CSE/ECE257 Final Exam TEST 3 December 17, 2005

Do five problems
Show your work

1. (a)Find the Newton form of the interpolation polynomial $p_3(x)$ which interpolates the data

i	xi	yi
0	-2	57
1	0	1
2	1	9
3	4	33

(b) Evaluate $p_3(2)$

2. (a) Use the composite Simpson's rule to estimate the value of $\int_{-1}^{1}(x+\cos^2(\pi x))dx$ using $h=\frac{1}{2}$.

(b) Estimate the error in part (a).

3. Use the error formula in the composite trapezoid rule to estimate the number of subintervals of [0,5] required to estimate the integral $\int_{0}^{5}(x^2+e^x)dx$ with an absolute error less than 10^{-4}. Do not calculate the actual composite trapezoid rule estimate.

4. Calculate the Romberg integration estimate $R_{3,3}$ to the integral $\int_{2}^{6}(x+\frac{1}{x})dx$.

5. Calculate the Euler method solutions y_0, y_1, y_2 for the initial value problem $y' = t - y^3$, $y(1) = 3$ for $h = 0.1$.

6. Show that the 2 stage Runge Kutta method:

$$k_1 = hf(t_i, y_i)$$
$$k_2 = hf(t_i + \frac{1}{4}h, y_i + \frac{1}{4}k_1)$$
$$y_{i+1} = y_i + 2k_2 - k_1$$

is $O(h^2)$ using the technique given in class.

7. Use the method given in number 6 to calculate y_0, y_1, y_2 for the initial value problem:

$$y' = -8y + 2t$$
$$y(1) = -3$$

for $h = 0.1$.