Signal Extraction Using Compressed Sensing for Passive Radar with OFDM Signals

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***Abstract*—Passive radar is a concept where possibly multiple non-cooperative illuminators are used in a multi-static setup. A freely available signal, like radio or television, is decoded and used to identify moving airborne targets based on their Doppler shift. New digital signals, like Digital Audio/Video Broadcast (DAB/DVB), are excellent candidates for this scheme, as they are widely available, can be easily decoded, and employ orthogonal frequency division multiplex (OFDM), a multicarrier transmission scheme based on channel equalization in the frequency domain using the Fast Fourier Transform (FFT). After successfully decoding the digital broadcast, the channel estimates can be used to estimate targets' bi-static range and range-rate by separating different multi-path components by their delay and Doppler shift. While previous schemes have simply projected available measurements onto possible Doppler shifts, we employ Compressed Sensing, a type of sparse estimation. This way we can enhance separation between targets, and by-pass additional signal processing necessary to determine the actual target within a "blotch" of signal energy smeared across different delays and Doppler frequencies.**

I. Introduction

In passive radar, one or several illuminators of opportunity are used in a multi-static setting to detect and track airborne targets. The general multi-static setup, see Fig. 1, is modified in the sense that we still measure bi-static range and range-rate, but the illuminators (senders) are not only dislocated from the receiver, but are also non-cooperative.

Although the concept of passive radar has been known for a long time (and has been used extensively by the military/intelligence communities), not much was published in the open literature until the last decade [1]. Now, technological advances have made this topic interesting to a broader audience [2], [3], [4], [5], mainly due to the widespread availability of high-performance signal processing equipment (see detailed discussion in [2]).

The concept of passive radar has appeal in many ways, e.g., multi-static benefits: it brings new electronic countermeasures challenges and the receiver operates in an entirely passive fashion. Furthermore, since the illuminators seem unsuspecting, the surveillance is in a sense covert and operation in the radio/television VHF/UHF frequency bands needs no frequency allocation, gives frequency diversity and has the potential to counter stealth efforts and to detect low-flying targets beyond the horizon [6], [7].

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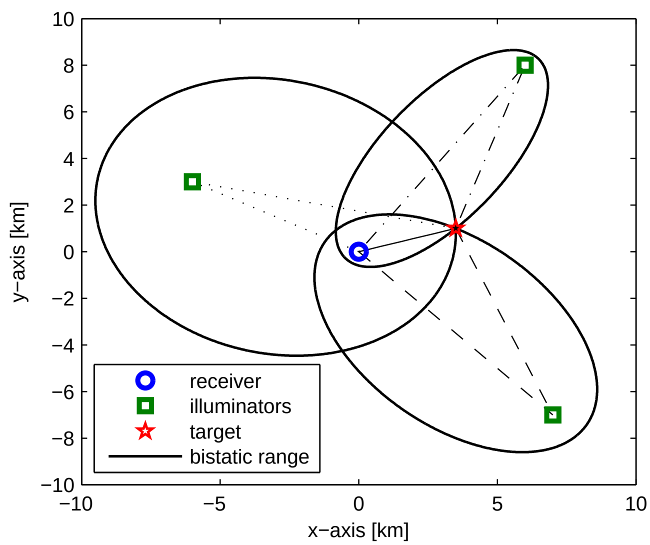

Fig. 1. In passive radar, illuminators of opportunity are used in a multi-static setup; weak target signatures can be extracted from the dominating "direct blast" radio/television signal based on their Doppler frequency, rendering bi-static range and range-rate information.

Since terrestrial radio/television signals were dominantly of analog modulation type, signal extraction of precise target information was extremely difficult. With the advent of widespread digital radio/television broadcasting, i.e., Digital Audio/Video Broadcast (DAB/DVB), a new generation of signal processing tools can be utilized to extract target information. This comes with the following advantages: i) both DAB and DVB employ orthogonal frequency division multiplexing (OFDM) as their modulation scheme[1]; ii) the digital signal can be easily decoded and then treated as a known signal for target signal extraction; iii) DAB and DVB are both wideband signals, giving ample frequency diversity; and iv) terrestrial broadcasting now operates in a single frequency network (SFN), i.e., all illuminators transmit in the same frequency band and their signals are combined at the receiver using the special OFDM structure – this results in each target being illuminated several times during each scan, while operating just in a single frequency band.

While the previously mentioned advantages are encouraging, they also lead to new implementation challenges. In OFDM, usually all arriving paths are assumed to have the

[1]OFDM is a multi-carrier scheme that uses the efficient Fast Fourier Transform (FFT) for frequency domain channel equalization; this greatly simplifies resolving multi-path arrivals (like the target reflections)

same Doppler frequency, making estimation of multipath arrivals a one-dimensional problem – now with additional varying Doppler frequencies, a two-dimensional plane has to be estimated. Also, the SFN generates one bi-static range and range-rate measurements per illuminator, per target at each scan; this offers more target information, but it introduces an additional association problem since we are not able to differentiate between the various illuminators. The association problem is not addressed in this paper, but can be handled within a tracking algorithm, e.g., see [5], [8].

In this paper, we want to focus on the challenge of extracting target information from the OFDM signal. To differentiate the weak target signatures, it is necessary to match the received signal to tentative range/range-rate combinations, corresponding to target detections. This is challenging, as to have a sufficient resolution in both range and range-rate, a very large number of combinations, many of which are highly correlated, have to be evaluated. This can be posed as a sparse estimation problem, as the number of tentative range/range-rate combinations is large compared to the number of actual target signatures and we can arrive at a linear formulation. Efficient sparse estimation approaches are being investigated under the name Compressed Sensing (CS) [9], [10], [11], [12], where a signal is disected into a discrete representation using a large dictionary of non-orthogonal signals. Using this sparse estimation approach, we can extract the signal information resulting in range/range-rate measurements, which can be used in a tracking algorithm.

We define the following notation: vectors and matrices are denoted as lower and upper case bold letters respectively, e.g., $\mathbf{x}$, $\mathbf{A}$; the conjugate complex, transpose and hermitian are $\mathbf{x}^*$, $\mathbf{x}^T$ and $\mathbf{x}^H$ respectively.

The paper has the following structure: In Section II we introduce the OFDM signal model; in Section III we relate this to the target signatures and bring it into a linear problem formulation in terms of unknown range/range-rate received power; in Section IV we go over a sparse estimation algorithm and its efficient implementation; in Section V we look at some numerical results and finally conclude in Section VI.

II. OFDM Signal Model

In OFDM transmission, the signal is divided into blocks of N_s symbols, which are modulated onto the subcarriers. One OFDM symbol is of duration T, then the subcarrier spacing is $\Delta f = 1/T$ and the bandwidth is $B = N_s \Delta f$. Let f_c denote the carrier frequency, and $f_m = f_c + m\Delta f$ denote the frequency for the mth subcarrier in the passband, where $m \in \mathcal{S} = \{-N_s/2, \dots, N_s/2-1\}$. To handle the multipath propagation in radio channels and avoid inter-symbol-interference (ISI), a cyclic prefix (CP) is used. Let T_{cp} denote the length of the CP, and define a rectangular window of length $T_{\text{cp}} + T_0$ as

$$q(t) = \begin{cases} 1 & t \in [-T_{\text{cp}}, T_0], \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

One symbol in baseband can be written as

$$x(t) = \sum_{m\in\mathcal{S}} s[m] e^{j2\pi m\Delta f t} q(t) \tag{2}$$

and the corresponding passband signal is

$$\tilde{x}(t) = \text{Re}\left\{ e^{j2\pi f_c t} \sum_{m\in\mathcal{S}} s[m] e^{j2\pi m\Delta f t} q(t) \right\} \tag{3a}$$

$$= \text{Re}\left\{ \sum_{m\in\mathcal{S}} s[m] e^{j2\pi f_m t} q(t) \right\}, \tag{3b}$$

where $s[m]$ is the transmitted symbol on the mth subcarrier.

The channel impulse response for a multipath radio channel can be described by

$$c(\tau, t) = \sum_p A_p \delta\left(\tau - \tau_p(t)\right), \tag{4}$$

where A_p is the path amplitude and $\tau_p(t)$ is the time-varying path delay caused by a moving sender, receiver or target. In this setup we assume both the senders and the receiver to be stationary, so all direct-blast arrivals have zero Doppler. We assume that for the duration of a number of OFDM symbols the Doppler-causing speeds can be assumed to be constant,

$$\tau_p(t) \approx \tau_p - a_p t, \tag{5}$$

where $a_p = \dot{r}_p / c$ and $\dot{r}_p$ is the range-rate of the pth path and c the speed of light. We sort the paths by their Doppler causing speeds, where if $a_{p_1} = a_{p_2}$, then $p_1, p_2 \in \mathcal{P}_k$, leading to a modified formulation,

$$c(\tau, t) = \sum_k \sum_{p\in\mathcal{P}_k} A_p \delta\left(\tau - \tau_p + a_k t\right). \tag{6}$$

When the passband signal in (3b) goes through the channel described in (6), we receive:

$$\tilde{y}(t) = \text{Re}\Bigg\{ \sum_k \sum_{m\in\mathcal{S}} s[m] e^{j2\pi f_m (1+a_k) t} \times \sum_{p\in\mathcal{P}_k} A_p q\big((1+a_k)t - \tau_p\big) e^{-j2\pi f_m \tau_p} \Bigg\} + \tilde{w}(t), \tag{7}$$

where $\tilde{w}(t)$ is additive noise. Define $\tau_{\max} = \max_p \tau_p$, which is usually less than the CP length T_{cp}. Using the definition of $q(t)$ in (1), we obtain

$$\tilde{y}(t) = \text{Re}\left\{ \sum_k \sum_{m\in\mathcal{S}} H_m^{(k)} s[m] e^{j2\pi f_m (1+a_k) t} \right\} + \tilde{w}(t) \quad t \in \mathcal{T}_{\text{c}} \approx \left[-(T_{\text{cp}} - \tau_{\max}), T\right], \tag{8}$$

where we define the channel transfer functions

$$C_k(f) = \sum_{p\in\mathcal{P}_k} A_p e^{-j2\pi f \tau_p} \tag{9}$$

and the frequency response on the mth subcarrier for each k as

$$H_m^{(k)} = C_k(f_m). \tag{10}$$

Converting the passband signal $\tilde{y}(t)$ to baseband, such that $\tilde{y}(t) = \text{Re}\left\{y(t) e^{j2\pi f_c t}\right\}$, we have:

$$y(t) = \sum_k \sum_{m\in\mathcal{S}} H_m^{(k)} s[m] e^{j2\pi(m\Delta f + a_k f_m)t} + w(t) \tag{11a}$$

$$\approx \sum_k e^{j2\pi a_k f_c t} \sum_{m\in\mathcal{S}} H_m^{(k)} s[m] e^{j2\pi m\Delta f t} + w(t), \tag{11b}$$

for $t \in \mathcal{T}_c$, where we used the narrowband approximated $a_k f_m \approx a_k f_c$ and $w(t)$ is the noise at baseband.

As expected for CP-OFDM, we observe a cyclic convolution between the signal and the channel in the specified interval, where each subcarrier is only multiplied by the corresponding frequency response. The narrowband approximation is very accurate in radio channels, since $(\max_m f_m - \min_m f_m)/f_c = B/f_c < 10^{-3}$, as the bandwidth is at most in the MHz, while the carrier is on the order of GHz.

The receiver correlates the received waveform with each subcarrier:

$$y_n = \frac{1}{T}\int_0^T e^{j2\pi n\Delta f t} y(t)\, dt \tag{12}$$

Inserting (11b) and carrying out the integration, we obtain

$$y_n = \sum_k \sum_{m\in\mathcal{S}} H_m^{(k)} s[m]\rho_{n,m}^{(k)} + w_n \tag{13}$$

where we define:

$$\rho_{n,m}^{(k)} = \frac{\sin(\pi[m-n+\epsilon_k])}{\pi[m-n+\epsilon_k]} \cdot e^{j\pi(m-n+\epsilon_k)} \tag{14}$$

and $\epsilon_k = a_k f_c T$. Rewriting (13) in vector form, we obtain:

$$\mathbf{y} = \sum_k \mathbf{\Gamma}(\epsilon_k)\mathrm{diag}(\mathbf{s})\mathbf{h}_k + \mathbf{w}, \tag{15}$$

where $[\mathbf{y}]_n = y_n$, $[\mathbf{\Gamma}(\epsilon_k)]_{nm} = \rho_{n,m}^{(k)}$, $[\mathbf{s}]_m = s[m]$, $[\mathbf{h}_k]_m = H_m^{(k)}$ and $[\mathbf{w}]_n = w_n$. Therefore we observe a linear superposition of the different $\mathbf{h}_k$, with dictionaries $\mathbf{\Gamma}(\epsilon_k)\mathrm{diag}(\mathbf{s})$ and the $\mathbf{h}_k$ contain the information about the targets we want to estimate.

When observing a series of packets, the phase shifts due to the Doppler frequencies accumulate, c.f. (11b); so the ith packet would have modified dictionaries as follows,

$$\mathbf{y}_i = \sum_k e^{j2\pi(i-1)a_k f_c T'}\mathbf{\Gamma}(\epsilon_k)\mathrm{diag}(\mathbf{s}_i)\mathbf{h}_k + \mathbf{w}, \tag{16}$$

where $T' = T + T_{\text{cp}}$ is the length of one OFDM symbol.

Using the definition of the frequency response,

$$H_m^{(k)} = \sum_{p\in\mathcal{P}_k} A_p e^{-j2\pi(m\Delta f + f_c)\tau_p} \tag{17a}$$

$$= \sum_{p\in\mathcal{P}_k} c_p e^{-j2\pi m\tau_p/T}, \tag{17b}$$

we define the complex target returns $c_p = A_p e^{-j2\pi f_c\tau_p}$, which also include the direct signal propagation. Putting this into a vector format, we have,

$$\mathbf{h}_k = \mathbf{F}_k^H \mathbf{c}_k, \tag{18}$$

with $[\mathbf{F}_k]_{nm} = e^{j2\pi m\tau_n/T}$, where τ_n is the nth element of $\mathcal{P}_k$ and $\mathbf{F}_k$ is of dimension $|\mathcal{P}_k| \times N_s$.

III. Signal Extraction for DAB/DVB-T

A. Narrowband Radio Model

We want to detect energy with non-zero Doppler frequencies, i.e., estimate the complex amplitudes $c_p = A_p e^{-j2\pi f_c\tau_p}$ belonging to some $\mathcal{P}_k$. Practically, we will later sample both τ and ϵ on a grid and estimate which grid points are non-zero.

In radio applications like DAB/DVB-T, the maximum normalized Doppler ϵ_k is limited by the range-rate and thereby the maximum speed. Assuming no targets are faster than 300 m/s, a carrier frequency of one GHz and a symbol duration of $T \approx 0.1$ ms, the maximum normalized Doppler is,

$$\epsilon_k = a_k f_c T = \dot{r}_k/c \cdot f_c T < \frac{300}{3\cdot 10^8}\cdot 10^9 * 10^{-4} = 0.1 \tag{19}$$

This keeps the matrix $\mathbf{\Gamma}(\epsilon_k)$ close to the identity matrix $\mathbf{I}_{N_s}$; additionally the paths reflected off targets are attenuated by about 10 dB with respect to the paths associated with the line of sight or reflections off large stationary objects like hills or buildings. Therefore, we approximate the marix $\mathbf{\Gamma}(\epsilon_k) \approx \mathbf{I}_{N_s}$. This leads to the following modified signal model,

$$\mathbf{y}_i = \sum_k e^{j2\pi(i-1)\epsilon_k T'/T}\mathrm{diag}(\mathbf{s}_i)\mathbf{F}_k^H\mathbf{c}_k + \mathbf{w}. \tag{20}$$

Further we assume a certain subset of each OFDM symbol $\mathbf{s}_i$ is known. This can be either pilot symbols or data symbols fed back from the decoding process, since we can assume the broadcast signal is using sufficient error-correcting codes to correctly decode the data. The elements of the vectors $\mathbf{y}_i$ corresponding to known symbols are compensated, while other components are dropped. This can be represented via a selector matrix $\mathbf{S}$, which is a diagonal matrix with $[\mathbf{S}]_{nn} = 1$ if $[\mathbf{s}]_n$ is known;

$$\bar{\mathbf{y}}_i = \sum_k e^{j2\pi(i-1)\epsilon_k T'/T}\mathbf{S}\mathbf{F}_k^H\mathbf{c}_k + \bar{\mathbf{w}}, \tag{21}$$

where $\bar{\mathbf{w}}$ is a noise vector of reduced size.

B. Sparse Estimation Problem Formulation

To arrive at a systematic problem formulation, we will estimate tentative c_p at a sampled grid of combinations between τ and ϵ. Since we can assume the CP is of sufficient length, we get,

$$\tau_p \in \{0,\, \Delta\tau,\, 2\Delta\tau, \ldots, T_{\text{cp}}\} \tag{22}$$

where $\Delta\tau$ is usually chosen as a fraction of the sampling time $1/(\alpha B)$ for $\alpha = 1, 2, \ldots$ and there are about $\alpha N_{\text{cp}} = \alpha B T_{\text{cp}}$ possible delays (N_{cp} is the length of the CP in samples). Furthermore, defining a maximum normalized Doppler $\epsilon_{\max}$,

$$\epsilon_k \in \{-\epsilon_{\max}, -\epsilon_{\max} + \Delta\epsilon, \ldots, \epsilon_{\max}\}, \tag{23}$$

and the Doppler spacing such that $2\epsilon_{\max}/(K-1) = \Delta\epsilon$. With this, all matrices $\mathbf{F}_k = \mathbf{T}^T\mathbf{F}$ are equal, with the DFT matrix $[\mathbf{F}_k]_{nm} = e^{j2\pi m(n-1)/\alpha N_s}$. This can be easily implemented via the FFT operation by inserting additional zero elements where needed, which we express via the zero insertion matrix $\mathbf{T}$.

Finally the signal model, using $i = 1, \ldots, I$ OFDM symbols, is given by:

$$\begin{bmatrix}\bar{\mathbf{y}}_1\\ \bar{\mathbf{y}}_2\\ \vdots\\ \bar{\mathbf{y}}_I\end{bmatrix} = \begin{bmatrix}\mathbf{S}\mathbf{F}^H\mathbf{T} & \cdots & \mathbf{S}\mathbf{F}^H\mathbf{T}\\ \nu_1\mathbf{S}\mathbf{F}^H\mathbf{T} & \cdots & \nu_K\mathbf{S}\mathbf{F}^H\mathbf{T}\\ \vdots & & \vdots\\ \nu_1^{I-1}\mathbf{S}\mathbf{F}^H\mathbf{T} & \cdots & \nu_K^{I-1}\mathbf{S}\mathbf{F}^H\mathbf{T}\end{bmatrix}\begin{bmatrix}\mathbf{c}_1\\ \vdots\\ \mathbf{c}_K\end{bmatrix} + \begin{bmatrix}\bar{\mathbf{w}}_1\\ \bar{\mathbf{w}}_2\\ \vdots\\ \bar{\mathbf{w}}_I\end{bmatrix} \tag{24}$$

where we define $\nu_k^{i-1} = e^{j2\pi(i-1)\epsilon_k T'/T}$ to abbreviate notation. This is a linear system of the general form $\mathbf{A}\mathbf{x}+\mathbf{w}=\mathbf{y}$, where we want to detect non-zero entries of the vector $\mathbf{x}$.

C. Solution via FFT based Projection

Current practice simply solves the problem by taking the following approach: the solution to $\mathbf{y}=\mathbf{A}\mathbf{x}+\mathbf{w}$ is simply the projection of the measurements onto all possible solution, $\hat{\mathbf{x}} = \mathbf{A}^H\mathbf{y}$ (equivalent to the correlation), which can be efficiently implemented via FFT operation. The dimensions of $\mathbf{A}$ are $(I\cdot\rho N_s)\times(K\cdot\alpha N_{\text{cp}})$, where ρ is the ratio of ones to zeros on the diagonal of matrix $\mathbf{S}$. Theoretically, if all data is known, $\mathbf{S}=\mathbf{I}_{N_s}$, only $K=I$ tentative ϵ_k are chosen with the right spacing, using $N_{\text{cp}}=N_s$ and $\alpha=1$, $\mathbf{A}$ is invertible and even hermitian. In this case the projection is optimal and can be efficiently implemented.

In practice signals have zero-tones, so even after data decoding $\mathbf{S}\neq\mathbf{I}_{N_s}$. Also to get a sufficient Doppler scale resolution, a large I is necessary, this increases complexity and conflicts with the assumption that the channel and target are constant during this time-period. This leads to the target signature being additionally “smeared” over several range/range-rate bins, as the target moves during one measurement cycle.

Instead, we would like to pose (24) as a sparse estimation problem, where $\mathbf{A}$ is a “fat” matrix, i.e., $(I\cdot\rho N_s)<(K\cdot\alpha N_{\text{cp}})$, and therefore there are many choices of $\mathbf{x}$ how to explain the observations $\mathbf{y}$. One possibility, e.g., would be the least squares solution, finding an $\mathbf{x}$ that satifies the equations and has minimum energy. This leads to many small, noise-like observations in $\mathbf{x}$ – instead we want to pose the problem as finding a solution $\mathbf{x}$ with the fewest possible non-zero entries. This is the focus of Compressed Sensing (CS) [10], [11], [12], which has been studied recently in many fields of signal processing.

IV. Orthogonal Matching Pursuit

A. Overview of the Algorithm

Orthogonal Matching Pursuit (OMP) is a greedy algorithm to solve the sparse estimation problem [9]. Given the observations

$$\mathbf{y}=\mathbf{A}\mathbf{x}+\mathbf{w}, \tag{25}$$

with $\mathbf{x}$ the desired unknown, $\mathbf{w}$ the noise and accordingly a signal-to-noise ratio (SNR) of $E\left\{|[\mathbf{y}]_k|^2\right\}/E\left\{|[\mathbf{w}]_k|^2\right\}$. We want to solve for the unknown $\mathbf{x}$ in the following sense,

$$\min_{\mathbf{x}} \operatorname{card}(\mathbf{x}), \quad \text{s.t. } |\mathbf{y}-\mathbf{A}\mathbf{x}|^2<\varepsilon \tag{26}$$

where “card” describes the cardinality operator, i.e., counting the number of non-zero entries in its argument $\mathbf{x}$, and ε is chosen depending on the SNR.

The algorithm goes through the following steps, starting with $\mathbf{x}=\mathbf{0}$, $\mathbf{r}_0=\mathbf{y}$, the current fitting error of the observations, $I_0=\emptyset$ the set of non-zero elements of $\mathbf{x}$: At the ith iteration, we choose a new index of a non-zero element of $\mathbf{x}$, b_i, as:

$$b_i=\arg\max_{k\notin I_{i-1}} \frac{|\mathbf{a}_k^H\mathbf{r}_{i-1}|^2}{\mathbf{a}_k^H\mathbf{a}_k}, \tag{27}$$

where $\mathbf{a}_k$ is the kth column of $\mathbf{A}$. Next b_i is added to the set $I_i=I_{i-1}\cup b_i$, and the new estimate of the non-zero entries of $\mathbf{x}$ is:

$$\hat{\mathbf{x}}_i=\arg\min_{\mathbf{x}_i}|\mathbf{y}-\mathbf{A}_{I_i}\mathbf{x}_i|^2=\left(\mathbf{A}_{I_i}^H\mathbf{A}_{I_i}\right)^{-1}\mathbf{A}_{I_i}^H\mathbf{y} \tag{28}$$

$\mathbf{A}_{I_i}$ is the matrix consisting of all columns $\mathbf{a}_k$ with $k\in I_i$, accordingly $\hat{\mathbf{x}}_i$ has only i elements. The new residual vector has to be calculated every time as

$$\mathbf{r}_i=\mathbf{r}_0-\mathbf{A}_{I_i}\hat{\mathbf{x}}_i, \tag{29}$$

and the algorithm stops when $|\mathbf{r}_i|^2<\varepsilon$.

B. Implementation

To minimize complexity, some operations can be combined, while other simplifications are based on the specific signal model, i.e., structure of $\mathbf{A}$ in (24), where we can take advantage of efficient implementations via FFT.

We define the projection vector after the ith iteration,

$$\mathbf{p}_i=\mathbf{A}^H\left(\mathbf{y}-\hat{\mathbf{y}}_i\right)=\mathbf{A}\mathbf{y}-\mathbf{A}^H\mathbf{A}_{I_i}\hat{\mathbf{x}}_i \tag{30}$$

and accordingly $\mathbf{p}_0=\mathbf{A}^H\mathbf{y}$. Also the matrix $\mathbf{A}^H\mathbf{A}$ can be computed analytically based on (24), then either be stored in memory or evaluated as needed.

One iteration of the algorithms can be implemented in the following steps:

$$b_i=\arg\max_{k\notin I_{i-1}}|[\mathbf{p}_{i-1}]_k| \tag{31}$$

$$\hat{\mathbf{x}}_i=\left(\mathbf{A}_{I_i}^H\mathbf{A}_{I_i}\right)^{-1}\left(\mathbf{p}_{i-1}\right)_{I_i} \tag{32}$$

$$\mathbf{p}_i=\mathbf{p}_0-\left(\mathbf{A}^H\mathbf{A}_{I_i}\right)\hat{\mathbf{x}}_i \tag{33}$$

$$|\mathbf{r}_i|^2=|\mathbf{r}_0|^2-\hat{\mathbf{x}}_i^H\left(\mathbf{p}_0\right)_{I_i} \tag{34}$$

Comments: (31) takes $K\cdot\alpha N_{\text{cp}}$ operations; in (32) $\left(\mathbf{p}_{i-1}\right)_{I_i}$ are the elements of the projection vector with indices in I_i and the matrix inversion can be accomplised via Gaussian elimination with complexity i^2; updating the projection vector in (33) takes $i\cdot K\alpha N_{\text{cp}}$ operations and (34) was simplified using the fact that $\hat{\mathbf{x}}_i$ is the least-squares solution when limiting to I_i, taking $i+1$ operations. Therefore the ith iteration is of complexity $(i+1)K\cdot\alpha N_{\text{cp}}+i^2+i+1$, assuming $\mathbf{A}^H\mathbf{A}$ is pre-computed and stored.

The highest complexity operation is therefore calculating $\mathbf{p}_0=\mathbf{A}^H\mathbf{y}$, which potentially takes $(I\cdot\rho N_s)\cdot(K\cdot\alpha N_{\text{cp}})$ operations. This can be reduced by using the special structure of $\mathbf{A}$, by taking $\mathbf{y}$ in parts, performing FFT operations and then adding them weighted with the respective ν_k^{i-1}. This reduces the complexity to $I\cdot N_s\log_2(N_s)+K\alpha N_{\text{cp}}I$.

V. Numerical Examples

A. Setup

We study a simple scenario of a single target moving on a straight line trajectory, while being illuminated by three stations, see Fig. 2. The signal parameters are a bandwidth of $B=4$ MHz, using $N_s=1024$ subcarriers, one quarter of which are deactivated as guard bands or zero tones, all other symbols are assumed to be known. The cyclic prefix

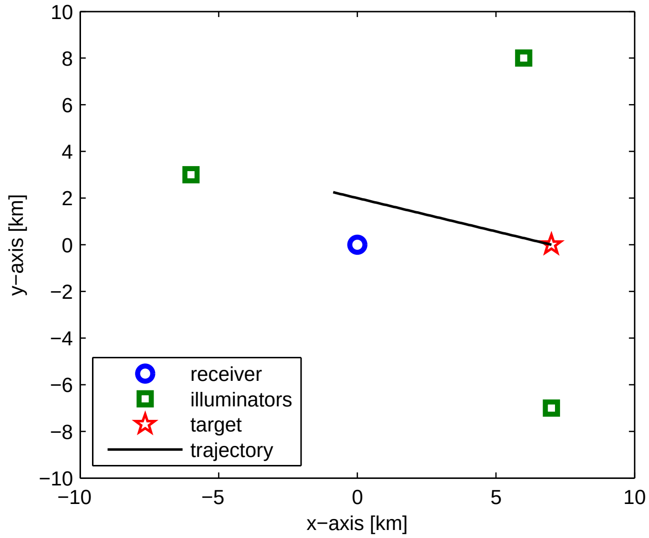

Fig. 2. Overview of the simulation scenario; the target moves on a straight line trajectory over a course of 45 s, being illuminated by three different stations.

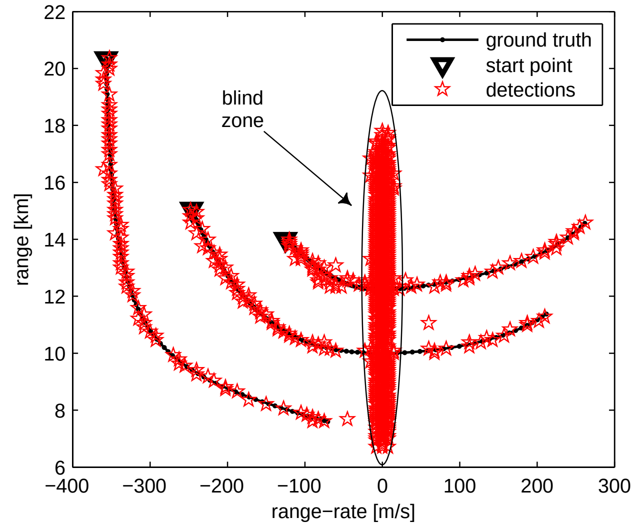

Fig. 3. Simultion results; the ground-truth in terms of range and range-rate are the lines, the superposition of all detections over the time interval are the red markers.

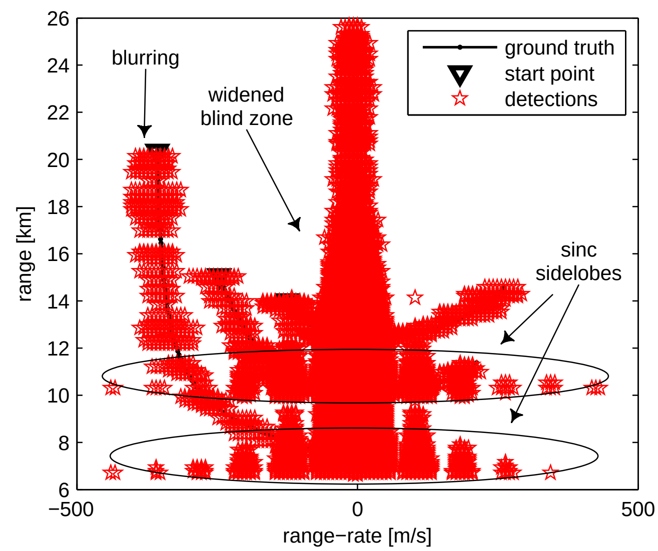

Fig. 4. Simultion results using projection via FFT; the superposition of all detections over the time interval are shown, we see that for few packets, $I = 12$, adjacent range/range-rate bins are highly correlated leading to "smearing", i.e., multiple adjacent detections.

is one quarter of the symbol duration $N_{\text{cp}} = 256$, leading to a total of $T' = 1.25 \cdot N_s/B = 0.32$ ms. The carrier frequency is 1 GHz and we observe $I = 12$ OFDM symbols. All stations are assumed to broadcast in the same frequency band, as defined for the single frequency network (SFN). This leads to the single target leaving three signatures, which still have to be associated to the correct direct signal. Although the positions of the stations and the receiver are generally assumed to be known, it might still be necessary to estimate the arrival of the direct signal. These issues are not addressed here.

The broadcast signal is modelled as multipath, where each path strength A_p in (4) is Rayleigh distributed and also the targets are similarly Swerling II targets. The direct arrival of each station has power inversely proportional to the distance squared, and is trailed by multipath arrivals that attenuate with increasing delay. The signal to noise ratio between the sum of all direct arrivals and the additive noise is 10 dB, while the target signatures are 10 dB weaker than a single direct arrival path of the signal, but about 20 dB belwo the sum of all multipath arrivals.

For simulation purposes, we generate the signal based on (21), were we assume the narrowband model, but evaluate the Doppler frequencies in continuous values, while our estimator assumes evenly sampled values. The target is observed over a time-frame of 45 s; moving at a speed of 182 m/s it covers a distance of about 8.2 km during this time interval. Although using $I = 12$ OFDM symbols we could theoretically generate measurements at a rate of $1/(IT') \approx 260$ Hz, we only generate estimates at 2 Hz. For simulation purposes the target positions and channel response are assumed constant during each of these updates (which last less than 4 ms).

B. Results

We run the algorithm as described in Section IV; the detection and false alarm probabilities can be controlled via the parameter ε in (26) or by applying an additional thresholding after the algorithm. The results of our algorithm are in Fig. 3, where we superimpose all detections over the 45 s period (90 scans). We find that the target can be well seperated from the signal and that the receiver noise plays little role as with a good detection performance, we observe almost no false alarms for this Swerling II target. Around the zero range-rate region, the dominating direct signal leads to a blind zone, which extends into a few range-rate bins on both sides. This is because neighboring range-rate bins are correlated, so the much stronger direct signal "spills over".

As comparison we plot results based on projection of the observations via FFT operation as described in Section III-C, see Fig. 4. This is in effect the same data the OMP algorithm operates on defined as $\mathbf{p}_0$. We see that directly applying thresholding to this data makes no sense, as it suffers from blurring and strong side-lobes of the direct signal. This could be improved by using more OFDM symbols (larger I), which would narrow the blurring, as with more observations the range-rate resolution becomes better. In any case, some additional processing would be necessary, e.g., peak picking to avoid multiple adjacent detections. Intuitively speaking, the OMP algorithm includes this, since starting from the projection, the algorithm iteratively picks a peak and subtracts it from the signal, removing also correlated energy on neighboring bins.

VI. Conclusion

We presented in detail the OFDM signal model as used in DAB/DVB for the purpose of target detection and tracking using passive radar. We suggested using CS to extract the target signals, as in our opinion this is a well suited approach to this problem. We described a possible CS implementation using the greedy OMP algorithm and detailed its implementation. Our numerical results show very good performanec in target detection and seperation. This shows especially for small range-rate measurements, as they are highly correlated with the direct blast signal.

In our opinion this setup warrants further investigation, including different CS algorithms, a more realistic generation of ground-truth for numerical simulation, possibly real experimental data, investiagtion of faster targets that cannot be modelled using the narrowband assumption and/or possibly multiple closely spaced targets.

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