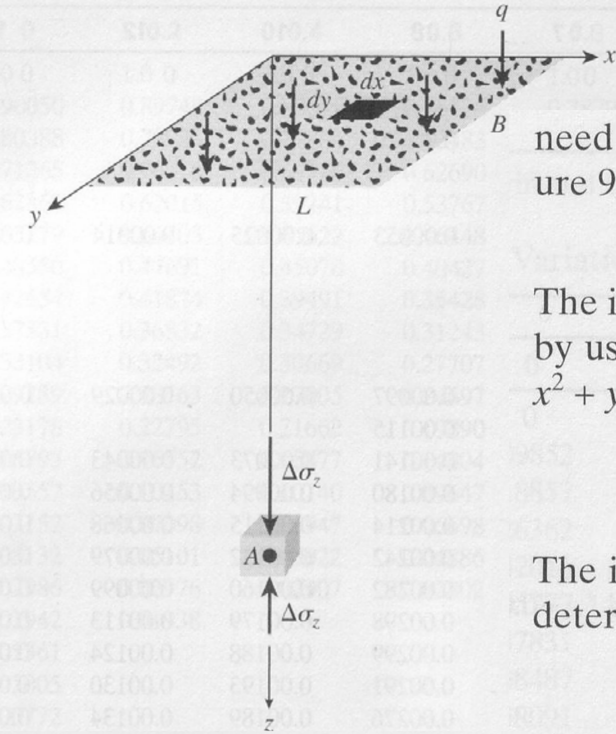
Class Outline

- Vertical stress caused by a rectangular areal load (cont.)
- Vertical stress due to embankment load
- Vertical stress caused by a circular areal load
- Influence chart for vertical pressure
- Example problems

Vertical stress caused by a rectangular areal load



Vertical stress caused by a rectangular areal load (at point bellow one corner of the loading patch)



need to consider a small elemental area $dx\,dy$ of the rectangle. (This is shown in Figure 9.23.) The load on this elemental area can be given by

$$dq = q\,dx\,dy \tag{9.32}$$

The increase in the stress ($d\sigma_z$) at point A caused by the load dq can be determined by using Eq. (9.12). However, we need to replace P with $dq = q\,dx\,dy$ and r^2 with $x^2 + y^2$. Thus,

$$d\sigma_z = \frac{3q\,dx\,dy\,z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}} \tag{9.33}$$

The increase in the stress, at point A caused by the entire loaded area can now be determined by integrating the preceding equation. We obtain

$$\boxed{\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_4} \tag{9.34}$$

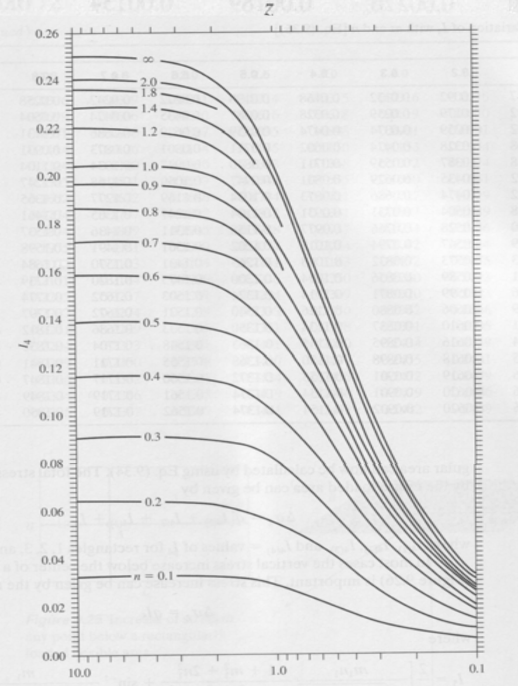
where

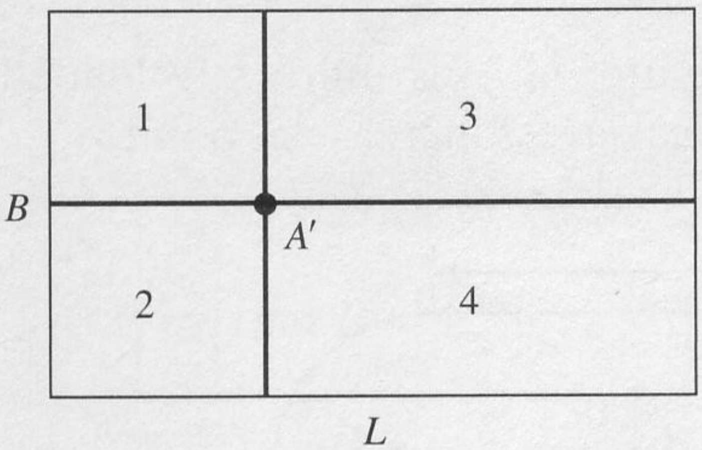
$$I_4 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \tag{9.35}$$

$$m = \frac{B}{z} \tag{9.36}$$

$$n = \frac{L}{z} \tag{9.37}$$

The variation of I_4 with m and n is shown in Table 9.9 and Figure 9.24.





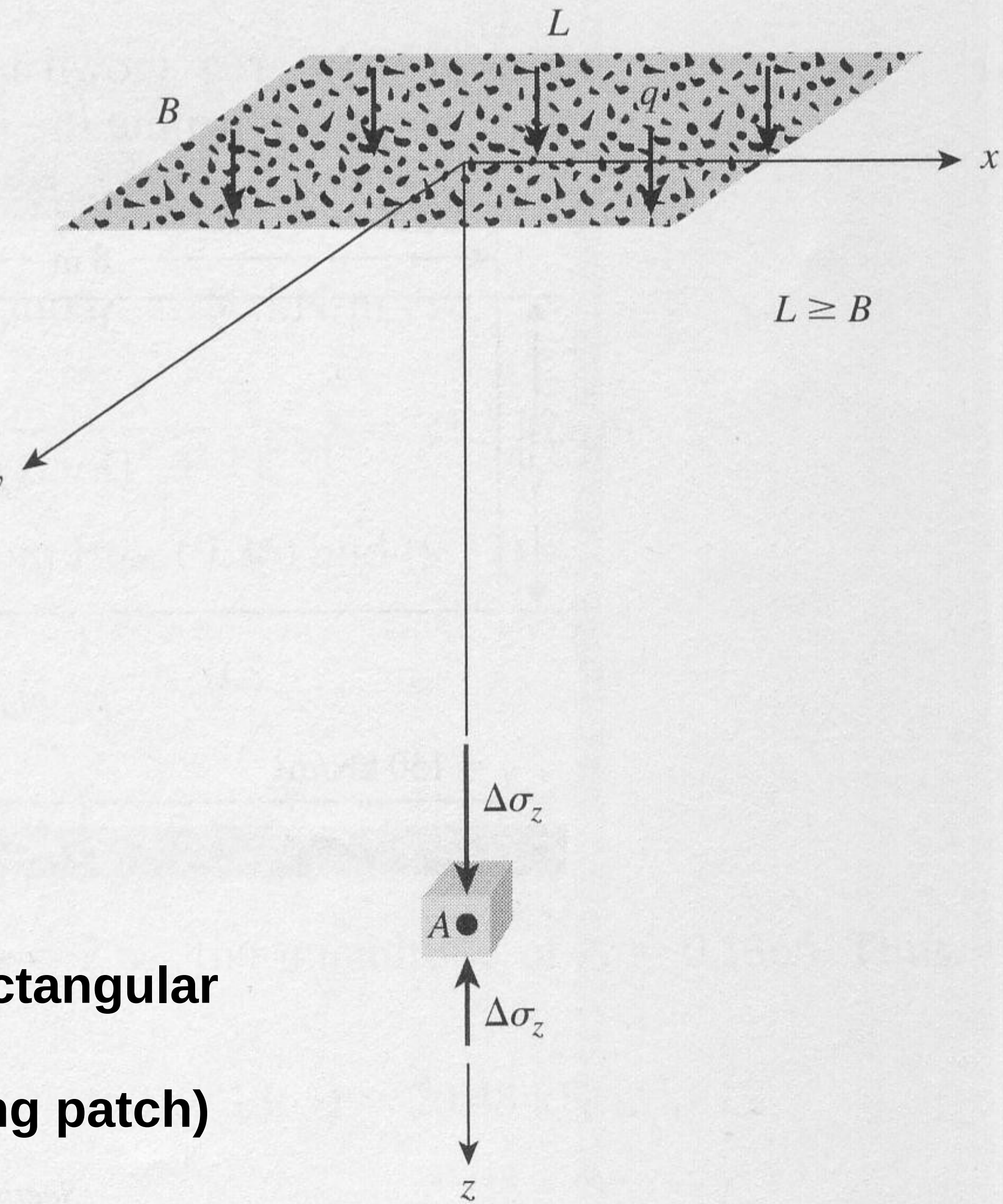


Figure 9.25 Increase of stress at any point below a rectangularly loaded flexible area

Vertical stress caused by a rectangular areal load
(at any point bellow the loading patch)

Vertical stress caused by a rectangular areal load (at any point bellow the loading patch)

$$\Delta\sigma_z = q[I_{4(1)} + I_{4(2)} + I_{4(3)} + I_{4(4)}] \tag{9.38}$$

where $I_{4(1)}$, $I_{4(2)}$, $I_{4(3)}$, and $I_{4(4)}$ = values of I_4 for rectangles 1, 2, 3, and 4, respectively.

In most cases the vertical stress increase below the center of a rectangular area (Figure 9.26) is important. This stress increase can be given by the relationship

$$\Delta\sigma_z = qI_5 \tag{9.39}$$

where

$$I_5 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}}\frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right] \tag{9.40}$$

$$m_1 = \frac{L}{B} \tag{9.41}$$

$$n_1 = \frac{z}{b} \tag{9.42}$$

$$b = \frac{B}{2} \tag{9.43}$$

Vertical Stress Due to Embankment Loading

Figure 9.17 shows the cross section of an embankment of height H. For this two-dimensional loading condition the vertical stress increase may be expressed as

$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right] \tag{9.25}$$

where $q_o = \gamma H$
γ = unit weight of the embankment soil
H = height of the embankment

$$\alpha_1 \text{ (radians)} = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right) \tag{9.26}$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) \tag{9.27}$$

For a detailed derivation of the equation, see Das (1997). A simplified form of Eq. (9.25) is

$$\Delta\sigma_z = q_o I_3 \tag{9.28}$$

where I_3 = a function of B_1/z and B_2/z.

The variation of I_3 with B_1/z and B_2/z is shown in Figure 9.18 (Osterberg, 1957).

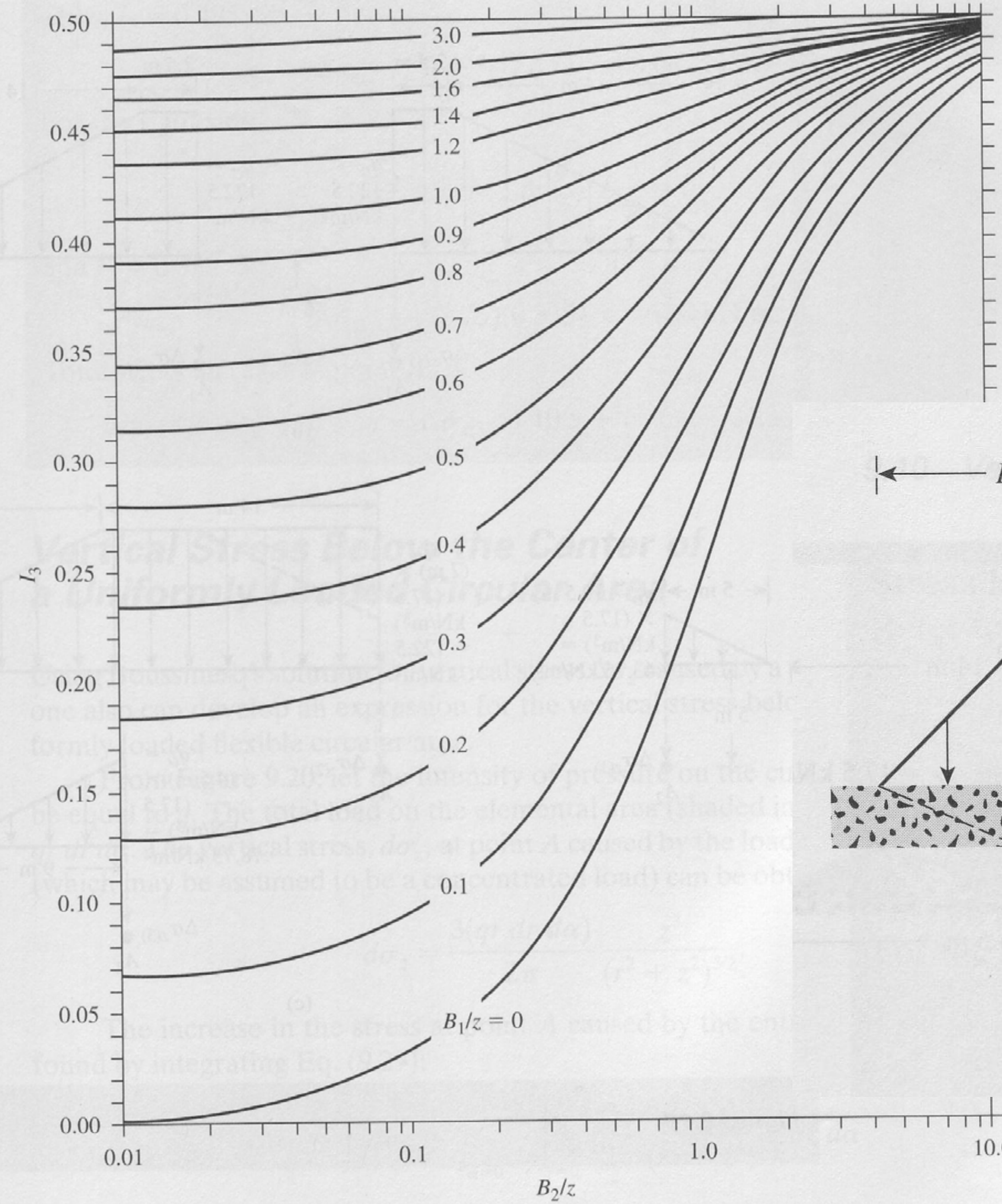


Figure 9.18

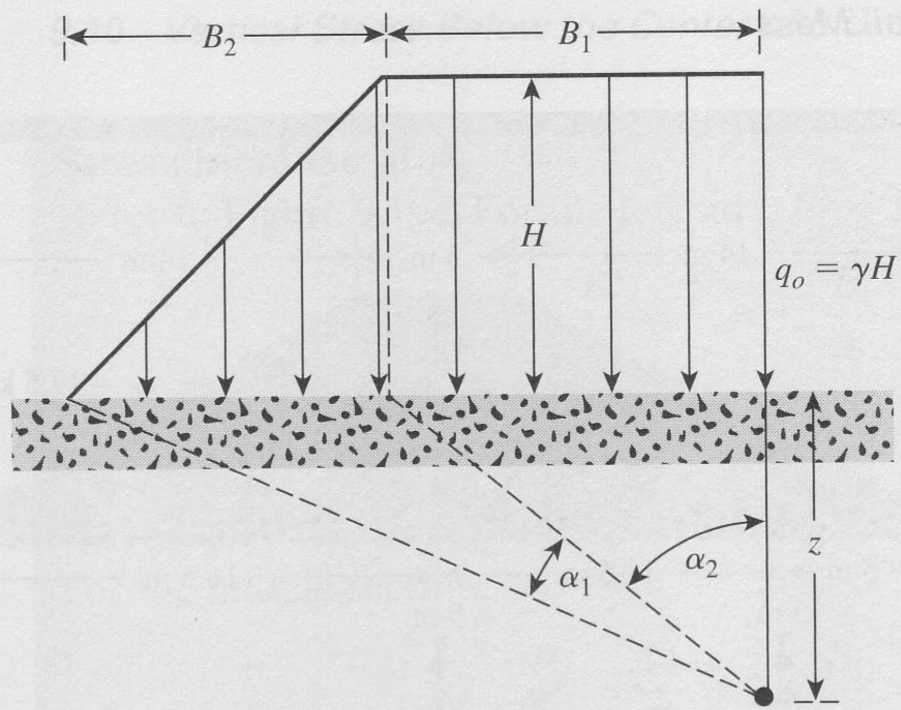
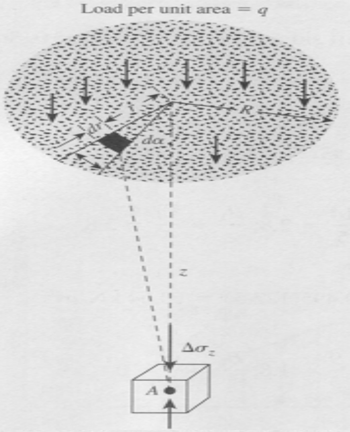


Figure 9.17



Vertical Stress Below the Center of a Uniformly Loaded Circular Area

Using Boussinesq's solution for vertical stress $\Delta\sigma_z$ caused by a point load [Eq. (9.12)], one also can develop an expression for the vertical stress below the center of a uniformly loaded flexible circular area.

From Figure 9.20, let the intensity of pressure on the circular area of radius R be equal to q. The total load on the elemental area (shaded in the figure) is equal to $qr\,dr\,d\alpha$. The vertical stress, $d\sigma_z$, at point A caused by the load on the elemental area (which may be assumed to be a concentrated load) can be obtained from Eq. (9.12):

$$d\sigma_z = \frac{3(qr\,dr\,d\alpha)}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}} \tag{9.29}$$

The increase in the stress at point A caused by the entire loaded area can be found by integrating Eq. (9.29):

$$\Delta\sigma_z = \int d\sigma_z = \int_{\alpha=0}^{\alpha=2\pi} \int_{r=0}^{r=R} \frac{3q}{2\pi} \frac{z^3 r}{(r^2 + z^2)^{5/2}}\, dr\, d\alpha$$

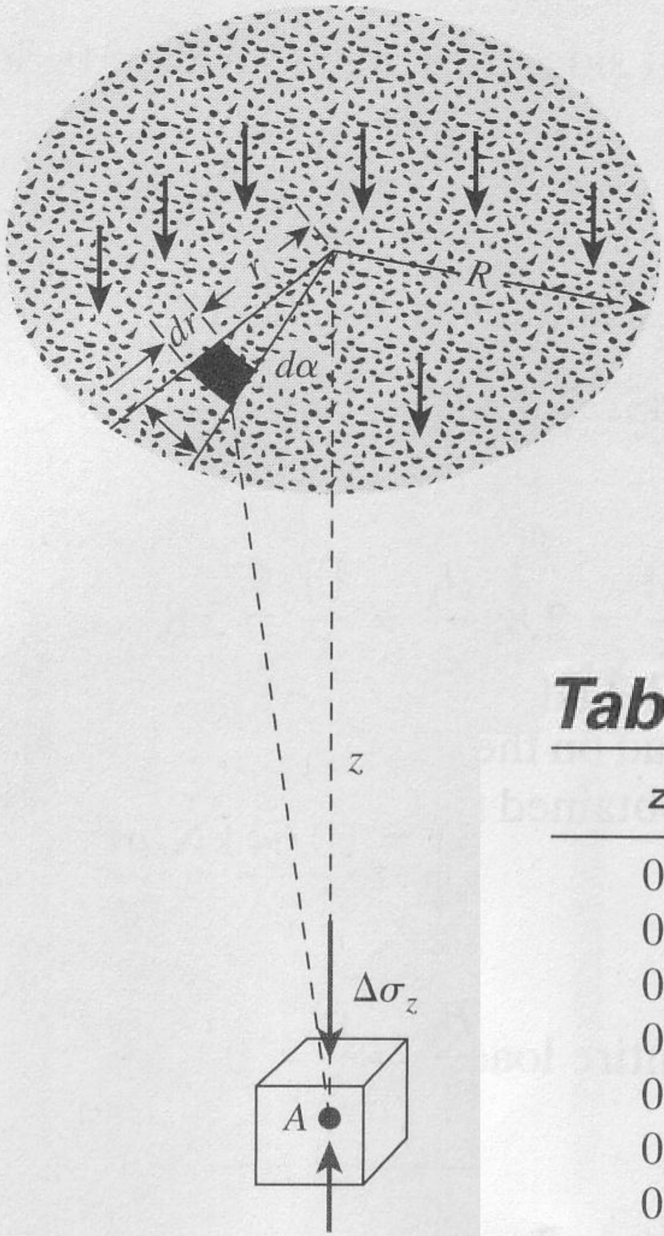


Figure 9.20

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} \quad (9.30)$$

Table 9.6 Variation of $\Delta\sigma_z/q$ with z/R

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

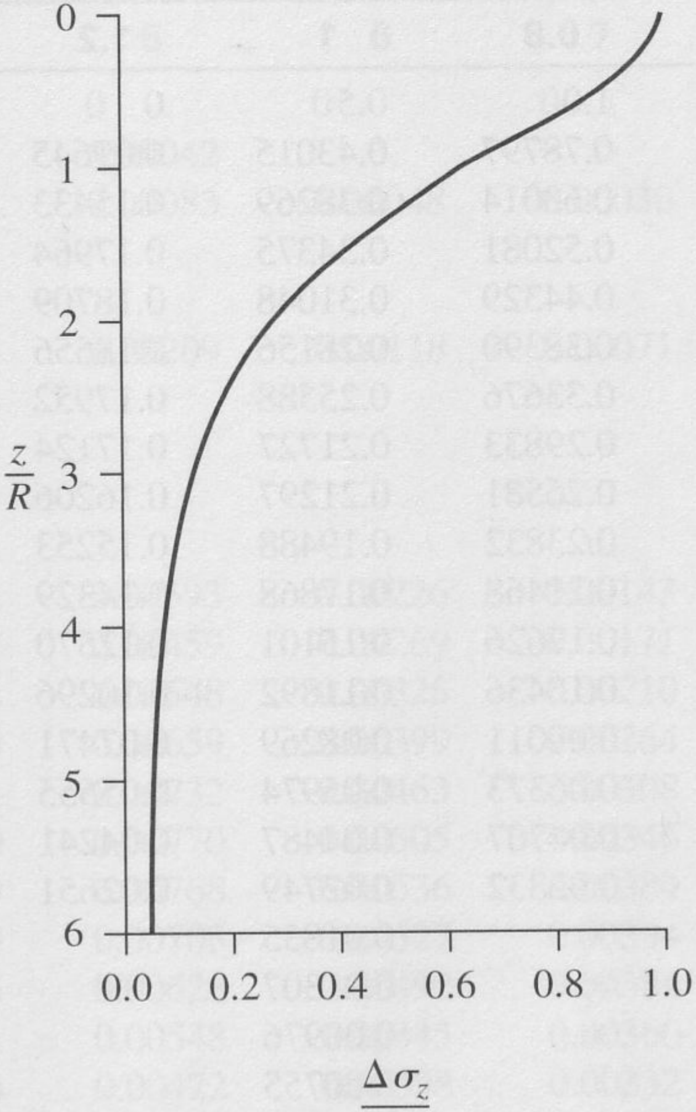


Figure 9.21 Stress under the center of a uniformly loaded flexible circular area

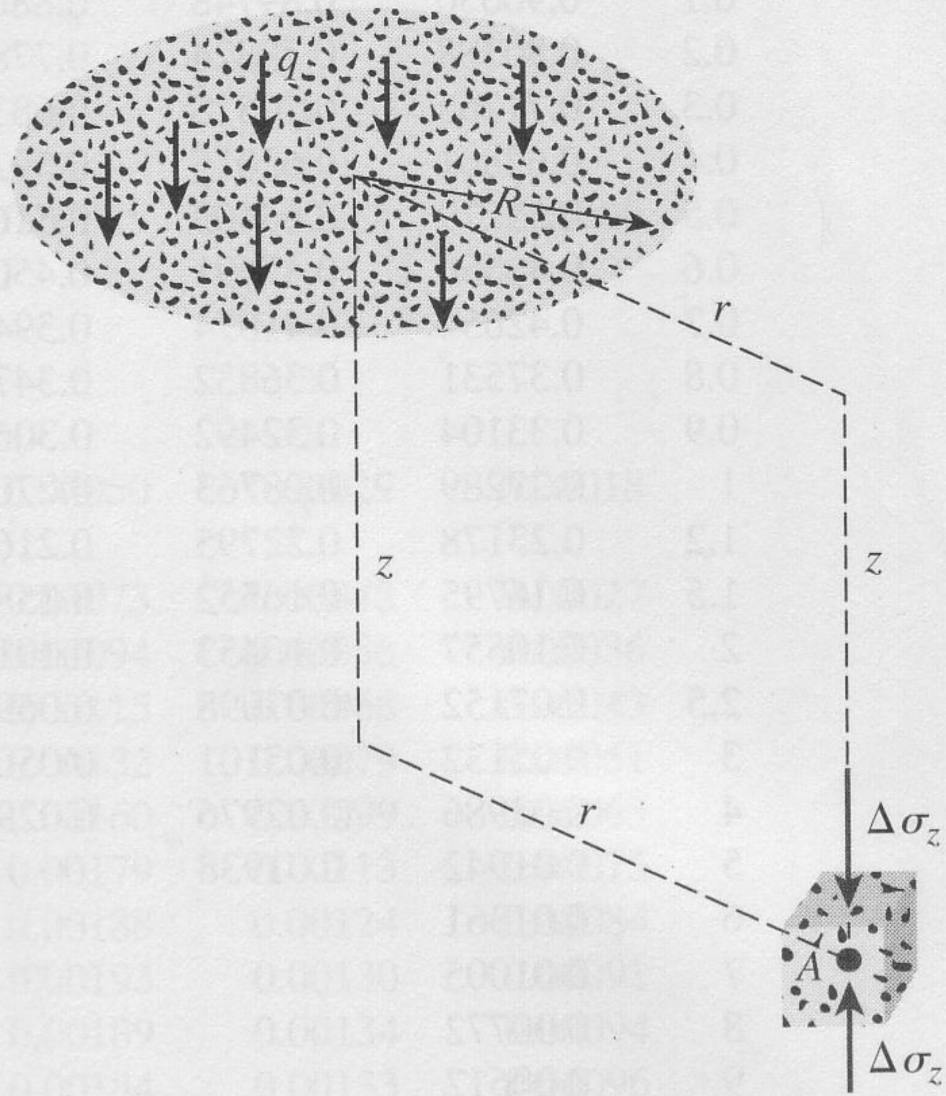


Figure 9.22 Vertical stress at any point below a uniformly loaded circular area

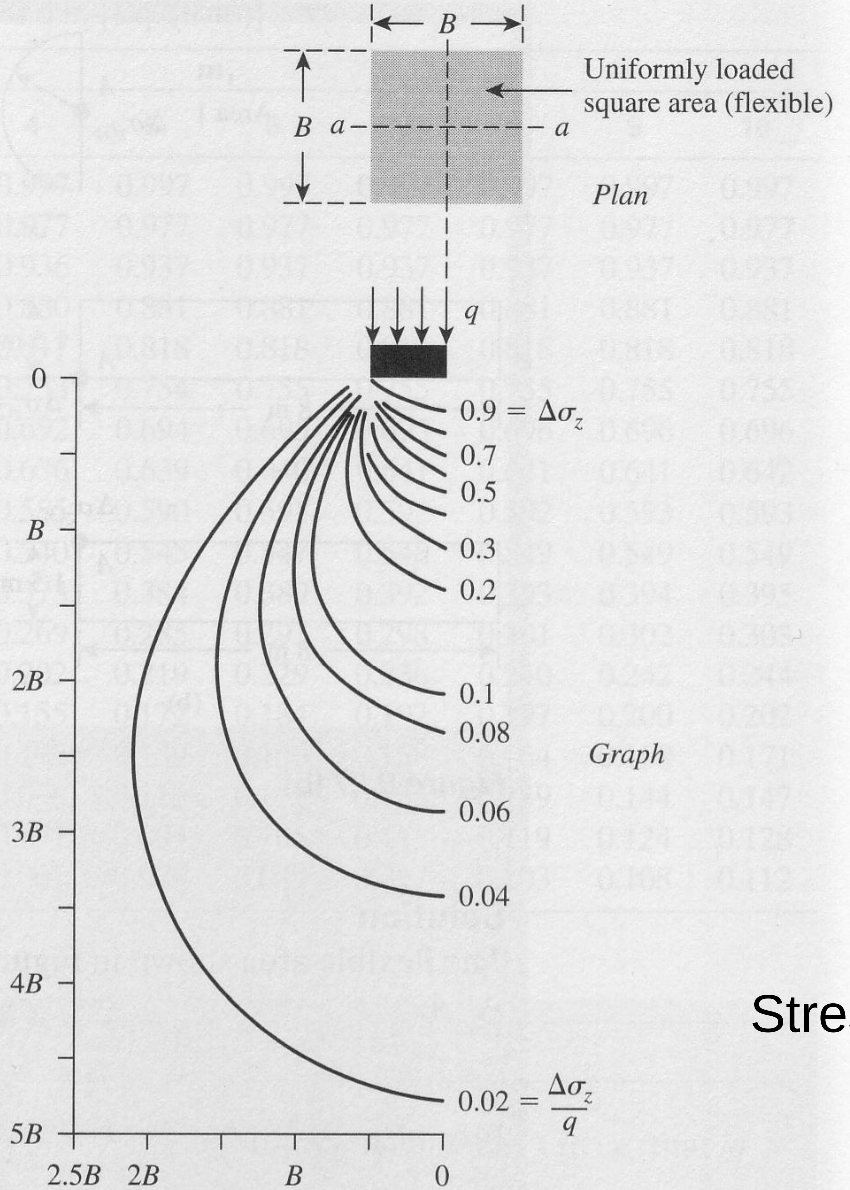


Figure 9.29 Vertical pressure isobars under a uniformly loaded square area (*Note:* Isobars are for line *a–a* as shown on the plan.)

Stress isobars

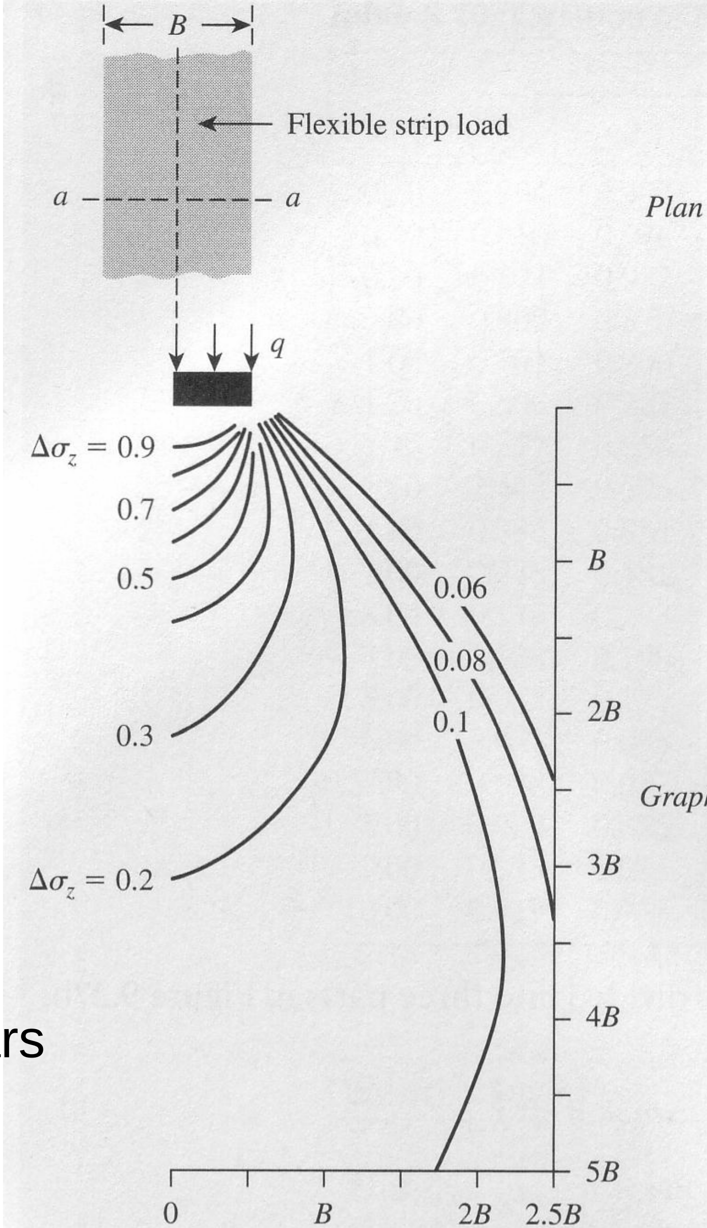


Figure 9.28 Vertical pressure isobars under a flexible strip load (*Note:* Isobars are for line *a–a* as shown on the plan.)

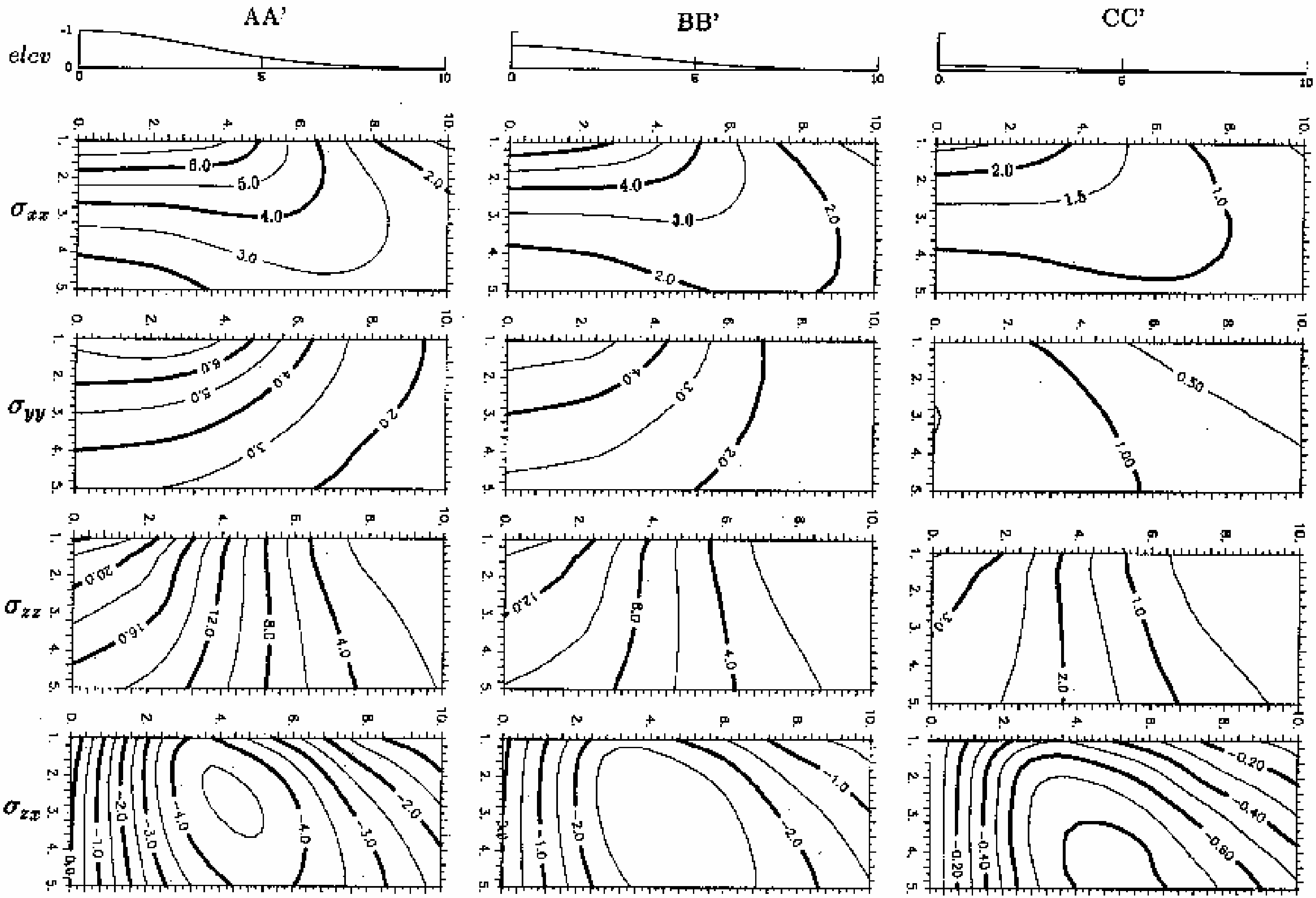
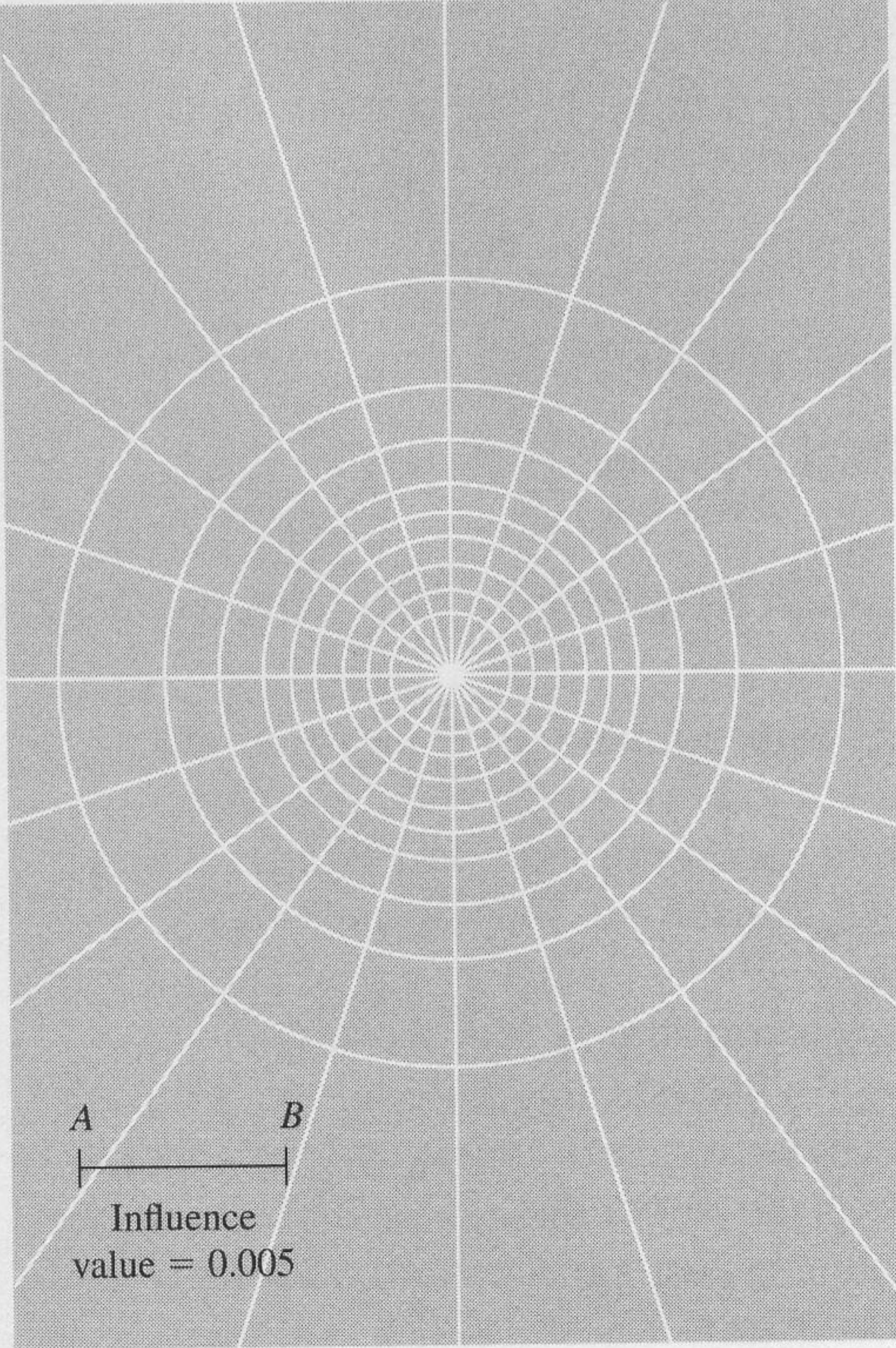


Fig. 2. Stress components of the profiles shown in Figure 1. (Left) Stresses in AA'. (Central) Stresses in BB'. (Right) Stresses in CC'. The unit of stress is megapascals. From top to bottom the rows illustrate σ_{xx}, σ_{yy}, σ_{zz}, and σ_{zx}, respectively.



Influence Chart for Vertical Pressure

Equation (9.30) can be rearranged and written in the form

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1}$$

1. Determine the depth z below the uniformly loaded area at which the stress increase is required.
2. Plot the plan of the loaded area with a scale of z equal to the unit length of the chart ($\overline{AB}$).
3. Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.
4. Count the number of elements (M) of the chart enclosed by the plan of the loaded area.

The increase in the pressure at the point under consideration is given by

$$\Delta\sigma_z = (IV)qM \tag{9.45}$$

where IV = influence value
q = pressure on the loaded area

Table 9.11 Values of R/z for Various Pressure Ratios [Eq. (9.44)]

$\Delta\sigma_2/q$	R/z	$\Delta\sigma_2/q$	R/z
0	0	0.55	0.8384
0.05	0.1865	0.60	0.9176
0.10	0.2698	0.65	1.0067
0.15	0.3383	0.70	1.1097
0.20	0.4005	0.75	1.2328
0.25	0.4598	0.80	1.3871
0.30	0.5181	0.85	1.5943
0.35	0.5768	0.90	1.9084
0.40	0.6370	0.95	2.5232
0.45	0.6997	1.00	∞
0.50	0.7664		

Problem solving Example

Example 9.10

The cross section and plan of a column footing are shown in Figure 9.31. Find the increase in vertical stress produced by the column footing at point A.

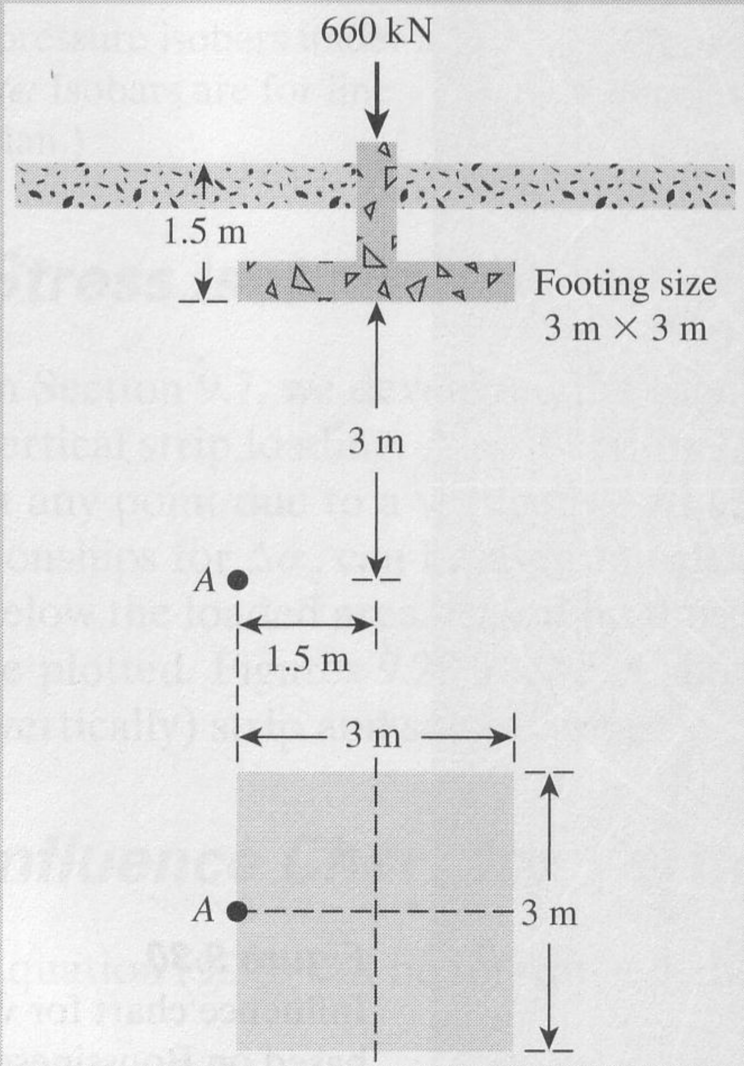
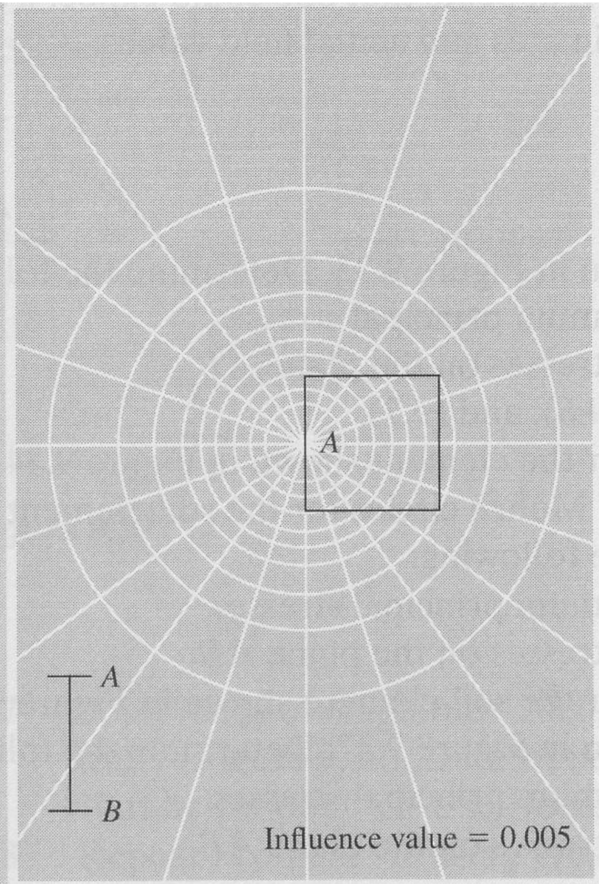


Figure 9.31
Cross section and plan of a column footing



The depth of interest is 3 m, make it equals to the unit scale of the Newmark's influence chart.

Figure 9.32 Determination of stress at a point by use of Newmark's influence chart

Solution

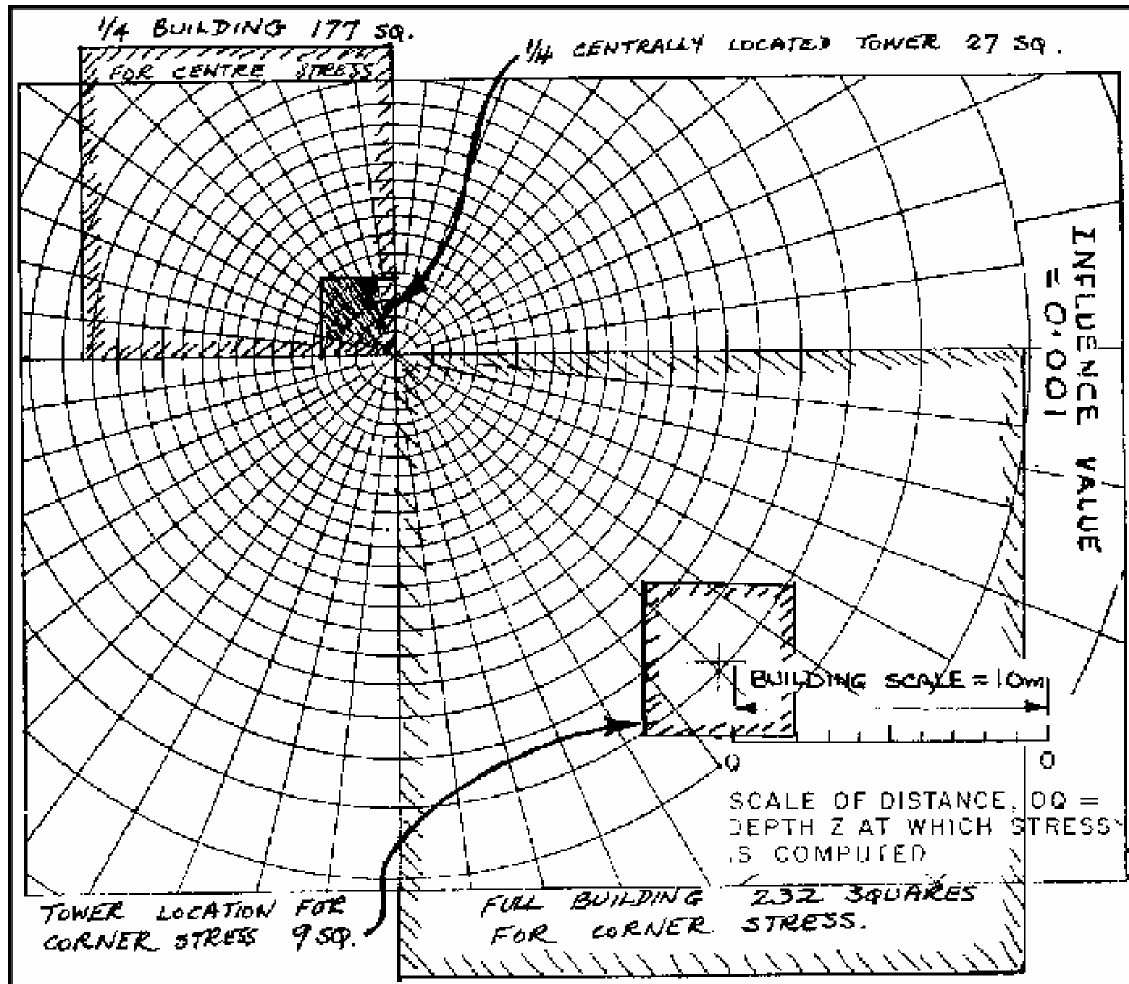
Point A is located at à depth 3 m below the bottom of the footing. The plan of the square footing has been replotted to a scale of $\overline{AB} = 3$ m and placed on the influence chart (Figure 9.32) in such a way that point A on the plan falls directly over the center of the chart. The number of elements inside the outline of the plan is about 48.5. Hence,

$$\Delta\sigma_z = (IV)qM = 0.005\left(\frac{660}{3 \times 3}\right)48.5 = \mathbf{17.78\ kN/m^2}$$ ■

EXAMPLE II

A building 20 m x 20 m results in a uniform surface contact pressure of 150 kPa. Using the Newmark Influence Chart obtain the vertical pressure depth of 10 m below (a) the centre of the building (b) a corner of the building. Estimate the additional pressure at both locations of a tower 5 m x 5 m placed at the centre of the building imposing 300 kPa uniform additional pressure.

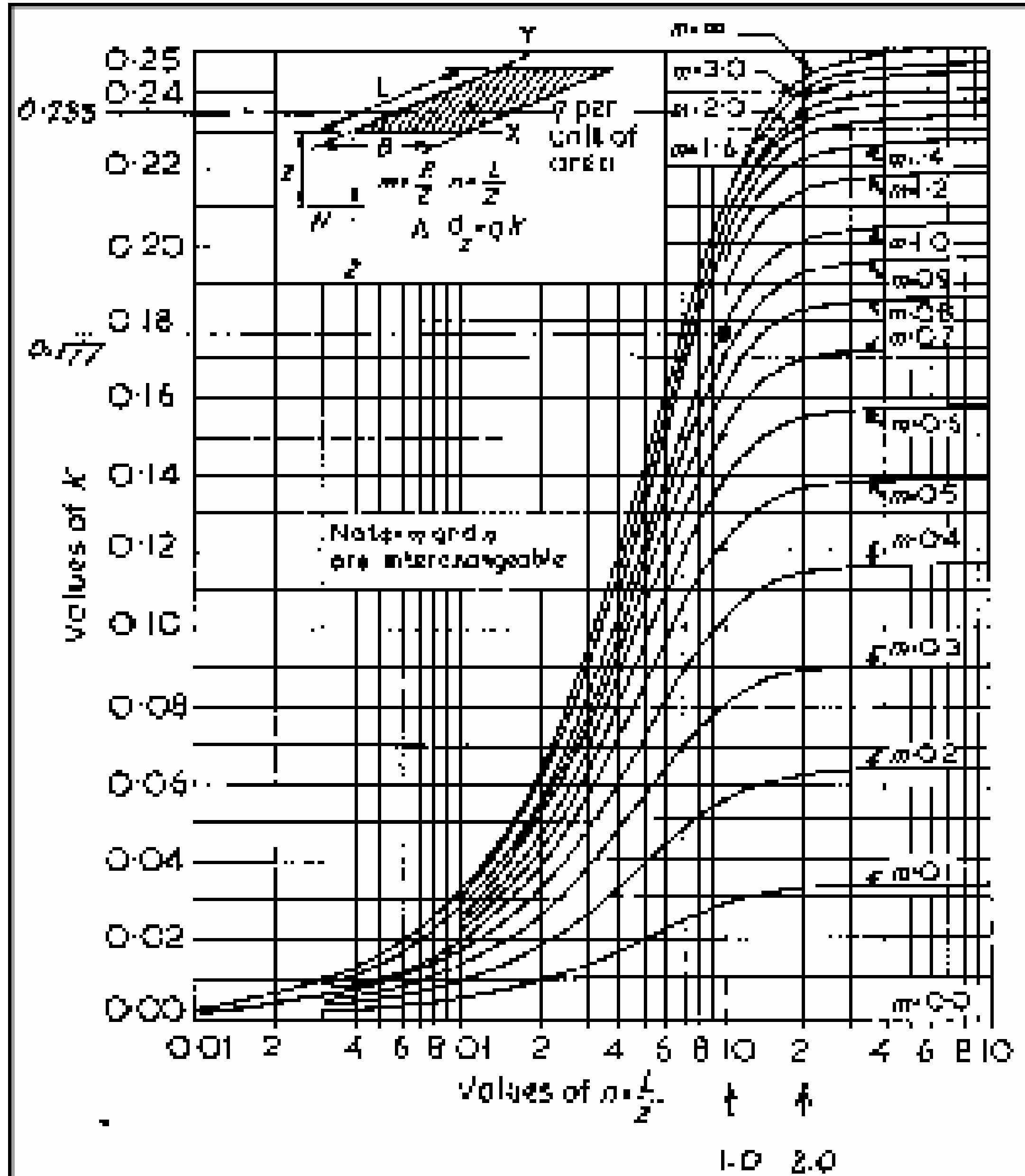
Increase in stress below centre of building:
= 4 x IV_x M x q
= 4 x 0.001 x 177 x 150
= 106 kPa
Increase in stress below corner of building:
= IV x_M x q
= 0.001 x 232 x 150 = 35 kPa
Increase in stress below centre due to tower:
= 0.001 x 4 x 27x300 = 32 kPa
Increase in stress below corner due to tower:
= 0.001 x 7 x 300 = 2 kPa



EXAMPLE by the Fadum Chart (Figure 9.24)
Using the Fadum Chart to obtain the vertical pressure depth of 10 m below (a) the centre of the building (b) a corner of the building.

For one quarter of building
L = 10 m; B = 10 m; z = 10 m
n = L/z = 10/10 = 1
m = B/z = 10/10 = 1
I_4 from chart = 0.177
σ= 4 x I_4 x q = 4 x 0.177 x 150
σ= 106 kPa

Increase in stress below corner
h = 20 m; B = 20 m; z = 10 m
n = L/z = 20/10 = 2
m = B/z = 20/10 = 2
I_4 from chart = 0.233
σ = I_4 x q = 0.233 x 150 = 35 kPa



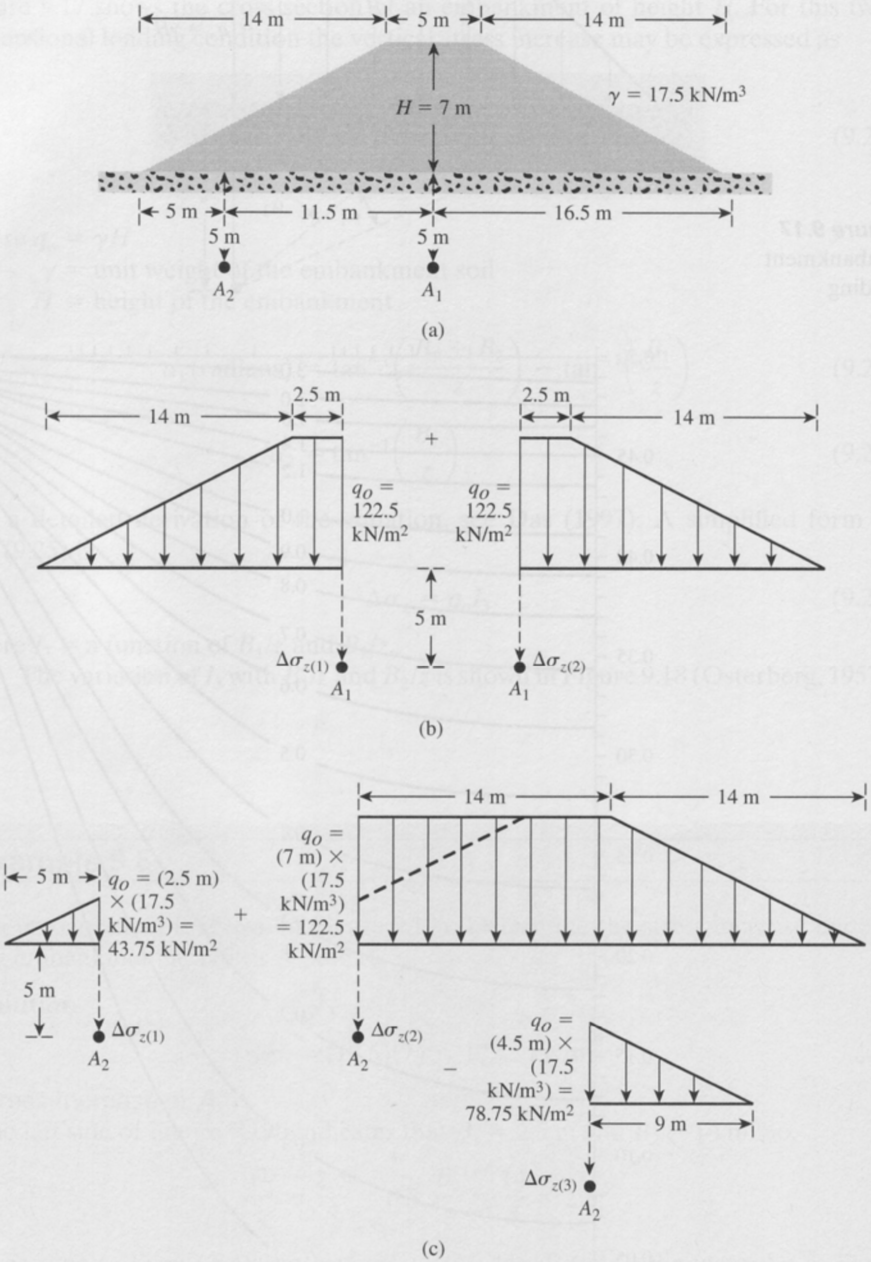


Figure 9.19

Example 9.8

An embankment is shown in Figure 9.19a. Determine the stress increase under the embankment at points A_1 and A_2.

Solution

$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

Stress Increase at A_1

The left side of Figure 9.19b indicates that $B_1 = 2.5$ m and $B_2 = 14$ m. So,

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

According to Figure 9.18, in this case, $I_3 = 0.445$. Because the two sides in Figure 9.19b are symmetrical, the value of I_3 for the right side will also be 0.445. So,

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{3(\text{Left})} + \text{I}_{3(\text{Right})}]$$

$$= 122.5[0.445 + 0.445] = \mathbf{109.03\ kN/m^2}$$

Stress Increase at A_2

Refer to Figure 9.19c. For the left side, $B_2 = 5$ m and $B_1 = 0$. So,

$$\frac{B_2}{z} = \frac{5}{5} = 1; \frac{B_1}{z} = \frac{0}{5} = 0$$

According to Figure 9.18, for these values of B_2/z and B_1/z, $I_3 = 0.24$. So,

$$\Delta\sigma_{z(1)} = 43.75(0.24) = 10.5 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I_3 = 0.495$. So,

$$\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$$

For the right side,

$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \frac{B_1}{z} = \frac{0}{5} = 0$$

and $I_3 = 0.335$. So,

$$\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.5 + 60.64 - 26.38 = \mathbf{44.76\ kN/m^2} \quad \blacksquare$$