Linking Drainage Front Morphology with Gas Diffusion in Unsaturated Porous Media

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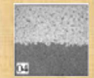
Theoretical Background

Immiscible displacement in porous media

- Immiscible displacement of one fluid by another porous media has been extensively studied, and various flow regimes have been identified. Here we focus on downward invasion of air into an initially saturated medium.



➢**Stable displacement**: front flattened by dominant gravitational forces. (Pictures from Meheust et al. [2003].)

➢**Capillary fingering**: at low drainage velocities, dominant capillary forces cause air to invade largest pore spaces preferentially; results in invasion-percolation-like behavior.

➢**Viscous fingering**: at high drainage velocities, dominant viscous forces create thin branching fingers.

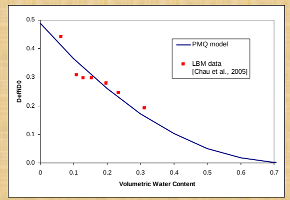
- Dominant forces can be quantified using dimensionless numbers: Bond number (Bo), capillary number (Ca), viscosity ratio, others. [Lenormand et al., 1988, Meheust et al., 2003]
- Meheust et al. [2003] propose a generalized Bond number **Bo*=Bo-Ca**. Their 2-D drainage experiments in glass beads show that Bo* is an excellent predictor of front morphology.

Gas diffusion in porous media

- **Effective diffusion coefficient D_{eff}** is a bulk measure of diffusive flux of gas through the air-filled portion of an unsaturated porous medium.
- For liquid uniformly distributed in a soil, D_{eff} depends on water content as follows [Moldrup et al., 2000]:

$$\frac{D_{eff}}{D_0} = 0.66n\left(\frac{n-\theta}{n}\right)^{(12-m)/3}$$

n = porosity
θ = volumetric water content
m = fitting parameter
D_0 = diffusion coeff. for θ = 0

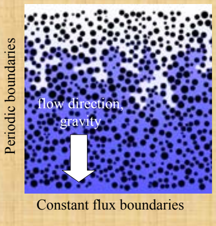


- Due to the complexity of multi-phase flow, no models exist for non-uniform phase distributions.

Numerical Modeling of Drainage in a 2-D Porous Medium

- Porous medium generated by randomly placing circles with normally-distributed radii in a 2-D domain.
- Immiscible displacement simulated using the **Lattice Boltzmann Method** (particle distributions evolve on lattice according to Maxwellian distribution function, influenced by external forces).
- 2 immiscible fluids with different viscosities were used; gravitational force applied to "water" was 1000 times the force applied to "air."

Drainage Simulation



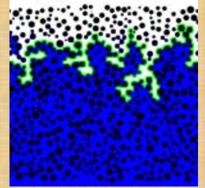
➢Domain filled with "water."

➢"Air" is forced in with a constant flux.

➢Simulation runs to breakthrough.

➢Front morphology depends on prescribed flux.

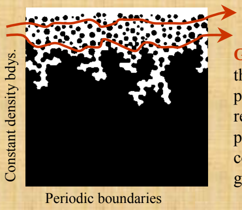
Post-processing



➢Density-based thresholding applied to sharpen front.

➢Front points (in green) extracted for calculation of fractal dimension.

Diffusion Simulation



Gas diffuses through air-filled pore space in response to prescribed concentration gradient.

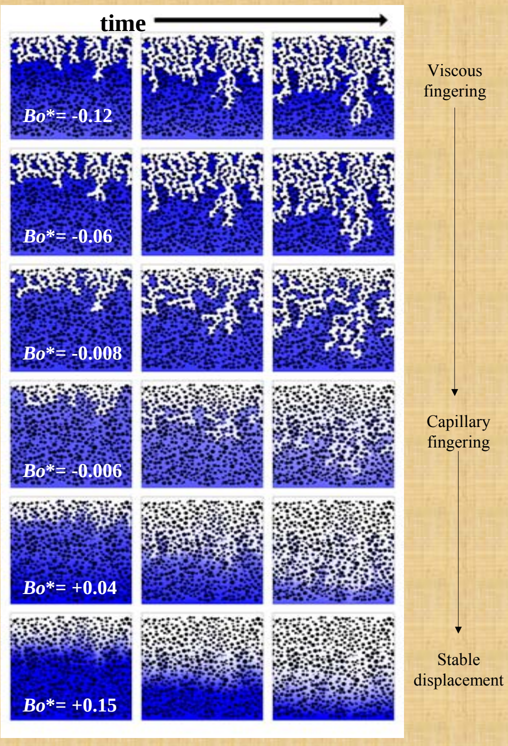
➢"Water" nodes are converted to solids for gas diffusion simulation.

➢Calculate D_{eff} for each water content:

$$q = D_{eff}\frac{dC}{dx}$$

Measure flux.   Prescribe gradient.

Modeling Results: Front Morphology



*Front morphology as a function of Bo**

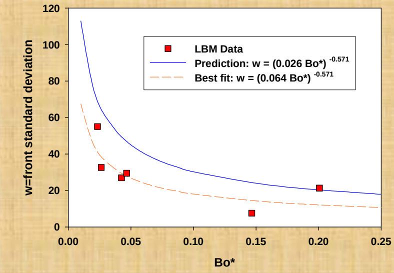
$$Bo^* = \frac{\text{Gravity}}{\text{Capillarity}} - \frac{\text{Viscous}}{\text{Capillarity}}$$

- Bo*=Bo-Ca quantifies the forces controlling instability of the front:
- Front morphologies followed expected progression as Bo* increased from negative (unstable fronts) to positive (stable fronts).
- Fractal dimensions were consistent with experimentally-determined values for viscous and capillary fingering.
- Width of stable fronts (quantified by the standard deviation of front position) compared well with theoretical scaling predictions.

Stable fronts: comparison with scaling law.

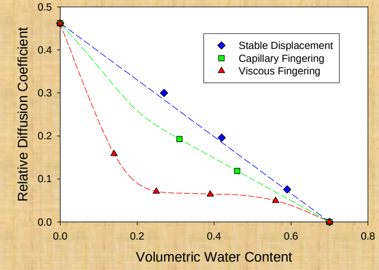
$$w = \left(\frac{\sigma}{W_t a} Bo^*\right)^{-\nu/(1+\nu)}$$

w = front width (stdev of front position)
σ = fluid surface tension
W_t = width of pore size distribution
a = average pore size
Bo* = Bo-Ca (generalized Bond number)
ν = percolation correlation length exponent (=4/3)



- LBM data correspond well to predicted exponent (*w* scales with $Bo^{*-0.57}$).
- Coefficient is higher than predicted but on the same order of magnitude.

Effect of Front Morphology on Gas Diffusion



- Diffusion coefficient reduced by up to 76 % in viscous fingering domain relative to stable.
- Reduction is less significant in capillary fingering domain (up to 25%).
- Reductions would be less significant in a 3-D system due to connectivity in the third dimension.

Conclusions

- Lattice Boltzmann model produced expected front morphologies at a range of gravities and drainage velocities (Bo* values).
- Widths of stable fronts follow scaling predictions of Meheust et al. [2003].
- Fingering flow creates significant reductions in diffusion coefficient through a 2-D porous medium.
- Effect is most significant with viscous fingering due to long liquid fingers which constrict gas diffusion; less significant effect with capillary fingering.

References

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