Compressed Sensing for OFDM/MIMO Radar

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***Abstract*—In passive radar, two main challenges are: mitigating the direct blast, since the illuminators broadcast continuously, and achieving a large enough integration gain to detect targets. While the first has to be solved in part in the analog part of the processing chain, due to the huge difference of signal strength between the direct blast and weak target reflections, the second is about combining enough signal efficiently, while not sacrificing too much performance. When combining this setup with digital multicarrier waveforms like orthogonal frequency division multiplex (OFDM) in digital audio/video broadcast (DAB/DVB), this problem can be seen to be a version of multiple-input multiple-output (MIMO) radar. We start with an existing approach, based on efficient fast Fourier transform (FFT) operation to detect target signatures, and show how this approach is related to a standard matched filter approach based on a piece-wise constant approximation of the phase rotation caused by Doppler shift. We then suggest two more applicable algorithms, one based on subspace processing and one based on sparse estimation. We compare these various approaches based on a detailed simulation scenario with two closing targets and experimental data recorded from a DAB network in Germany.**

***Index Terms*—Multi-static radar, subspace algorithms, sparse estimation.**

I. Introduction

A. Passive Radar: Motivation & Challenges

In passive radar, illuminators of opportunity are used to detect and locate airborne targets. This is essentially the same as a bi-static radar setup, as sender and receiver are dis-located, and time difference of arrival (TDoA) measurements place targets on ellipses around the sender-receiver axis. It is the differences though, what makes passive radar attractive; i) as the illuminators are not part of the radar system, its presence is virtually undetectable; ii) illuminators of opportunity are often radio and tv stations, broadcasting in the VHF/UHF frequency bands otherwise not available to radar applications. The benefits of VHF/UHF radar are discussed in [1], [2]

Challenges connected to implementing a passive radar system are mostly due to using broadcast signals not under control for illumination. The transmitted signals are not known a priori, so a regular matched filter based receiver cannot be implemented easily. Second, although broadcast antennas are sectorized at times, since broadcast signals have to cover a large area, the send antennas are approximately isotropic and there is no significant transmitter gain. This leads to passive radars operating at a very low signal-to-noise ratio (SNR). Last, since the illumination is continuous, there is no easy time-domain way to separate the direct blast from target reflections, as is typically done in bi-static settings.

B. Current State-of-the-Art

Earlier systems working with analog broadcast (TV/FM) used the direct blast as a noisy template to implement an approximate matched filter [3]–[5]. Newly available *digital* broadcast systems allow passive radar receivers to perfectly reconstruct the transmitted signal after successful demodulation [6]–[9]. A practical concern is that received signals can have a dynamic range of easily 100 dB between direct blast and targets, which cannot be handled by analog-to-digital converters, and this makes additional analog pre-compensation of the direct blast necessary. A current state-of-the-art system has the following structure:

1) The broadcast signal is decoded and perfectly reconstructed based on the dominant direct blast.
2) Analog attenuation of the direct blast is used to bring the dynamic range below 70 dB.
3) The signal is divided into segments.
4) Matched filtering is performed efficiently in the frequency domain using the fast Fourrier transform (FFT).
5) A second Fourier transform in executed across segments to separate low SNR targets from the dominant direct blast based on their non-zero Doppler rate.

The output of such a processing chain are bi-static range and range-rate. This implementation is especially applicable in multi-carrier digital broadcast systems, such as digital audio/video broadcast (DAB/DVB), as the transmit signal is specifically designed for frequency domain equalization.

C. Our Work

We investigate passive radar starting from the DAB scenario considered in [7], [9]. The transmitted multi-carrier signal uses orthogonal frequency division multiplex (OFDM), which is especially amenable to the FFT based approach outlined above. Our contributions are the following,

1) We thoroughly derive the exact matched filter formulation and show that it is equivalent to the practical approach outlined above based on several approximations.
2) We suggest several super-resolution approaches, including sparse estimation (Basis Pursuit) and sub-space methods (MUSIC).

We use a detailed simulation study and experimental data from a recording campaign in Germany to test our approaches.

II. Signal Model

A. Transmitted Signal

The digital audio broadcast (DAB) standard [10] uses orthogonal frequency division multiplex (OFDM) using N frequencies that are orthogonal given a rectangular window of length T at the receiver,

$$x_i(t) = \sum_{n=-N/2}^{N/2-1} s_i[n] e^{j2\pi n \Delta f t} q(t). \quad (1)$$

Each block carries N data symbols $s_i[n]$; the frequencies are orthogonal because the frequency spacing is $\Delta f = 1/T$, whereby the transmitted waveform is extended periodically by $T_{\rm cp}$ to maintain a cyclic convolution with the channel, i.e.,

$$q(t) = \begin{cases} 1 & t \in [-T_{\rm cp}, T], \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

We define a symbol duration as $T' = T + T_{\rm cp}$. The broadcast signal is continuous as,

$$x(t) = \sum_{i=-\infty}^{\infty} x_i(t - iT'), \quad (3)$$

where the data symbols $s_i[n]$ vary with each block, but we assume they can always be decoded without error for our purposes. Some of the data symbols $s_i[n]$ might be deactivated for various reasons (protection of bandwidth edges, Doppler estimation, etc.) and, also, a complete Null symbol is inserted periodically for synchronization (all $s_i[n]$ are zero).

B. Target/Channel Model

When a waveform is emitted by a transmitter, we expect to receive a direct blast as well as reflections off targets that are characterized by a delay τ and a Doppler shift f_d. We adopt a narrow-band model here where a signal $x(t)$ of center frequency f_c, will only experience a phase rotation in base band, i.e., a Doppler shift $f_d = af_c$; time compression or dilation is assumed negligible and a is related to the range-rate. Indexing the return of the pth arrival and its associated Doppler shift and delay, the received signal in baseband is

$$y(t) = \sum_p A_p e^{j2\pi a_p f_c t} x(t - \tau_p) + w(t). \quad (4)$$

In most radar applications, the delay τ and Doppler shift af_c are assumed to be constant, but we will revisit this later when we discuss the integration time of receiver processing.

C. Matched Filter Receiver

The standard approach to "search" for targets is to use a bank of correlators tuned to the waveform given a certain Doppler shift and delay, i.e., a matched filter. As an example, the kth correlator will produce for every $\hat{\tau}$ and a fixed Doppler shift $\hat{a}_k f_c$,

$$z_k(\hat{\tau}) = \int_0^{T_i} e^{-j2\pi \hat{a}_k f_c t} x^*(t - \hat{\tau}) y(t)\, dt. \quad (5)$$

Due to limitations in signal processing complexity, the delay dimension $\hat{\tau}$ is usually only evaluated at discrete points.

As the transmission $x(t)$ is divided into blocks of length T', see (3), each consisting of a signal of length T and a cyclic extension of length $T_{\rm cp}$, assuming that the largest possible delay is smaller than the cyclic extension $\tau_{\max} < T_{\rm cp}$, the correlator in (5) can be implemented as,

$$z_k(\hat{\tau}) = \sum_{i=0}^{T_i/T'} \int_{iT'}^{iT'+T} e^{-j2\pi \hat{a}_k f_c t} x^*(t - \hat{\tau}) y(t)\, dt \quad (6)$$

$$= \sum_{i=0}^{T_i/T'} e^{-j2\pi \hat{a}_k f_c iT'} z_k^{(i)}(\hat{\tau}). \quad (7)$$

The integration time T_i is chosen as an integer multiple of T', which means we coherently combine a certain number of OFDM blocks, and we define the correlator output of the ith block as,

$$z_k^{(i)}(\hat{\tau}) = \int_0^T e^{-j2\pi \hat{a}_k f_c t} x^*(t + iT' - \hat{\tau}) y(t + iT')\, dt. \quad (8)$$

For OFDM signals the block correlation operation in (8) can be efficiently implemented using the fast Fourier transform (FFT). This is further simplified since due to the cyclic prefix, the correlation operation is actually cyclic in an interval of length T. We write this as,

$$z_k^{(i)}(\hat{\tau}) = \int_0^T e^{-j2\pi \hat{a}_k f_c t} x_i^*(t - \hat{\tau}) y(t + iT')\, dt \quad (9)$$

$$= \sum_{n=-N/2}^{N/2-1} e^{j2\pi n \Delta f \hat{\tau}} s_i^*[n] \times \int_0^T e^{-j2\pi n \Delta f t} \left[e^{-j2\pi \hat{a}_k f_c t} y(t + iT') \right] dt \quad (10)$$

In words, there are four steps, corresponding to the parentheses, from inside out:

1) Compensation for the phase rotation in the time domain caused by the Doppler shift;
2) integration over t - in practice an FFT operation of the sampled signal - giving N outputs for each subcarrier;
3) compensation of the (assumed known) data symbols $s_i^*[n]$; and
4) inverse FFT operation across various delays.

The output will be correlation values for given delay $\hat{\tau}$ and Doppler $\hat{a}_k f_c$ for the ith OFDM block, the outputs for all blocks have to be combined as given in (7) to produce the final correlation value.

III. Signal Approximation and Efficient Implementation

A. Small Doppler Approximation

Since the target returns are of very low SNR, often the integration time is on the order of a second. This means that a very large number of OFDM blocks are combined to achieve a sufficient SNR for target detection, $T' \ll T_i$. When T' is small compared to the Doppler shifts, we can approximate the phase rotation within one OFDM block as constant,

$$e^{-j2\pi a_p f_c t} \approx e^{-j2\pi a_p f_c (i+1/2)T'} \forall t \in [iT', (i+1)T']. \quad (11)$$

Then the Doppler shift has to be estimated based on the increasing accumulated phase shift between consecutive blocks, and (10) can be simplified to

$$z_k^{(i)}(\hat{\tau}) = \sum_{n=-N/2}^{N/2-1} e^{j2\pi n \Delta f \hat{\tau}} s_i^*[n] \int_0^T e^{-j2\pi n \Delta f t} y(t+iT') dt \quad (12)$$

$$= \sum_{n=-N/2}^{N/2-1} e^{j2\pi n \Delta f \hat{\tau}} H_n^{(i)}, \quad (13)$$

where $H_n^{(i)}$ corresponds to the channel transfer function estimate of the nth frequency in the ith block (ignoring inter-carrier-interference (ICI)). We see that the output of all correlators for the ith block are identical, as there is no dependence on k. We simplify (7), by defining $z^{(i)}(\hat{\tau})$ without the index k, as only one correlator is necessary,

$$z_k(\hat{\tau}) = \sum_{i=0}^{T_i/T'} e^{-j2\pi \hat{a}_k f_c i T'} z^{(i)}(\hat{\tau}). \quad (14)$$

We see that the inverse Fourier transform of the channel transfer function, $z^{(i)}(\hat{\tau})$, is exactly the estimate of the channel impulse response, and the optimal combining across all i is a Fourier transform across packets for constant $\hat{\tau}$.

B. Efficient Implementation and Link to Array Processing

We can rewrite the matched filter output as

$$z_k(\hat{\tau}) = \sum_{i=0}^{T_i/T'} \sum_{n=-N/2}^{N/2-1} e^{j2\pi(i\hat{a}_k f_c T' + n\Delta f \hat{\tau})} H_n^{(i)}, \quad (15)$$

which is a two-dimensional DFT of the channel transfer function estimates, efficiently implemented as an FFT.

As the operation in (15) is identical to beamforming with a uniform rectangular array (URA), we take a closer look at the channel estimates. Assuming no noise and only a single target present with amplitude A_0, delay τ_0 and Doppler $a_0 f_c$, we calculate

$$H_n^{(i)} = s^*[n] \int_0^T e^{-j2\pi n \Delta f t} y(t+iT')\, dt \quad (16)$$

$$= A_0 \sum_{m=-N/2}^{N/2-1} s[m] s^*[n] e^{-j2\pi m \Delta f \tau_0}$$

$$\times \int_0^T e^{-j2\pi (n-m)\Delta f t} e^{j2\pi a_0 f_c (t+iT')}\, dt \quad (17)$$

Using the approximation in (11), all frequencies are orthogonal except $m = n$ (no ICI),

$$H_n^{(i)} \approx A_0 T \left|s[n]\right|^2 e^{j2\pi(i a_0 f_c T' - n\Delta f \tau_0)}, \quad (18)$$

and the channel estimates have the same form as the receiver elements of an URA. This means the problem, given the small-Doppler approximation, can be seen as sinusoids impinging on an URA with equivalent element spacing of T' in Doppler and Δf in delay, the total array aperture size is T_i and B respectively.

IV. Implementation of Algorithms

A. Matched Filter / Beamforming

This approach has been used in previous work, e.g. [7], and follows exactly our formulation in (15). It needs to be pointed out that the ambiguity function of the target will be similar to a sinc-shape in both delay and Doppler, due to the rectangular array. Since even after analog attenuation the direct blast is still 60 dB above the target signatures, the sinc sidelobes will lead to strong clutter in the non-zero Doppler region. Therefore a second digital attenuation of the direct blast has to be included, either via a simple high-pass filter or by subtracting identified targets of zero Doppler before further processing.

Approximately 2 dB can be gained choosing a fine delay and/or Doppler resolution, as is common when evaluating the ambiguity function. This can be easily implemented by using zero-padded FFT operation in (15).

B. 2D-FFT MUSIC

As outlined above, the signal model is completely equivalent to the one of p wavefronts impinging on a grid of sensors, see (18). For being able to use subspace methods we apply spatial smoothing, i.e., using shifted versions of rectangular subarrays in order to construct independent snapshots. As the dimension of the snapshots is much larger than the dimension of the signal subspace, we express the MUSIC pseudo spectrum in terms of the signal subspace. Stacking the signal subspace eigenvectors in matrices of dimensions equal to the subarrays, the computation essentially boils down to a two-dimensional DFT of these signal subspace matrices. Therefore, the main part of the computational effort (apart from the short SVD) is mainly N_{ev} times that of the matched filter approach, where N_{ev} is the dimension of the signal subspace. We use the same pre-processing step for removing the direct blast as in the beamforming approach.

C. Compressed Sensing

The algorithm works as follows; all observations are stacked into an observation vector $\mathbf{b}$, which are assumed to be a superposition of additive noise $\mathbf{w}$ and an unknown number of point targets. As in our matched filter operation, the waveforms given a certain pair of τ_p and $a_p f_c$ are known. These waveforms form the columns of a dictionary $\mathbf{A}$, which is used to construct a superposition using weights stacked in a vector $\mathbf{x}$. The corresponding model is

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w}. \quad (19)$$

Since $\mathbf{x}$ is of dimension enumerating all possible combinations of τ_p and $a_p f_c$, most entries will be zero, as there is only a

carrier frequency	f_c	227.36 MHz
subcarrier spacing	Δf	1 kHz
no. subcarriers	N	1537
bandwidth	B	1.537 MHz
symbols length	T	1 ms
cyclic prefix	T_{cp}	0.246 ms
block length	T'	1.246 ms
blocks per frame	L	76
Null symbol	T_{NULL}	1.297 ms
frame duration	T_F	96 ms

TABLE I
OFDM SIGNAL SPECIFICATIONS OF DAB ACCORDING TO ETSI 300 401 [10]

much smaller number of point targets present. As a link to the matched filter operation,

$$\hat{\mathbf{x}}_{MF} = \mathbf{A}^H \mathbf{b}, \tag{20}$$

is exactly the output of matched filters corresponding to the entries in $\mathbf{A}$. When no zero-padding is used in the FFT operations of the matched filter, this corresponds to picking the columns of $\mathbf{A}$ such that the matrix is orthogonal. This means we adopted the model to make the estimator optimal. Since generally targets don't appear on neat grid spacings, we expect to gain performance by considering more possible pairs of τ_p and $a_p f_c$. This leads to a non-invertible $\mathbf{A}$, which is addressed by regularization using the l_1-norm,

$$\hat{\mathbf{x}}_{BP} = \arg\min_{\mathbf{x}} |\mathbf{A}\mathbf{x} - \mathbf{b}|^2 + \lambda |\mathbf{x}|_1, \tag{21}$$

where λ determines the "sparsity" of the solution.

V. Numerical Simulation

To compare the various suggested algorithms, we use numerical simulation, where we can control our assumptions. The signal is simulated as

$$y(t) = \sum_p A_p x\left(t - \tau_p(t)\right) + w(t) \tag{22}$$

where $\tau_p(t)$ is the exact bi-static delay. The target is assumed to be a point target for now, although the auto-correlation of $A_p(t)$ from OFDM block to block is modeled based on a five-point extended target assumption. The signal parameters correspond to DAB as given in [10], for convenience the OFDM parameters are listed Tab. I. For the simulation we run 200 DAB frames of 76 OFDM blocks each plus one Null symbol, where the Null symbol length is chosen such that the total frame duration adds to 96 ms; with this the simulation covers 19.2 s. In the simulation, two targets are flying almost to parallel, closing in slowly. All algorithms integrate over one DAB frame, rendering an integration gain of about 50 dB ($T_F \cdot B$).

The first interesting observation is in Fig. 1, where the beamforming clearly shows the target RCS fluctuation over the simulation time, featuring several fades. In Fig. 2 we see that the advanced algorithms improve resolution significantly, whereby the Basis Pursuit has higher complexity, but is the only approach that can handle the direct blast.

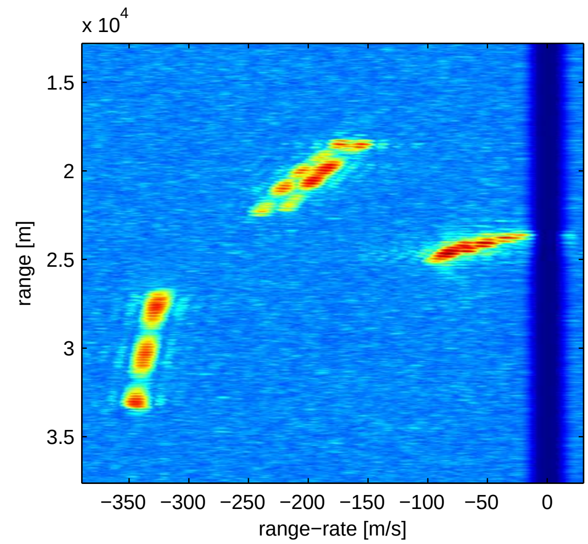


Fig. 1. Super-imposed results of simulation scenario with two closing targets using beamforming; the targets are clearly visible, although not well separated; there is a gap at zero Doppler due to the suppression of the direct blast; strong target RCS fluctuations can be observed over time.

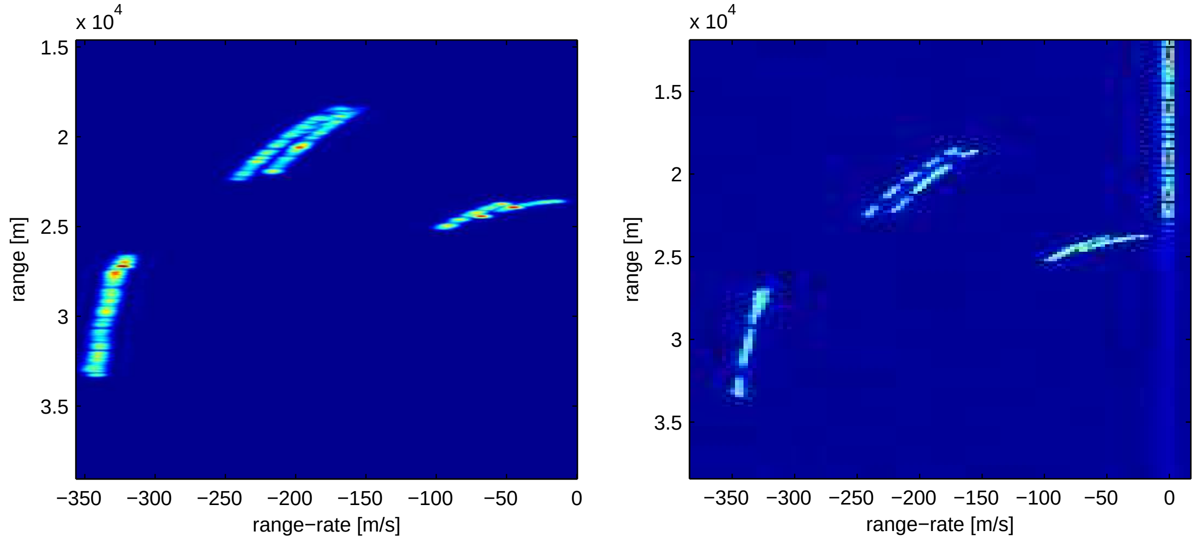
VI. Experimental Study

The experimental data was recorded by the German Research Establishment (FGAN) in using the local DAB network, which follows the same specifications as in Tab. I. We were able to test our algorithms on approximately 60 s of data. Unfortunately no ground truth is available at this point, but it is still interesting to see how the algorithms perform, so we include results for illustration.

We include results for beamforming and Basis Pursuit, the MUSIC approach was not working with the experimental data at the time of publication. The results are in Fig 3, where the beamforming applies a higher resolution and combines two DAB frames for extra integration gain. Although the same is possible for the Basis Pursuit approach, complexity is a limiting factor, as to process one frame of data takes several minutes vs. several seconds for the FFT implementation of beamforming.

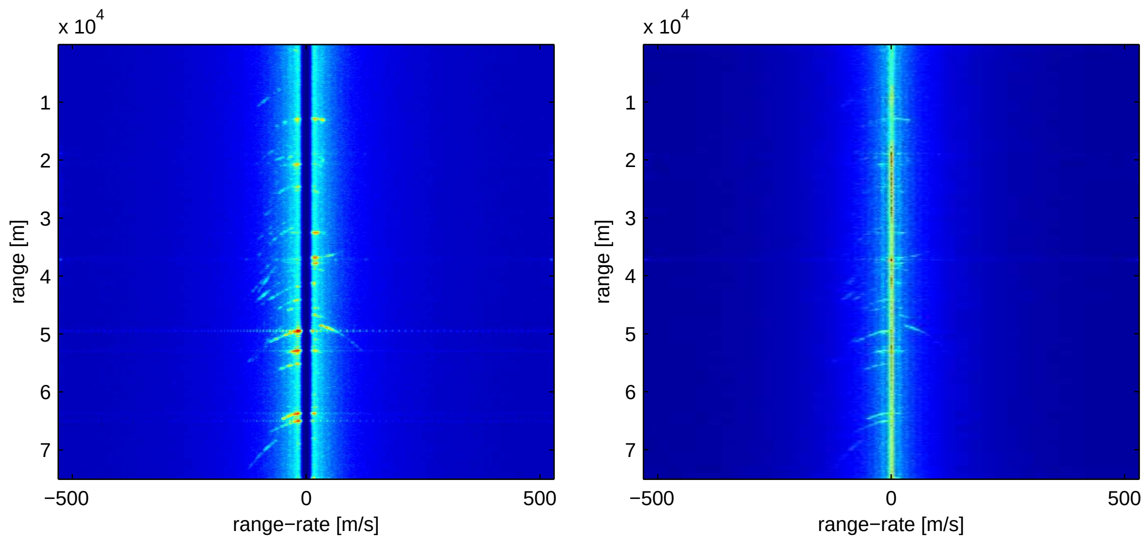
VII. Conclusion

We presented several algorithms to process OFDM based passive radar data, based on matched filter, subspace, and sparse estimation. In our simulation scenario, the latter two algorithms show significantly improved resolution, but with practical improvements like direct blast suppression, the matched filter approach can perform reasonably as well. In the experimental data, due to the extremely low target SNR, the ability to handle enough data for a large integration gain efficiently seems to be the dominant factors. We will continue to pursue advanced signal processing schemes for OFDM passive radar.



(a) MUSIC (b) Basis Pursuit

Fig. 2. Simulation scenario with two closing targets using 2D-FFT MUSIC and Basis Pursuit; both methods show signigicant improvement over beamforming.



(a) Beamforming (b) Basis Pursuit

Fig. 3. Results for experimental data recorded by the German Research Establishment (FGAN); although no ground truth is available, numerous tracks can be observed.

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