CSE/ECE257 TEST2 November 8, 2005

Do three problems
Show your work

1. (a) Calculate the first two Gauss-Seidel iteration estimates $\mathbf{x}^1$ and $\mathbf{x}^2$ to the linear system A**x**=**b** where the matrix A is given by $A=\begin{bmatrix} 5 & -3 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -2 & 4 & -1 \\ 0 & 0 & -3 & 4 \end{bmatrix}$ and the column vector **b** is given by $\mathbf{b}=\begin{bmatrix} -1 \\ 3 \\ -5 \\ 4 \end{bmatrix}$. Use $\mathbf{x}^0=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ as the start vector for the iteration.

(b)Will the Jacobi and Gauss-Seidel methods converge to the solution of this system? Explain your answer.

2. (a)Find the Lagrange form of the interpolation polynomial $p_3(x)$ which interpolates the data

i	xi	yi
0	-1	1
1	1	-1
2	2	8
3	3	39

(b) Evaluate $p_3(-2)$.

3. For $f(x)=\cos\left(\frac{\pi x}{4}\right)$ on the interval [0,1], use the error formula for polynomial interpolation to find a value of n which will guarantee that the interpolating polynomial p_n to f at n+1 equally spaced points in [0,1] will satisfy $|f(x)-p_n(x)|<10^{-5}$ for all x in [0,1].

4. Show that for $n \geq 3$, $2x^3 + x + 1 \equiv \sum_{k=0}^{n} (2x_x^3 + x_k + 1) l_{n,k}(x)$ where $l_{n,k}(x)$ is the basic Lagrange polynomial for the data. The formula is $l_{n,k}(x) = \dfrac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$.

5. (a) Write out the Hermite cubic polynomial $p_3(x)$ which satisfies $p_3(-3) = -57$, $p_3(1) = -1$, $p_3'(-3) = 58$, $p_3'(1) = 2$.

(b) Evaluate $p_3(-2)$.