Experiments on Heat Transfer in a Thin Liquid Film Flowing Over a Rotating Disk

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An experimental study of heat transfer into a thin liquid film on a rotating heated disk is described. Deionized water was introduced at the center of a heated, horizontal disk with a constant film thickness and uniform radial velocity. Radial distribution of the disk surface temperatures was measured using a thermocouple/slip ring arrangement. Experiments were performed for a range of liquid flow rates between 3.0 lpm and 15.0 lpm. The angular speed of the disk was varied from 0 rpm to 500 rpm. The local heat transfer coefficient was determined based on the heat flux supplied to the disk and the temperature difference between the measured disk surface temperature and the liquid entrance temperature onto the disk. The local heat transfer coefficient was seen to increase with increasing flow rate as well as increasing angular velocity of the disk. Effect of rotation on heat transfer was largest for the lower liquid flow rates with the effect gradually decreasing with increasing liquid flow rates. Semi-empirical correlations are presented in this study for the local and average Nusselt numbers. [DOI: 10.1115/1.1652044]

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1 Introduction

Heat transfer and fluid flow in thin liquid films with rotation is of particular interest in several mechanical engineering and chemical processing applications. Thin liquid films on rotating surfaces are encountered in gas-turbine engines, manufacturing of micro-electronics, vapor absorption heat pumps, and heat exchangers. In recent years, interest in thin liquid film heat transfer on rotating surfaces has increased due to potential applications in the microgravity thermal-fluid handling systems. These systems are attractive since rotational forces cause thinning and acceleration of liquid films, resulting in high heat transfer performance independent of gravitational forces.

Traditionally, thin liquid films on rotating surfaces have been studied using an impinging jet to deliver the liquid onto a rotating disk surface. The liquid spreads outward from the impingement zone in a high velocity thin film until a hydraulic jump occurs. An excellent review of these studies was compiled by Webb and Ma [1]. The first major contribution to the study of impinging jet flow phenomenon was made by Watson [2] in 1964, who considered the case of a free liquid jet, impinging on a stationary flat plate. Watson divided the flow into four distinct regions. The first region was the central stagnation zone immediately beneath the impinging jet. In this region, both momentum and thermal boundary layer thicknesses are independent of radial location. In the second region, a growing momentum boundary layer reached a point where the boundary layer is of sufficient thickness to influence the free surface. The third region consisted of the transition of the velocity profile from where the free surface was first influenced (Blasius type profile), to a fully developed velocity profile that includes the free surface. Finally, in the fourth region, a fully developed velocity profile existed for which Watson developed a similarity profile. This region extended through the remainder of the supercritical flow, and was bounded by the hydraulic jump diameter. This regional segregation methodology has been adopted by numerous subsequent investigators including Chadhury [3], who extended Watson’s approach to include heat transfer phenomena. Wang et al. [4] developed a unique analytical approach in which, the heat transfer in the disk and liquid film were treated separately and then the solutions were matched at the disk surface. Thomas et al. [5] developed a one-dimensional analytical solution to the quasi-steady momentum equation. The model assumed solid body rotation in predicting film thickness, including the hydraulic jump. The resulting velocity distribution was coupled with conservation of energy to predict the heat transfer performance. Rahman et al. [6] studied the influence of turbulence on the free surface height and hydraulic jump for the case of controlled impinging jets in both plane and radial flows. A numerical model was developed that employed a body-fitted moving grid method and a k-ε turbulence model. Rahman and Faghri [7] studied heating and evaporation from a rotating disk by considering three distinct regimes. For developing flow and heat transfer, a three dimensional numerical model was utilized. Also, a two dimensional analytical solution was formulated for developing heat transfer and fully developed flow assuming solid body rotation. For the case of fully developed heat transfer and fluid flow, a closed form solution was developed. This solution predicted that the Nusselt number, based on film thickness, approached a constant value in the fully developed regime.

In an experimental study, Carper and Deffenbaugh [8] evaluated the convective heat transfer from a jet of cooling oil to an approximately isothermal rotating disk. Correlations were presented for the average Nusselt number as influenced by rotational Reynolds number, jet Reynolds number and radius of impingement. In a later study, Carper et al. [9] extended this study to consider Prandtl number effects. Average heat transfer coefficients were presented with no information on its local values. Vadar et al. [10] studied the effects of jet velocity and temperature on the heat transfer between a planar water jet and a stationary heated plate. They concluded that the heat transfer performance was affected by the free stream turbulence intensity and the Prandtl number. A similar study was presented by Stevens and Webb [11] in which the influence of jet Reynolds number, nozzle to plate spacing and jet diameter were evaluated. Empirical correlations were developed for stagnation point, local and average Nusselt numbers.

Mudawwar et al. [12] investigated convective heat transfer for a rotating thin film flowing through an internal channel within a high speed rotating disk. The study was motivated by gas turbine
blade cooling applications. A semi-empirical turbulence model was developed to predict heat transfer performance. Faghri et al. [13] presented heat transfer results for a controlled liquid impinging jet on a stationary disk. They presented a numerical study of the system showing good agreement between heat transfer predictions and experimental data. Auone and Ramshaw [14] performed heat and mass transfer experiments of a rotating disk for controlled jet impingement. They predicted the heat transfer coefficients analytically by adapting the solution that Nusselt used for the film condensation under the influence of gravity.

From the above literature review, it is evident that comprehensive experimental data on heat transfer for controlled jet impingement on a rotating disk is not available. Additionally, the relation between the liquid film flow characteristics and heat transfer performance has not been studied simultaneously in the past investigations.

The present study considers measurement of local and average heat transfer coefficients for a thin liquid film flowing on a rotating heated disk. The thin water film is generated via a unique flow collar device that provides a circumferentially uniform, radial flow onto the disk. This flow collar device differs from the impinging jet system described by Watson [2]. Watson’s stagnation region does not exist for the flow collar system. Instead the water exits the flow collar with a fully developed radial velocity distribution and is suddenly bounded by a free surface. In this study, the inlet liquid flow rate, the collar gap height through which the fluid enters the disk surface and the rotational speed of the disk are used as the experimental control parameters. A film Reynolds number is defined based on the inlet gap height and the entrance velocity of the film onto the disk. Film thickness measurements are made with and without rotation. Local Nusselt numbers are calculated from temperature measurements taken from the rotating disk. In this study, angular velocity and flow rate on the heat transfer performance are presented. Semi-empirical correlations are derived for the local and average Nusselt numbers.

2 Experimental

2.1 Rotating Disk Setup. The experimental setup described by Faghri et al. [13] was modified for the present study. A basic description of the apparatus is provided here with emphasis on the design modifications. A schematic of the experimental set-up is shown in Fig. 1. The 40.6 cm diameter, 0.58 cm thick aluminum design modifications. A schematic of the experimental set-up is provided here with emphasis on the experimental apparatus is described here with emphasis on the design modifications. A schematic of the experimental set-up is provided here with emphasis on the experimental setup for heat transfer experiments for a liquid film flowing over a rotating disk. (1) Pulley assembly, (2) High precision spindle, (3) Flow collar, (4) Disk, (5) Etched foil heater, (6) Annular tank, (7) Thermocouples, (8) Motor, (9) Variable speed motor control, (10) 0-5000V heater control, (11) Thermocouple transmitters, (12) Precision slip ring, (13) Cooling air, (14) Rotating coupling, (15) External process tank, (16) Heat exchanger, (17) Pump, (18) Bypass valve, (19) Large metering valve, (20) Small metering valve, and (21) Flow meter.

position the CCD camera accurately, a precision linear slide was used. Quantitative comparison of these images with reference images of the dry disk surface allowed determination of the film thickness with an uncertainty of ± 0.025 mm as discussed in Ref. [17].

3 Results and Discussion

3.1 Liquid Film Visualization and Measurements. Visualization of the liquid film was performed with the purpose of identifying the characteristics of the liquid film flow over stationary and rotating disk surfaces. Figure 3 shows photographs of the liquid film at different conditions. In Fig. 3(a), the liquid film on the stationary disk surface at a flow rate of 3.0 lpm (Re = 238) undergoes a sudden increase in its thickness right at the edge of the collar and the liquid film is in the subcritical region downstream of the hydraulic jump all over the disk surface. The hydraulic jump is the sudden increase in the film thickness and it is located where the momentum is balanced between the supercritical and subcritical flows. At a higher flow rate of 7.0 lpm (Re = 555), as shown in Fig. 3(b), the hydraulic jump location moves to a larger radius on the disk and the waviness of the liquid’s free surface increases indicating a more turbulent nature of the film surface. Figures 3(c) through 3(f) present the liquid film topography for varying rotation speeds. With rotation, the hydraulic jump is pushed outwards and is not present on the disk surface at a rotation speed of 100 rpm. Spiral waves begin to emerge due to centrifugal forces acting on the liquid film. Additionally, as the rotation speed is increased from 100 rpm to 500 rpm, the wave amplitudes decrease because of the thinning of the liquid film due to increased centrifugal effects.

Figure 4 shows the film thickness for Reynolds numbers between Re = 238 and 1188 for the stationary case and for the rotation speeds of 100 rpm and 300 rpm. As the Reynolds number increases, the hydraulic jump moves towards the outer edge of the disk because the liquid leaves the collar with a higher radial momentum. For Re = 238, the hydraulic jump is attached to the collar. However at Re = 555, the hydraulic jump occurs between r/r_i = 1.6 and 2.5. At Re = 1188, it moves near the outer edge of the disk. For Re = 555, the film thickness is nearly uniform in part of the subcritical region between r/r_i = 2.5 and 3.5 but then drops off near the outer edge of the disk. This decrease in the film thickness is due to the acceleration of the flow as the liquid leaves the disk surface. Also the liquid film thickness in the subcritical region is an order of magnitude greater than that in the supercritical region.

At moderate rotation speeds such as 100 rpm shown in Fig. 4(b), the film thickness exhibits a maximum whose location propagates outward with increasing flow rate. For these cases, the liquid film behavior can be divided into three zones; the inner inertia-dominated region, the outer rotation-dominated region and the transition region in between. In the inertia-dominated region, the inertia, and friction forces are dominant. Because of the friction forces, the liquid tends to slow down and the film thickness increases as reported by Thomas et al. [18]. In the rotation-dominated region, centrifugal forces cause the film thinning of the liquid film. Meanwhile in the transition region, both of these forces are in play. Thus, the film thickness reaches a maximum in the transition region, where deceleration of the velocity due to friction is compensated and eventually balanced by the acceleration due to centrifugal effects.

As apparent in both Figs. 4(b) and 4(c), the location where the maximum film thickness occurs, changes as a function of Reynolds number and rotational speed. An increase in the Reynolds number causes an increase in the film inertia. Therefore, higher centrifugal forces are required to overcome higher inertial forces. If the rotation speed is kept constant and the Reynolds number is increased, then the maximum film thickness location travels towards the edge of the disk. Since radial location influences the magnitude of centrifugal effects, the region where the centrifugal
and inertia forces are comparable, moves towards the edge of the disk. Figure 5 shows the change in the film thickness, when the Reynolds number is kept constant and the rotation speed is increased. In this situation, the point where the centrifugal and the inertia forces are comparable moves towards the collar. Thus, the maximum film thickness location travels towards the center of the disk. The film thickness decreases as the rotation speed increases. For 300 rpm, the centrifugal forces are dominant over the whole disk surface such that the film thickness decreases with increasing radial distance.

3.2 Heat Transfer Measurements. Measurements of the disk surface temperature distribution were made for different flow rates and different rotation speeds. The heat transfer coefficient is defined in terms of the heat input per unit area supplied by the heater and the temperature difference between the liquid inlet temperature and the determined disk surface temperatures. The total heat input was chosen as 3000 W for the flow rates between 3.0 lpm and 9.0 lpm and 4500 W for the flow rates between 12.0 lpm and 15.0 lpm. These variables were chosen to obtain sufficiently high disk surface temperatures so that the maximum error in the heat transfer coefficient is limited to approximately 10 percent.

The Nusselt number distributions are shown for different rotation speeds and Reynolds numbers between $Re_i=238$ (3.0 lpm) and $Re_i=1188$ (15.0 lpm). In all cases, the Nusselt numbers increase with increasing liquid film flow rate and the rotation speed. The Nusselt numbers are higher at the inner portions of the disk close to the collar and decrease towards the outer edge. For the supercritical region, close to the collar exit, inertial forces have a significant effect on the flow and the maximum temperature differences exist in this region. Thus, the highest heat transfer rates are also obtained in this area. Away from the collar, due to the radial spread, the flow begins to slow down which causes a decrease in the Nusselt number. Also, the developing thermal boundary layer causes a decrease in the Nusselt number. For the case where the disk is stationary and the subcritical flow of the liquid film is present over the disk, the behavior of the Nusselt number profiles changes. As shown in Fig. 3(a), the hydraulic jump is attached to the collar and the disk is cooled by the subcritical region of the liquid film for the stationary case at $Re_i=238$. The liquid velocities within the subcritical region are quite low and result in low Nusselt numbers over the disk. The Nusselt numbers for the stationary disk are significantly lower than the rotating disk cases. Increasing rotation progressively results in an increase of the heat transfer coefficient and the Nusselt number as expected due to thinning of the liquid film and more effective heat transfer.

For $Re_i=555$, the hydraulic jump exists only for the stationary case and it resides between $r/r_i=1.5$ and 2.5. The Nusselt number
variation shown in Fig. 6(b) exhibits this effect as the Nusselt number is significantly higher in the supercritical region ($r/r_i < 2.0$) as compared to the subcritical region ($r/r_i > 2.5$). For the rotating disk cases, the hydraulic jump is not present on the disk surface and consequently the heat transfer coefficients are consistently higher. The effect of higher liquid flow rate and higher liquid film velocity is to enhance the heat transfer in the inner regions as seen in Figs. 6(a) and 6(b). Additionally, the Nusselt number in the inner disk regions approaches a constant value at high flow rates. The magnitude of Nusselt number increases slightly at the high rotation speeds. The approach to a constant Nusselt number at high flow rates is expected as the inner regions of the disk are cooled by the thin liquid film which has a high velocity in these regions. It is interesting that the rotational effects are still present in this region. Figure 6(c) shows the heat transfer data for 15.0 lpm liquid flow rate corresponding to $Re_i=1188$. The Nusselt numbers are higher than the previous two cases (3.0 lpm and 7.0 lpm) and the constancy of the Nusselt number in the inner regions is more apparent. In fact, it is evident from all these
data that the character of the Nusselt number distribution changes from that of a continuous decrease from the center to the outer edge of the disk to that of a constant inner part and a decrease towards the outer edge. As explained before, this trend is a result of the interplay between the effects of the high velocity liquid film in the inner regions and the pronounced effects of the rotation in the outer regions.

The Nusselt number distributions are shown in Fig. 7 at different flow rates for stationary disk and two rotational speeds of 100 and 300 rpm. In these experiments the flow rates are 3.0, 5.0, 7.0, 9.0, 12.0, and 15.0 lpm with the corresponding Re of 238, 396, 555, 713, 951, and 1188. Also shown in Fig. 7(a) is the data obtained earlier by Faghri et al. [13] for the stationary disk case. The agreement between those and the current data is quite good with a maximum deviation of 9 percent which is within the uncertainty of each data set. As the liquid flow rate increases, the Nusselt numbers also increase in all cases. However, the increase is most pronounced for the stationary case, particularly in the inner regions of the disk. For Re = 238 and 396, the hydraulic jump is very close to the collar exit and as such the Nusselt numbers are low and have similar magnitude for the subcritical liquid film flow. For Re = 555, the hydraulic jump resides between \( r/r_i = 1.5 \) and 2.5 and the data exhibit a significantly different variation as discussed earlier. At higher liquid flow rates, the inner part of the disk attains a uniformly higher cooling from the high velocity thin liquid film. As the rotational speed of the disk increases, the variation of the Nusselt number with respect to flow rate decreases.

In Fig. 8, the previously shown data are plotted at several locations on the disk to determine the effect of flow rate (or inlet Reynolds number) on the Nusselt number. At all rotation speeds, the Nusselt number increases as the Re increases. At the inner region of the disk \( (r/r_i = 1.4) \) shown in Fig. 8(a), the Nusselt number increases with the inlet Reynolds number with a power law dependence for the rotating disk data. For the stationary disk case, the presence of the hydraulic jump influences the Reynolds number dependence. Hydraulic jump is present in the vicinity of \( r/r_i = 1.4 \) between Re = 400 and 600 for the stationary case. Below Re = 400 the liquid film is in the subcritical region and the Nusselt number appears to exhibit a stronger Reynolds number dependence as compared to the thin film region. In the thin liquid film regime, the Nusselt number dependence on the inlet Reynolds number is similar for all cases including the supercritical

Fig. 7 Nusselt number distributions along the radial direction for a thin liquid film flowing on a stationary and rotating disk for (a) 0 rpm, (b) 100 rpm, and (c) 300 rpm

Fig. 8 Local Nusselt Number distribution for a thin liquid film flowing on a stationary and rotating disk as a function of Reynolds number for Pr = 4.2 at (a) \( r/r_i = 1.4 \), (b) \( r/r_i = 2.1 \), and (c) \( r/r_i = 2.9 \)
regions of the film on the stationary disk. However, Nusselt number shifts to higher values with increasing rotational speed for all cases.

Figure 9 shows the dependence of local Nusselt number on the rotational Reynolds number at three locations on the disk surface. This dependence becomes stronger and more pronounced at larger radii on the disk surface as expected since the influence of rotation becomes progressively more dominant at larger radii. The power law dependence on rotational Reynolds number is similar for the two outer locations shown in this figure. In a similar fashion, the Nusselt number dependence on the Rosby number is shown in Fig. 10. It is clearly seen that data also exhibit a power law dependence on the Rosby number, which is a measure of the relative importance of the radial and tangential components of the film flow. As the rotation speed increases for a given film flow rate or Rosby number decreases, the heat transfer coefficient is enhanced as expected.

In Fig. 11, the effect of the collar gap height through which the liquid film emerges, is shown for the stationary disk. There is a substantial dependence of the local heat transfer rate and Nusselt number on the initial gap height at small radii \( r/r_i < 2.5 \). With increasing gap height, the Nusselt number decreases, particularly in the supercritical region. It is also interesting to note that this effect persists for sometime into the subcritical region.

The heat transfer results presented here could have been influenced to some degree by the conjugate heat transfer effects in the disk although the experimental setup was designed to minimize such effects. However, earlier work by Faghri et al. [13] suggests that conjugate heat transfer effects can be significant under certain conditions. In light of this, we are currently investigating the influence of the conjugate heat transfer effects by computational modeling to better assess these effects on the experimental findings.

### 3.3 Heat Transfer Correlation

A heat transfer correlation was developed by dividing the domain into two regions. Region I represents the thermal entry length over the disk, and thus extends from the flow collar to the radius where the free surface temperature is first influenced by the wall heat flux. Region II extends from this point downstream but does not include the hydraulic jump. A scale analysis is presented here to identify the controlling parameters. For Region I, heat transfer coefficient can be written as

\[
h = \frac{q^*}{T_{w} - T_i} = \frac{k_i}{\Delta T} \frac{\partial T}{\partial Z}
\]

where \( \partial T/\partial Z = T_w - T_i / \delta_T \), \( \Delta T = T_w - T_i \), thus,

\[
h = \frac{k_i}{\delta_T} \quad \text{or} \quad \text{Nu}_{\delta_T} = 1
\]

To predict the heat transfer coefficient, a knowledge of the thermal boundary layer is required. For Prandtl numbers of order one, thermal boundary layer development is highly dependent on the nature of the momentum boundary layer. The relationship between the thermal and hydrodynamic boundary layers can be written as

\[
\frac{\delta_T}{\delta} = C\rho r^b
\]

Since the velocity distribution is in a transitional mode, the prediction of \( \delta \) requires some simplifying assumptions. Namely, the inlet velocity profile can be neglected and a uniform radial velocity distribution at the inlet can be assumed. This is similar to Watson’s second region [2] with the addition of rotational effects. For this case, a developing radial boundary layer exists in a rotating flow field for which a scale analysis of the momentum equation yields

\[
\delta = C_1 \frac{V_i}{\sqrt{\rho}} \left( \frac{\text{Ro}}{\text{Re}^*} \right)^{1/2}
\]

The first term in the above equation relates to the viscous growth of the hydrodynamic boundary layer, which is the significant term for radial flow regime. The second term results from rotational forces and has the effect of accelerating and thinning the boundary layer. This centrifugal thinning term governs the relation for rotationally dominant regimes. Thus, a correlation for Region I can be formulated as

![Fig. 9 Local Nusselt Number distribution for a thin liquid film flowing on a rotating disk as a function of rotational Reynolds number for Re_i=555 at Q=3000 W](image1)

![Fig. 10 Local Nusselt Number distribution for a thin liquid film over a rotating disk as a function of Rosby number for Re_i=555 at Q=3000 W](image2)

![Fig. 11 Comparison of Nusselt number distributions along the radial direction for a liquid film flowing over a stationary disk for Re_i=555 at Q=3000 W for \( \delta_i = 0.254 \text{ mm}, \ 0.381 \text{ mm} \) and 0.508 mm](image3)
Average Nusselt number: \( \overline{\text{Nu}} = 1.595 \frac{Re_i^{0.79}}{Pr^{0.33}} + 2.692 \left( \frac{Re_{ig}/Ro}{Pr} \right)^{0.30} \frac{Pr}{3} 
\tag{9}\n\]

Note that in the correlation for local Nusselt number, Nu is defined using the local radius and local heat transfer coefficients while in the case of average Nusselt number, it is defined by the outer disk radius and average heat transfer coefficient based on the arithmetic mean of the disk surface temperature. The validity of these data fits are shown in Figs. 12 and 13 for the local and overall Nusselt numbers respectively. The correlations for the local and the overall Nusselt numbers were both found to be accurate to within \( \pm 20 \) percent. In developing these correlations only the data in the thin liquid film region (i.e., supercritical regions for those cases having a hydraulic jump) were taken into account. Also, due to possible cooling effects near the outer edge of the disk as the liquid exits the disk surface, the data at the outermost location closest to the disk outer edge were not included in the correlations.

4 Conclusions

The characteristics of a thin liquid film with a free surface on a stationary and rotating disk have been examined experimentally. The change in the position of the hydraulic jump was observed qualitatively by a photographic study. Also, the film thickness was measured for different Reynolds numbers and rotational speeds with the laser light reflection technique. On the stationary disk surface, the liquid film experienced a hydraulic jump in which the liquid velocity decreased and the film thickness increased. With increasing flow rates the hydraulic jump migrated towards the outer edge of the disk. It was also observed that the hydraulic jump washed off the disk surface under rotation. It was found that three regions existed in the rotating thin liquid film: the inertia dominated region, where the inertia and the frictional forces were prevalent; the rotation dominated region, where the centrifugal forces had a more significant effect; and a transition region which contained elements of both the inertia and the rotation dominated regions.

The experimental study of heat transfer of a thin water film on a rotating disk was also presented. It was found that the heat transfer was affected by different flow conditions over the disk surface. Nusselt number was higher at the inner portions of the disk, close to the collar, and decreased towards the outer edge due to radial spread, convective heating of the liquid and the developing thermal boundary layer. When hydraulic jump occurred on the disk, low Nusselt numbers were observed in the subcritical region due to the significant increase in the film thickness and reduced liquid velocities. However, the Nusselt numbers attained high values in the supercritical region. The increase in the rotation speed caused an increase in the Nusselt numbers because of the thinning effect. As the liquid flow rate increased, Nusselt numbers also increased. This increase was more pronounced for the stationary case, especially on the inner parts of the disk surface. With the increasing flow rate, the hydraulic jump migrated to outer radii and the larger portions of the disk were cooled by the supercritical region of the liquid film. In addition, the increase in the flow rate caused a more uniform temperature and Nusselt number distribution at the inner regions due to the high liquid velocities.

Finally, semi-empirical correlations were presented for the local and overall Nusselt numbers. Both of the correlations represented the experimental data within \( \pm 20 \) percent. It should be also noted here that the conjugate heat transfer effects may influence the experimental results. Currently, a computational study of such effects is being carried out in this configuration to assess their contribution.
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Nomenclature
\[ a, b, c, d, C_1, C_2 = \text{correlation coefficient} \]
\[ A_h = \text{area of the heater, m}^2 \]
\[ d_h = \text{hydraulic diameter, (2}h, \text{m) } \]
\[ h = \text{local heat transfer coefficient, } (Q/A_h)/(T_w - T_i), \text{W/m}^2\text{K} \]
\[ \bar{h} = \text{average heat transfer coefficient, } (Q/\langle A_h \rangle)/(T_m - T_i), \text{W/m}^2\text{K} \]
\[ k_l = \text{thermal conductivity of the liquid, W/mK} \]
\[ \text{Nu}_l = \text{local Nusselt number, } (h r/k_l) \]
\[ \text{Nu}_{in} = \text{Nusselt number based on inlet radius, } (h r/k_l) \]
\[ \text{Nu}_{i-o} = \text{average Nusselt number, } (h r_o/k_i) \]
\[ \text{Pr} = \text{Prandtl number, } (\nu a) \]
\[ Q = \text{total heat input power, W} \]
\[ r = \text{local radius, m} \]
\[ r_o = \text{outer disk radius, m} \]
\[ \text{Re}_i = \text{inlet Reynolds number, } (V_{in} \delta_i/\nu) \]
\[ \text{Re}_j = \text{jet Reynolds number, } (V_{in} \delta_j/\nu) \]
\[ \text{Re}_c = \text{local Reynolds number, } (V_{in} r/\nu) \]
\[ \text{Re}_{i-o} = \text{rotational Reynolds number, } (\omega r^2/\nu) \]
\[ \text{Re}_{i-o} = \text{rotational Reynolds number based on the outer disk radius, } (\omega r_o^2/\nu) \]
\[ \text{Ro} = \text{Rossby number, } (V_{in}/\omega r) \]
\[ \text{Ro}_{i-o} = \text{Rossby number based on the outer disk radius, } (V_{in}/\omega r_o) \]
\[ T_{av} = \text{average disk surface temperature, } ^\circ\text{C} \]
\[ T_{bulk} = \text{bulk liquid film temperature, } ^\circ\text{C} \]
\[ T_w = \text{local disk surface temperature, } ^\circ\text{C} \]
\[ T_i = \text{inlet fluid temperature, } ^\circ\text{C} \]
\[ T_s = \text{free surface temperature of the liquid film, } ^\circ\text{C} \]
\[ V_{in} = \text{radial velocity component at the exit of the collar, } (V/2\pi r \delta), \text{m/s} \]
\[ \nu = \text{volumetric flow rate, m}^3/\text{s} \]

Greek Symbols
\[ \alpha = \text{thermal diffusivity, m}^2/\text{s} \]
\[ \delta = \text{momentum boundary layer thickness, m} \]
\[ \delta_j = \text{film thickness, m} \]
\[ \delta_i = \text{inlet gap height, m} \]
\[ \delta_f = \text{thermal boundary layer thickness, m} \]
\[ \nu = \text{kinematic viscosity of the liquid, m}^2/\text{s} \]
\[ \omega = \text{rotational speed, rad/s} \]

References