Large-Girth Nonbinary QC-LDPC Codes of Various Lengths

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Abstract—In this paper, we construct nonbinary quasi-cyclic low-density parity-check (QC-LDPC) codes whose parity check matrices consist of an array of square sub-matrices which are either zero matrices or circulant permutation matrices. We propose a novel method to design the shift offset values of the circulant permutation sub-matrices, so that the code length can vary while maintaining a large girth. Extensive Monte Carlo simulations demonstrate that the obtained codes of a wide range of rates (from 1/2 to 8/9) with length from 1000 to 10000 bits have very good performance over both AWGN and Rayleigh fading channels. Furthermore, the proposed method is extended to design multiple nonbinary QC-LDPC codes simultaneously where each individual code can achieve large girth with variable lengths. The proposed codes are appealing to practical adaptive systems where the block length and code rate need to be adaptively adjusted depending on traffic characteristics and channel conditions.

Index Terms—LDPC, Galois field, nonbinary, quasi-cyclic, girth, variable length.

I. INTRODUCTION

Binary low-density parity-check (LDPC) codes proposed by Gallager [1] are excellent error-correcting codes that achieve performance close to the benchmark predicted by the Shannon capacity [2]. The extension of LDPC codes to nonbinary Galois field GF(q) was first investigated empirically by Davey and MacKay over the binary-input AWGN channel [3]. Better performance than binary LDPC codes has been reported with nonbinary LDPC codes [3].

Since then, nonbinary LDPC codes have been actively studied, including code design and reduced-complexity decoding algorithms [4]–[20]. The interest in nonbinary LDPC codes is motivated by the following two factors.

• First, nonbinary LDPC codes can potentially have better performance than their binary counterparts, especially when the block length is small to moderate and the code rate is high [3]–[6]. The selection of the operating Galois field provides another degree of freedom to achieve both good water-fall and error-floor performance. It has been proven in [4] that nonbinary cycle codes with fixed column weight 2 can achieve the capacity when q increases. A special class of cycle codes — regular cycle codes, which have fixed column and row weights in their parity check matrices — are well structured and can be encoded in linear time and in a parallel fashion [6]. Therefore, the family of regular cycle codes are attractive when a large q is selected. Later to address the problematic error-floor issue with small to moderate q, References [5], [7] have proposed a code structure with a near-regular column weight distribution profile. The proposed structure allows for linear time encoding in parallel, good performance, easy code design procedure and better tradeoff between the code’s water-fall and error floor performance. Codes with this near-regular structure, including those of high rates over small to moderate Galois fields, are also shown to have good performance over fading channels [5].

• Second, nonbinary LDPC codes can be combined with high order modulation seamlessly, avoiding the operation of bit-to-symbol conversion and its inverse. Its applications to radio and underwater acoustic communications combined with high order modulation and multiple-input multiple-output techniques have been reported [7], [9], [10], [12], [21]. Lower decoding complexity than its binary counterpart has been achieved [9], [19], [20].

Quasi-cyclic LDPC (QC-LDPC) codes are particularly appealing to practical systems as the quasi-cyclic structure of the parity check matrix (PCM) allows for linear time encoding using only shift registers, renders efficient routing for decoding implementation, and enables the storage of the coding matrix with only a few memory units. However, short cycles in the PCM H introduce correlation of the extrinsic information during iterative decoding, and cause decoding performance degradation. One active research direction on QC-LDPC codes has been the construction of the H matrix with large girth, which is the length of the shortest cycle in the code’s Tanner graph representation.

The design of QC-LDPC codes, binary or nonbinary, can be categorized into two types: random-like method and structured method. The shift offset values for component circulant permutation sub-matrices are obtained via the help of computer search for random-like methods [11], [22]–[24] whereas special algebraic or geometrical structures are utilized for code
design in structured methods [13]–[17], [25], [26]. Although the aforementioned methods can achieve large girth, the code block lengths are often not flexible. This inflexibility may limit their use in practical systems. As different multimedia services (e.g., voice, video conference, streaming media, web browsing) have different quality of service (QoS) requirements and the wireless channels are constantly time-varying, the physical and link layer parameters of an adaptive communication system such as the length of a data frame, the code rate, and the block length need to be adjusted adaptively based on traffic characteristics and channel conditions [27]. Hence, it is desirable that the block length and code rate can be fine adjusted while the LDPC codes still possess good properties.

In this paper, we construct nonbinary QC-LDPC codes which can achieve large girth with various lengths\(^1\). The parity check matrices of the proposed nonbinary QC-LDPC codes consist of an array of \(L \times L\) sub-matrices which are either zero matrices or circulant permutation matrices. We propose to first search for appropriate solution for the shift offset value of each circulant permutation sub-matrix to meet the girth requirement by relaxing the modulo \(L\) operation, and then find a threshold \(L_{\text{min}}\) on \(L\) such that the obtained codes can achieve the target girth whenever \(L \geq L_{\text{min}}\). This way, the obtained nonbinary QC-LDPC codes can achieve large girth while the code length can be flexibly changed. Further, we propose a novel formula for the shift offset values which combines algebraic structure and random search. An algorithm on the search of the shift offset values is presented.

The contributions of this paper are as follows.

- We propose a novel method to design nonbinary QC-LDPC codes which can achieve large girth with various lengths.
- We present very good performance of the proposed codes with a wide range of rates (from 1/2 to 8/9) and block lengths (from 1000 bits to 10000 bits) over both AWGN and Rayleigh fading channels.
- We further extend the proposed method to design multiple nonbinary QC-LDPC codes possessing large girth with various lengths.

The rest of the paper is organized as follows. We first state the design of nonbinary QC-LDPC codes and our objective in Section II. We present the proposed method in Section III. Section IV presents extensive simulation results of the proposed codes over both AWGN and Rayleigh fading channels. We extend the proposed method to design multiple nonbinary QC-LDPC codes of large girth and various lengths in Section V and we draw conclusions in Section VI.

II. LARGE-GIRTH NONBINARY QC-LDPC CODES OF VARIOUS LENGTHS

In this section we first review the structure of QC-LDPC codes, then present design steps for nonbinary QC-LDPC codes and state our design objective.

\(^1\)As pointed out by one anonymous reviewer, the design of parity check matrices for LDPC codes can be performed off-line for different lengths or rates separately. Instead of separate designs, this paper provides an alternative method to construct QC-LDPC codes with large girth and various lengths. Yet the proposed codes have similar performance to those codes designed separately, as shown in the numerical results.

A. Preliminaries

Consider an \(m \times n\) matrix \(\mathbf{M}(H)\), which is called the mother matrix (or base matrix) in this paper. With cyclic expansion, that is, replacing entries “0” and “1” in \(\mathbf{M}(H)\) with zero sub-matrices of size \(L \times L\) and circulant permutation sub-matrices of size \(L \times L\), respectively, one can obtain a PCM \(\mathbf{H}\) of size \(mL \times nL\) which defines a binary QC-LDPC code [22], [23]. Specifically, let \(\mathbf{P}\) be an \(L \times L\) permutation matrix as

\[
\mathbf{P} = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & \ldots & \ddots & \ldots & \vdots \\
0 & 0 & 0 & \ldots & 1 \\
1 & 0 & 0 & \ldots & 0
\end{bmatrix}
\]

For a finite \(a\), \(0 \leq a < L\), \(\mathbf{P}^a\) denotes a circulant permutation sub-matrix of size \(L \times L\) which is obtained by cyclically shifting the identity matrix \(\mathbb{I}_L\) to the right by \(a\) times. For simple notation, \(\mathbf{P}^\infty\) denotes the zero matrix of size \(L \times L\).

Applying cyclic expansion to the mother matrix \(\mathbf{M}(H)\), a PCM of size \(mL \times nL\) for a binary QC-LDPC code can be obtained [22], [23]

\[
\mathbf{H} = \begin{bmatrix}
\mathbf{P}^{a_11} & \mathbf{P}^{a_12} & \ldots & \mathbf{P}^{a_1n} \\
\mathbf{P}^{a_21} & \mathbf{P}^{a_22} & \ldots & \mathbf{P}^{a_2n} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{P}^{a_m1} & \mathbf{P}^{a_m2} & \ldots & \mathbf{P}^{a_mn}
\end{bmatrix}
\]

where the shift offset value \(a_{ij} \in \{0, 1, \ldots, L - 1, \infty\}\), \(i = 1, 2, \ldots, m\), \(j = 1, 2, \ldots, n\); and \(a_{ij} = \infty\) when the corresponding entry in \(\mathbf{M}(H)\) is “0”.

Given a binary PCM as in (2), a nonbinary PCM \(\mathbf{H}_q\) can be obtained by replacing each nonzero entry in \(\mathbf{H}\) with a nonzero element from a Galois field \(\text{GF}(q)\). However, generally speaking, the obtained code is not qualified as a quasi-cyclic code over \(\text{GF}(q)\). To address this issue special structures have been imposed on the selection of nonzero entries for each sub-matrix \(\mathbf{P}^{a_{ij}}\) for the cases of \(L = q - 1\) in [13] and \(L/(q - 1)\) in [11], respectively.

B. Design of Nonbinary QC-LDPC Codes

In this paper, we adopt the following three steps to construct a nonbinary QC-LDPC code.

- **Step 1**: Construct a mother matrix \(\mathbf{M}(H)\).
- **Step 2**: Specify the shift offset value (\(a_{ij}\) in (2)) for each nonzero entry of \(\mathbf{M}(H)\). After cyclic expansion, we obtain a binary PCM \(\mathbf{H}\) as in (2).
- **Step 3**: Specify the nonzero elements of \(\mathbf{H}_q\) by replacing each “1” entry in \(\mathbf{H}\) by an element from \(\text{GF}(q)\)\(\setminus\{0\}\).

In this paper, we focus on Step 2, while using existing methods\(^2\) on Step 1 and Step 3. We summarize some design issues in the following.

\(^2\)Here the selection of shift offset values and the selection of nonzero elements are carried out separately. A joint selection is possible, but is subject to further research.
1) Construction of A Mother Matrix: Since the size of the mother matrix $M(H)$ is usually small, one can even design the matrix by hand. Otherwise, a computer-based search algorithm, such as the well-known progressive-edge-growth (PEG) algorithm [28] can be adopted. Note that some special patterns in the mother matrix can induce inevitable cycles of length larger than 10 in the expanded PCM $H$ [29], thus special effort are needed to avoid such patterns in $M(H)$ if the target girth is larger than 10.

2) Selection of Nonzero Elements from $GF(q)$: Given a binary PCM $H$ shown in (2), special treatment is needed in Step 3 to render the obtained nonbinary code $H_2$, qualified as a quasi-cyclic code over $GF(q)$. Here we present a method to address this problem which works for any $L$.

Our method follows the same line as in References [11], [13]. Specifically, let $\rho_{ij}$ denote the nonzero element in the first row of $P^{\alpha j}$ which can be drawn randomly from $GF(q) \backslash \{0\}$, the nonzero elements for the remaining rows of $P^{\alpha j}$ are obtained by multiplying the one in the row above by $\alpha^k$, where $\alpha$ is a primitive element of $GF(q)$ and $\lambda$ is an integer. However, generally speaking, the nonzero element in the first row is not equal to the nonzero element in the last row multiplied by $\alpha^\lambda$.

To address this issue, Reference [13] introduces a so-called $\alpha$-multiplied circulant permutation matrix which translates to $L = q - 1$ and $\lambda = 1$. Later Reference [11] has generalized it to a $\beta$-multiplied circulant permutation matrix which translates to $L/(q - 1)$ and $\lambda = (q - 1)/L$. Here we select $\lambda$ once $L$ is given. Specifically, we choose the smallest $\lambda$ for any given $L$ such that $L \cdot \lambda = (q - 1)/\gamma$ where $\gamma$ is an integer.

After obtaining a binary PCM, the above structure is applied to each sub-matrix $P^{\alpha j}$. Following the same reasoning in [11], [13], the obtained code is qualified as a nonbinary QC-LDPC code.

C. Design Objective

To achieve large girth, special linear inequalities in terms of $a_{ij}$ and $L$ have to be satisfied [22], [23]. However, in general the solutions for $a_{ij}$ are different for different $L$, which is not convenient when the block length needs to change. In this paper, we present a design which allows for fixed $a_{ij}$ with guaranteed girth $g$ for any $L$ no less than a threshold $L_{\text{min}}$. This way, the constructed codes can have various lengths while achieving a target girth.

III. THE PROPOSED METHOD

We first review the requirements to achieve large girth. Then we present the proposed method to achieve our design objective. Finally we give an example design.

A. Girth Property

Due to the fact that each sub-matrix $P^{\alpha j}$ in (2) is either a zero matrix or a circulant permutation matrix, a cycle in the Tanner graph representation of a QC-LDPC code can be considered as a sequence of nonzero sub-matrices, or equivalently as a sequence of nonzero entries in $M(H)$ [22], [23]. Specifically, we can express a cycle of length $2w$ as an ordered series of circular permutation matrices

$$\mathbf{S} := \{(i_0,j_0); (i_1,j_1); \ldots; (i_k,j_k); \ldots; (i_{w-1},j_{w-1}); (i_0,j_0)\}$$

(3)

where $(i_k,j_k)$ is used to index the nonzero sub-matrix $P^{\alpha i_k j_k}$, and semicolon between $(i_k,j_k)$ and $(i_{k+1},j_{k+1})$ can be considered as the nonzero sub-matrix $P^{\alpha i_{k+1} j_{k+1}}$. Certainly the conditions $i_k \neq i_{k+1}$ and $j_k \neq j_{k+1}$ shall hold for the sub-matrix sequence in (3) to form a valid cycle. Note that some sub-matrices can appear more than once in (3).

For the sequence $\mathbf{S}$ in (3), define

$$\Delta_\mathbf{S} \triangleq \sum_{k=0}^{w-1} (a_{i_k j_k} - a_{i_{k+1} j_{k+1}}) = \sum_{k=0}^{w-1} \Delta_{i_k,i_{k+1}}(j_k)$$

(4)

where $i_w = i_0$ and $\Delta_{i_k,i_{k+1}}(j_k) \triangleq a_{i_k j_k} - a_{i_{k+1} j_{k+1}}$.

As proved in References [22], [23], a necessary and sufficient condition for the Tanner graph representation of $H$ in (2) to have a girth at least $g$ ($g$ is even) is that $\Delta_\mathbf{S}$ is nonzero modulo $L$ for all sequences in (3) with $2 \leq w \leq g/2 - 1$. Let $B_w$ denote the set of all possible sequences in (3) of length $2w$, then the requirements to achieve a girth at least $g$ can be mathematically represented as

$$\Delta_\mathbf{S} \neq 0 \mod L, \quad \forall \mathbf{S} \in \bigcup_{w=2}^{g/2-1} B_w.$$  

(5)

B. The Proposed Method

To achieve the design objective, our approach is as follows. First, suppose that $L$ is no less than a certain threshold $L_{\text{min}}$ so that the modulo $L$ operation can be relaxed to simplify (5) as

$$\Delta_\mathbf{S} \neq 0, \quad \forall \mathbf{S} \in \bigcup_{w=2}^{g/2-1} B_w.$$  

(6)

After obtaining the solutions for $a_{ij}$, $L_{\text{min}}$ can be chosen as

$$L_{\text{min}} = \max \left\{ \left\lfloor \frac{\Delta_\mathbf{S}}{L} \right\rfloor \in \bigcup_{w=2}^{g/2-1} B_w \right\} + 1.$$  

(7)

That is, $L_{\text{min}}$ is chosen to be just larger than the maximum absolute value of $\Delta_\mathbf{S}$ among all $B_w$ with $w < g/2$. This way, eq. (6) is equivalent to eq. (5) for any $L \geq L_{\text{min}}$, thus girth $g$ is achieved whenever $L \geq L_{\text{min}}$.

Next we provide one explicit design of the shift offset values $\{a_{ij}\}_{i,j=1}^{m,n}$ to fulfill this objective.

1) Formula for Shift Offset Values: The first method one can investigate is computer-based random search. Trial-and-error tests using (6) can be used to search for appropriate $a_{ij}$ given a target girth $g$. The dimension of the search space increases exponentially in terms of the number of nonzero entries in $M(H)$. Another choice is to impose an algebraic structure for $a_{ij}$ to facilitate the design. Different formulas including summation, product and power have been explored in the literature [22], [26]. However, the girth that can be achieved with an algebraic structure is usually limited [22], [26].
We propose a novel formula which combines algebraic structure and random search. Specifically, we set
\[ a_{ij} = r_{ij}l_j, \]  
where \( r_{ij} \) is drawn from a difference set \{0, 1, 3, 7, 12, \ldots \} sequentially for each nonzero entry in the \( j \)-th column of \( M(H) \). The difference set \{0, 1, 3, 7, 12, \ldots \} guarantees that the difference between any two elements from the set is distinguishable (See e.g., [30] and references therein), which renders the possibility of achieving large girth even in the presence of many short cycles in the mother matrix. Note that a similar form with \( l_j = 37^{j-1} \) has been adopted in [29] to achieve girth 14 for a specific code, however the reported size of the circulant permutation sub-matrices is extremely large (\( L = 37^5 \)) and the formula used in [29] is algebraically structured.

For any two nonzero entries in the \( j \)-th column of \( M(H) \) with row indices \( i_x \) and \( i_y \), we have
\[ \Delta_{i_x,i_y}(j) = a_{i_x,j} - a_{i_y,j} = \Delta_{i_x,i_y}(j)r_{ij}, \]  
(9)
where \( \Delta_{i_x,i_y}(j) = r_{ij} - r_{ij} \). Note that once the mother matrix \( M(H) \) is given, \( \Delta_{i_x,i_y}(j) \) is determined. Next we propose a computer search procedure to determine the value of \( l_j \).

2) Computer Search: Since \( L_{min} \) plays a significant role, reducing its value as much as possible is desirable. With the proposed formula for the shift offset values as in (8), we propose a progressive search procedure to find solutions for \( l_j \) and also the value of \( L_{min} \). The proposed search procedure works in a sequential manner. At the \( j \)-th step, a partial matrix comprised of the first \( j \) columns of the mother matrix, denoted as \( M(H)^{1\rightarrow j} \), is constructed. The relaxed constraints and the value of \( L_{min} \) restricted to this partial matrix \( M(H)^{1\rightarrow j} \) become
\[ \Delta_{S} \neq 0, \quad \forall S \in \bigcup_{w=2}^{w=g/2-1} B_w(j), \]  
(10)
and correspondingly we can define
\[ L_{min}(j) = \max \left\{ |\Delta_{S}| : S \in B_w(j) \right\} + 1. \]  
(11)
Note that \( L_{min} = L_{min}(n) \).

The following algorithm takes the target girth, the mother matrix, an initial value \( l_1 \) and a parameter \( \Gamma_{max} \) (its meaning will be clear soon) as inputs, and provides the values for \( l_j \) and \( L_{min} \) as outputs.

Algorithm 1

INPUTS: the target girth \( g \), the mother matrix, an initial value \( l_1 \) and the search bound \( \Gamma_{max} \).

OUTPUTS: \( l_j, 1 \leq j \leq n \) and \( L_{min} \).

LOOP FROM \( j = 2 \) TO \( j = n \)

LOOP FROM \( l_j = -\Gamma_{max} \) TO \( \Gamma_{max} \)

Check whether \( l_j \) satisfies the relaxed constraints (10) of the partial matrix \( M(H)^{1\rightarrow j} \). If satisfied, then for this \( l_j \), calculate \( L_{min}(j) \) according to (11). Record the pair of \( l_j \) and \( L_{min}(j) \).

END OF LOOP

Select the \( l_j \) whose corresponding parameter \( L_{min}(j) \) is minimum from all recorded pairs of \( l_j \) and \( L_{min}(j) \). If there are multiple choices, randomly pick up one. If there is no available solution for \( l_j \) STOP (a failure is reached).

END OF LOOP

The search bound \( \Gamma_{max} \) in Algorithm 1 is used to limit the search time for each \( l_j \). For a given mother matrix and a given target girth \( g \), one can try a set of combinations of \( l_1 \) and \( \Gamma_{max} \), then choose the one with minimum \( L_{min} \). The number of steps needed in Algorithm 1 equals the number of columns in \( M(H) \). Thus the dimension of the search space is considerably reduced compared with a fully random search method where the number of steps needed equals the number of nonzero entries in \( M(H) \) [24].

Note that in Algorithm 1 we have considered only one target girth. It is straightforward to generalize Algorithm 1 to consider multiple target girths during the search procedure with each target girth having a corresponding parameter \( L_{min}(j) \). Other than a single \( L_{min}(j) \), the average value of \( L_{min}(j) \) across different target girths can be used as the criterion to select \( l_j \) in the \( j \)-th step.

C. An Example Design

A mother matrix of size \( 6 \times 12 \) with mean column weight \( 2.5 \) is shown below:

\[
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

The design results are listed in Table I where \( \Gamma_{max} \) is chosen to be less than 40 for the first four designs and it is set to be 150 for ‘Design 5’. Further, ‘Design 1’ is considering only one target girth \( g = 6 \) whereas other four designs are obtained considering several target girths as shown in Table I. For example, in ‘Design 4’ we have considered four target girths, namely, 6, 8, 10 and 12. In the \( j \)-th step of the search procedure, each target girth has one associated parameter \( L_{min}(j) \) for a tentative value of \( l_j \). The average value of \( L_{min}(j) \) across different target girths is used as the criterion to select the solution for \( l_j \) from a set of candidate lists of \( l_j \) and \( L_{min}(j) \). After running the search procedure, one can get the solution for \( l_j \) and the \( L_{min} \) for the four target girths as well. Note that the solutions of \( l_j \) are shared for different target girths.

In Fig. 1 we compare the girth property between the proposed ‘Design 4’ codes and codes whose parity check matrices \( H \) are constructed by the well-known PEG algorithm [28], where the size of the sub-matrix \( L \) varies from 5 to
TABLE I
PARAMETERS OF RATE-1/2 NONBINARY QC-LDPC CODES OF VARIOUS LENGTHS. THE SIZE OF THE MOTHER MATRIX IS 6 × 12 WITH MEAN COLUMN WEIGHT OF 2.5.

<table>
<thead>
<tr>
<th>Design</th>
<th>Target Girth, Symbol</th>
<th>(L_{\text{min}})</th>
<th>Symbol Length (k = 0, 1, \ldots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(l_1 - l_6)</td>
<td>6</td>
<td>48 + 12k</td>
</tr>
<tr>
<td>2</td>
<td>(l_7 - l_{12})</td>
<td>6</td>
<td>96 + 12k</td>
</tr>
<tr>
<td>3</td>
<td>(l_1, l_7)</td>
<td>6</td>
<td>92 + 12k</td>
</tr>
<tr>
<td>4</td>
<td>(l_1, l_7)</td>
<td>8</td>
<td>92 + 12k</td>
</tr>
<tr>
<td>5</td>
<td>(l_1, l_7)</td>
<td>10</td>
<td>228 + 12k</td>
</tr>
<tr>
<td>6</td>
<td>(l_1, l_7)</td>
<td>8</td>
<td>192 + 12k</td>
</tr>
<tr>
<td>7</td>
<td>(l_1, l_7)</td>
<td>10</td>
<td>192 + 12k</td>
</tr>
</tbody>
</table>

Note that the PEG algorithm works in a column-by-column manner which determines one nonzero entry of \(H\) in each step [28]. The advantage of the proposed method over the PEG algorithm probably comes from the fact that the number of steps needed for the proposed method (which equals the number of columns in the mother matrix \(M(H)\)) is much smaller than the number of steps needed for the PEG algorithm (which equals the number of nonzero entries in \(H\)). The more search steps, the smaller the chance of hitting the optimal solution is. In a nutshell, when the girth is concerned the proposed method can achieve similar or better design compared with the PEG algorithm. Another advantage of the proposed method over the PEG algorithm is that codes constructed by the proposed method are guaranteed to achieve a target girth \(g\) whenever \(L\) is no less than the corresponding \(L_{\text{min}}\).

IV. SIMULATION RESULTS

Monte Carlo simulations are used to evaluate the performance of the proposed nonbinary QC-LDPC codes of various lengths. The near-regular column weight distribution profile presented in [5], [7] is adopted. That is, the column weight of a mother matrix \(M(H)\) is either 2 or \(t \geq 3\). Further, the criterion presented in [7], [8] is applied to select the nonzero element \(p_{ij}\) for each sub-matrix \(P^{a_{ij}}\) in Section II.B which can lower the code’s error floor, especially when the mean column weight is small.

For encoding, the constructed nonbinary QC-LDPC codes can be encoded in linear time using shift registers as done in [31]. Furthermore, having mixed column weights of 2 and \(t \geq 3\) as constructed in [5], [7], a parallel encoding algorithm with total linear complexity can also be applied [5], [7]. In short, the proposed codes with the near-regular profile can be encoded in linear time in two different ways.

For decoding, we use the mixed domain decoding algorithm presented in [9] where messages are transformed between probability domain representation and log domain representation to utilize Fast Fourier Transform in the check node.
a coding gain of more than 10 bits is only from 684 bits to 7488 bits. Further, the code of length 7488 bits is only 1.830 dB away from the Shannon limit at BLER of $10^{-4}$. For simulations over the i.i.d. Rayleigh fading channel, a coding gain of more than 1 dB is achieved at BLER of $10^{-4}$ as the block length increases from 684 bits to 7488 bits. Further, the code of length 7488 bits is only 1.6 dB away from the Shannon limit at BLER of $10^{-4}$.

For comparison we have constructed five codes using the PEG algorithm [28]. The column weight distribution adopted by the PEG algorithm is the same near-regular profile [5], [7] as that used by the proposed method. The girths for these PEG codes are 4, 6, 8, 8, and 10 for symbol lengths of 228, 492, 756, 1596, and 2496 respectively. Their performance are also reported in Figs. 2 and 3. We observe that the proposed non-binary QC-LDPC codes have similar performance compared with codes constructed by the PEG algorithm separately.

Compared with existing QC-LDPC codes in the literature, codes constructed by the proposed method achieve performance gains. Specifically, we have the following observations:

- In Reference [16], a rate-1/2 16-ary (300,150) nonbinary QC-LDPC code has been constructed, which has block length of 1200 bits. This code achieves BLER of $10^{-4}$ at about 2.6 dB over AWGN channel. Our code over GF(8) of length 684 bits, as shown in Fig. 2, achieves similar performance compared with the 16-ary (300,150) code reported in [16] at BLER above $10^{-4}$, but with almost half of the block length. Further, our code over GF(8) of length 1476 bits, as shown in Fig. 2, achieves BLER of $10^{-4}$ at about 2.0 dB, which is 0.6 dB better than the 16-ary (300,150) code reported in [16].
- In Reference [25], a rate-0.5036 (3060,1544) binary QC-LDPC code has been constructed, which has block length of 3060 bits. This code achieves BLER of $10^{-4}$ at about 2.0 dB over AWGN channel. Our code over GF(8) of length 2268 bits, as shown in Fig. 2, achieves BLER of $10^{-4}$ at about 1.8 dB, which is 0.2 dB better than the (3060,1544) code reported in [25], even though the block length of our code is smaller.
- In Reference [15], a rate-1/2 128-ary (2032,1016) non-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Performance of rate-1/2 nonbinary QC-LDPC codes of various lengths over AWGN channel. The Shannon limit for AWGN channel is 0.188 dB.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Performance of rate-1/2 nonbinary QC-LDPC codes of various lengths over Rayleigh fading channel. The Shannon limit for Rayleigh fading channel is 1.830 dB.}
\end{figure}

A. Codes of Rate 1/2

The rate-1/2 mother matrix is shown in (12) which has a near-regular profile with $t = 3$. The rightmost 6 columns correspond to parity check symbols and are used to facilitate linear time encoding [7]. We choose the 'Design 4' listed in Table I and simulate codes with symbol length of 228, 492, 756, 1596 and 2496 over GF(8), whose girths are lower bounded by 6, 8, 10, 12 and 12, respectively. All the parity check matrices constructed share the same shift offset values. The bit lengths are 684, 1476, 2268, 4788 and 7488, respectively.

Figs. 2 and 3 show the performance of these codes over the AWGN and the independent and identically distributed (i.i.d.) Rayleigh fading channels respectively. For simulations over the AWGN channel, a coding gain of more than 1 dB is achieved at BLER of $10^{-4}$ as the block length increases from 684 bits to 7488 bits. Further, the code of length 7488 bits is only 1.2 dB away from the Shannon limit at BLER of $10^{-4}$. For simulations over the i.i.d. Rayleigh fading channel, a coding gain of more than 1.5 dB is achieved at BLER of $10^{-4}$ as the block length increases from 684 bits to 7488 bits. Further, the code of length 7488 bits is only 1.6 dB away from the Shannon limit at BLER of $10^{-4}$.

For simulations we run until 20 block errors are found or 1,000,000 blocks are transmitted. Performance in terms of block-error-rate (BLER) is reported.

 decoding while avoiding multiplication and normalization operation in the variable node decoding.

For simulations, the maximum number of iterations is set to be 80. Binary phase shift keying (BPSK) modulation is used. We consider both AWGN and Rayleigh fading channels. For all simulations we run until 20 block errors are found or 1,000,000 blocks are transmitted. Performance in terms of block-error-rate (BLER) is reported.
binary QC-LDPC code has been constructed, which has block length of 14224 bits. This code achieves BLER of $10^{-8}$ at about 2.0 dB over AWGN channel. Our code over GF(8) of length 7488 bits, as shown in Fig. 2, achieves BLER of $10^{-4}$ at about 1.4 dB, which is 0.6 dB better than the 128-ary (2032,1016) code reported in [15], even though the block length of their code is about two times larger than that of our code.

B. Codes of Rate 3/4

A mother matrix of size $6 \times 24$ with mean column weight 2.75 and $t = 3$ is shown in (13). The rightmost 6 columns correspond to parity check symbols and are used to facilitate linear time encoding [7]. The design results are listed in Table II. The ‘Design 1’ is obtained considering only one target girth $g = 6$ whereas the ‘Design 2’ is obtained considering two target girths 6 and 8. $\Gamma_{\max}$ is set to 20 in the two designs.

We choose the ‘Design 2’ listed in Table II and simulate codes with symbol length of 864, 1080, 2016 and 3024 over GF(8), whose girths are lower bounded by 6, 8, 8 and 8 respectively. All the parity check matrices constructed share the same shift offset values. The bit lengths are 2592, 3240, 6048, 9072 respectively.

Fig. 4 shows the performance of these codes over both AWGN and i.i.d. Rayleigh fading channels. For simulations over the AWGN channel, a coding gain of more than 0.4 dB is achieved at BLER of $10^{-4}$ as the block length increases from 2592 bits to 9072 bits. Further, the code of length 9072 bits is only 0.9 dB away from the Shannon limit at BLER of $10^{-4}$. For simulations over the i.i.d. Rayleigh fading channel, a coding gain of more than 0.8 dB is achieved at BLER of $10^{-4}$ as the block length increases from 2592 bits to 9072 bits. Further, the code of length 9072 bits is only 1.6 dB away from the Shannon limit at BLER of $10^{-4}$.

Compared with existing QC-LDPC codes in the literature, codes constructed by the proposed method achieve performance gains. Specifically, we have the following observations.

- In Reference [25], a rate-0.75 (8176,6162) binary QC-LDPC code has been constructed, which has block length of 8176 bits. This code achieves BLER of $10^{-4}$ at about 2.8 dB over AWGN channel. Our code over GF(8) of length 6048 bits, as shown in Fig. 4, achieves BLER of $10^{-4}$ at about 2.6 dB, which is 0.2 dB better than the (8176,6162) code reported in [25], even though the block length of our code is smaller.

- In Reference [15], a rate-0.75 256-ary (4080,3063) non-binary QC-LDPC code has been constructed, which has block length of 32640 bits. This code achieves BLER of...
The Shannon limit for AWGN channel is 1.6 dB and the Shannon limit for Rayleigh fading channel is 4.9 dB.

Fig. 4. Performance of rate-3/4 nonbinary QC-LDPC codes of various lengths. The Shannon limit for AWGN channel is 1.6 dB and the Shannon limit for Rayleigh fading channel is 4.9 dB.

\[ \text{Block Error Rate (dB)} \]

<table>
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<tr>
<th>Block Error Rate (dB)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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</thead>
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<td>4374</td>
<td>6480</td>
<td>8748</td>
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<td></td>
<td></td>
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</tbody>
</table>

\[ \text{Fig. 5. Performance of rate-8/9 nonbinary QC-LDPC codes of various lengths. The Shannon limit for AWGN channel is 3.1 dB and the Shannon limit for Rayleigh fading channel is 8.6 dB.} \]

\[ \text{Block Error Rate (dB)} \]

<table>
<thead>
<tr>
<th>Block Error Rate (dB)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWGN</td>
<td>2106</td>
<td>4374</td>
<td>6480</td>
<td>8748</td>
</tr>
</tbody>
</table>

\[ \text{C. Codes of Rate 8/9} \]

A mother matrix of size $6 \times 54$ with mean column weight 2.8333 and $t = 3$ is shown in (14). The rightmost 6 columns correspond to parity check symbols and are used to facilitate linear time encoding [7]. The design results for a target girth of 6 are listed in Table III with $\Gamma_{\text{max}}$ set to 15.

From Table III, we simulate codes with symbol length of 702, 1458, 2160, and 2916 over GF(8), whose girths are lower bounded by 4, 6, 6 and 6 respectively. All the parity check matrices constructed share the same shift offset values. The bit lengths are 2106, 4374, 6480, and 8748, respectively.

Fig. 5 shows the performance of these codes over both AWGN and i.i.d. Rayleigh fading channels. For simulations over the AWGN channel, a coding gain of more than 0.55 dB is achieved at BLER of $10^{-4}$ as the block length increases from 2106 bits to 8748 bits. Further, the code of length 8748 bits is only 0.8 dB away from the Shannon limit at BLER of $10^{-4}$. For simulations over the i.i.d. Rayleigh fading channel, a coding gain of more than 1.15 dB is achieved at BLER of $10^{-4}$ as the block length increases from 2106 bits to 8748 bits. Further, the code of length 8748 bits is only 1.8 dB away from the Shannon limit at BLER of $10^{-4}$.

Compared with existing nonbinary LDPC codes in the literature, nonbinary QC-LDPC codes constructed by the proposed method achieve similar performance. Specifically, we have the following observations.

- In Reference [5], a rate-8/9 nonbinary LDPC code of length 2016 bits over GF(8) achieves BLER of $10^{-4}$ at about 4.3 dB over AWGN channel. Our code over GF(8) of length 2106 bits, as shown in Fig. 5, has similar performance as the code reported in [5] at BLER above $10^{-4}$.
- In Reference [5], a rate-8/9 nonbinary LDPC code of length 4000 bits over GF(8) achieves BLER of $10^{-4}$ at about 4.1 dB over AWGN channel. Our code over GF(8) of length 4374 bits, as shown in Fig. 5, has similar performance as the code reported in [5] at BLER above $10^{-4}$.

Compared with the Shannon limit, we have the following results on different code rates.

- For rate 1/2, at BLER of $10^{-4}$ the proposed code of length 7488 bits is 1.2 dB from the Shannon limit of AWGN channel and is 1.6 dB from the Shannon limit of i.i.d. Rayleigh fading channel.
- For rate 3/4, at BLER of $10^{-4}$ the proposed code of length 9072 bits is 0.9 dB from the Shannon limit of AWGN channel and is 1.6 dB from the Shannon limit of i.i.d. Rayleigh fading channel.
- For rate 8/9, at BLER of $10^{-4}$ the proposed code of length 8748 bits is 0.8 dB from the Shannon limit of AWGN channel and is 1.8 dB from the Shannon limit of i.i.d. Rayleigh fading channel.

Hence the proposed nonbinary QC-LDPC codes can achieve close-to-capacity performance for a wide range of code rates (from 1/2 to 8/9). To further close the gap, one can either move to high order Galois field or fine tune the column weight...
very good performance over correlated fading channels. We see that the proposed nonbinary QC-LDPC codes have channels with the product of maximum Doppler shift simulations is Jakes’ model [32]. We consider two correlated fading channels. The correlated channel model used for sim-
binary QC-LDPC codes over binary input correlated Rayleigh fading channel. The Shannon limit for rate 1/2 is 1.830 dB.

Fig. 6. Performance of rate-1/2 nonbinary QC-LDPC codes over correlated fading channel. The Shannon limit for rate 1/2 is 1.830 dB.

D. Performance Over Correlated Fading Channel

Next we investigate the performance of the proposed non-
binary QC-LDPC codes over binary input correlated Rayleigh fading channels. The correlated channel model used for sim-
ulations is Jakes’ model [32]. We consider two correlated channels with the product of maximum Doppler shift $f_d$ and sampling time interval $T$ being 0.01 and 0.1.

Fig. 6 shows the performance of the rate-1/2 codes of length 684 and 7488 bits presented in Section IV-A. Fig. 7 shows the performance of the rate-8/9 codes of length 2106 and 8748 bits presented in Section IV-C. Several observations can be made.

- For the rate-1/2 code of length 684 bits, at BLER of $10^{-4}$ there is only 0.9 dB loss when the product $f_d T$ is 0.1 compared with the i.i.d. Rayleigh fading channel.
- For the rate-1/2 code of length 7488 bits, at BLER of $10^{-4}$ there is only 1.3 dB loss when the product $f_d T$ is 0.1 compared with the i.i.d. Rayleigh fading channel. Another 0.4 dB penalty is paid when the product $f_d T$ further decreases from 0.1 to 0.01.
- For the rate-8/9 code of length 2106 bits, at BLER of $10^{-4}$ there is only 0.7 dB loss when the product $f_d T$ is 0.1 compared with the i.i.d. fading channel.
- For the rate-8/9 code of length 8748 bits, at BLER of $10^{-4}$ there is only 1 dB loss when the product $f_d T$ is 0.1 compared with the i.i.d. Rayleigh fading channel. Another 0.8 dB penalty is paid when the product $f_d T$ further decreases from 0.1 to 0.01.

We see that the proposed nonbinary QC-LDPC codes have very good performance over correlated fading channels.

V. DESIGN OF MULTIPLE NONBINARY QC-LDPC CODES

The proposed method can be readily extended to design multiple codes together, including rate-compatible LDPC codes using puncturing/ extending or shortening/appending techniques [33]–[36] and multiple-rate LDPC codes with constant blocklength [37], [38]. For illustration we apply the proposed method to design rate-compatible nonbinary QC-LDPC codes of various lengths using shortening/appending techniques. With respect to shortening/appending, lower-rate codes are obtained by shortening higher-rate codes [35], that is, some information symbols of higher-rate codes are set to be zeros for lower-rate codes (This is referred to as information nulling in [35]). Equivalently higher-rate codes are obtained by appending more information symbols to lower-rate codes.

A. Rate-Compatible Nonbinary QC-LDPC Codes of Various Lengths

The same procedure as presented in Section II can be applied to design rate-compatible nonbinary QC-LDPC codes of various lengths using the shortening/appending technique. The difference is that there is only one code under consideration in Section II whereas here we have to consider multiple codes. The following three steps are used to construct nonbinary rate-compatible QC-LDPC codes using the shortening/appending technique.

- **Step 1**: Construct mother matrices for all code rates
- **Step 2**: Specify the shift offset values for all code rates. After expanding, we obtain a binary PCM for each code rate.
- **Step 3**: Specify the nonzero elements for all code rates by replacing each “1” in its binary PCM by an element from GF$(q)\setminus\{0\).
When designing the mother matrices for a bunch of rates, we can first design the mother matrix of lowest rate, then apply the information appending technique sequentially to obtain the mother matrices of all other rates (This amounts to adding columns to the mother matrix). When searching for the shift offset values we need to take a bunch of codes into consideration. For each individual code, there exists a parameter $L_{\text{min}}(j)$ associated with each of its target girth $g$ in the $j$-th step of the search procedure. Instead of a single $L_{\text{min}}(j)$, the average value of $L_{\text{min}}(j)$ across all target girths for all codes is used as the criterion to select $l_j$. When determining nonzero elements from a Galois field, codes of different rates can share the same $\rho_{ij}$—the nonzero element in the first row corresponding to $\mathbf{P}^{a_{ij}}$, in positions where they share the same nonzero entry in their mother matrices.

### B. An Example Design

We consider a design with five rates—1/2, 2/3, 3/4, 5/6 and 8/9. The mother matrix $\mathbf{M}(\mathbf{H})$ of size $6 \times 54$ is shown in (15), where the first $6d$ columns form the PCM of size $6 \times 6d$ for the code of rate $r = (d-1)/d$, $d = 2, 3, 4, 6, 9$. The mean column weights are 2.5, 2.667, 2.75, 2.8333 and 2.8333 respectively. The 6 columns inside the dashed box correspond to parity check symbols and are used to facilitate linear time encoding for all codes [7]. Now when searching for the solutions of $l_j$ we need to consider a set of codes instead of a single code. After obtaining the solutions of $l_j$ for a low rate code, these values are fixed while continuing searching for residual $l_j$ for the next-higher-rate code. The design results are listed in Table IV with $\Gamma_{\text{max}}$ set to 40.

We have simulated two sets of codes with $L$ set to 30 and 60 respectively. With $L = 30$ the constructed codes over GF(8) have block lengths of 1080, 1620, 2160, 3240 and 4860 bits respectively and their girths are lowered bounded by 6, 6, 4, 4 and 4 respectively. With $L = 60$ the constructed codes over GF(8) have block lengths of 2160, 3240, 4320, 6480 and 9720 bits respectively and their girths are lowered bounded by 10, 8, 8, 6 and 6 respectively. Fig. 8 shows the performance of these codes over the AWGN channel and Fig. 9 shows the performance of these codes over the i.i.d. Rayleigh fading channel.

Figs. 8 and 9 show that there is a coding gain of 0.2 to 0.5 dB when the block lengths get doubled at BLER of $10^{-4}$. Consider the three code rates 1/2, 3/4 and 8/9. Comparing the simulation results in Figs. 8 and 9 with those designed for each code rate separately in Sections IV.A–C, we can see that codes of the same rate and similar block lengths have similar performance over both the AWGN and i.i.d. Rayleigh fading channels. Further, it can be seen from Fig. 8 that at BLER of $10^{-4}$ the five codes with $L = 60$ are about 1.6, 1.3, 1.1, 0.9 and 0.7 dB from the Shannon limit of the AWGN channel. It can be seen from Fig. 9 that at BLER of $10^{-4}$ the five codes with $L = 60$ are about 2.2, 1.95, 1.9, 1.9 and 1.7 dB from the Shannon limit of the Rayleigh fading channel.

### VI. Conclusion

In this paper, we proposed a novel method to design nonbinary QC-LDPC codes that have large girth and flexible code lengths, including codes of multiple rates. Simulations over both AWGN and Rayleigh fading channels showed that the proposed codes perform very well, thus providing a family of codes suitable for practical adaptive systems.

### References


### Table IV

<table>
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<tr>
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<th>$l_{11} - l_{20}$</th>
<th>$l_{21} - l_{30}$</th>
<th>$l_{31} - l_{40}$</th>
<th>$l_{41} - l_{50}$</th>
<th>$l_{51} - l_{54}$</th>
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The Shannon limits for rates 1/2, 2/3, 3/4, 5/6 and 8/9 over AWGN channel are 0.188, 1.0, 1.6, 2.4 and 3.1 dB, respectively. Solid curves correspond to $L = 30$ whereas dashed curves correspond to $L = 60$.

![Fig. 8. Performance of rate-compatible nonbinary QC-LDPC codes of various lengths over AWGN channel.](image)

![Fig. 9. Performance of rate-compatible nonbinary QC-LDPC codes of various lengths over Rayleigh fading channel. The Shannon limits for rates 1/2, 2/3, 3/4, 5/6 and 8/9 over Rayleigh fading channel are 1.83, 3.65, 4.9, 6.7 and 8.6 dB, respectively. Solid curves correspond to $L = 30$ whereas dashed curves correspond to $L = 60$.](image)


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