I. INTRODUCTION

Characteristics of buoyant plumes are of significant interest in many engineering applications and in natural phenomena. They occur in a wide variety of circumstances including accidental fires, release of buoyant gases in industry from exhaust stacks, and geophysical occurrences such as volcanic eruptions and alike. These flows are predominately driven by buoyancy forces arising from either injection of a lower or higher (than ambient) density fluid into a uniform density or density stratified medium or density changes as a result of combustion. It is also known that the initially momentum dominated jets and flames can eventually become buoyancy dominated farther downstream. Momentum dominated nonuniform density jets have been studied extensively. Papanicolaou and List studied turbulent buoyant liquid jets by measuring velocity and scalar concentration fields spanning axial distances to 120 jet diameters. These measurements, along with dimensional analysis for the self-similar regions of the flow, provide a complete description of velocity and concentration fields including mean and fluctuating quantities. Their results cover whole range of conditions from momentum dominated jets to buoyant plumes. A similar investigation of downwardly propagating heavy gaseous jets was carried out by Dai et al. with particular emphasis on scalar properties of buoyant jets. They had made concentration measurements in the “fully developed” region of turbulent buoyant plumes and showed that self-similarity is attained at farther distances from the nozzle than it has been previously thought, with narrower plume widths and higher mean and fluctuating mixture fractions of the jet fluid. Panchapakesan and Lumley measured turbulence structure of air and helium jets and compared their turbulence characteristics. They also presented approximate budgets for turbulent kinetic energy, scalar variance, and scalar fluxes.

Becker and Massaro presented one of the earlier experimental studies of jet instability. They identified vortex shedding regimes in nonreacting, constant density jets with and without external acoustic forcing. Their results suggest that vortex shedding undergoes a number of changes as the jet Reynolds number increases. Their results showed that the Strouhal number of oscillations is proportional to the square root of the jet Reynolds number, revealing an inverse square root diameter dependence of the oscillation frequency. This scaling was justified on the basis that the boundary layer thickness determines the most amplified wavelength of the shear layer instability. Mollendorf and Gebhart conducted a stability analysis of jet boundary layer flow using Orr–Sommerfeld equations within the Boussinesq approximation. They found that nonbuoyant jets were stable to axisymmetric disturbances, while asymmetrical disturbances were found to be amplified. In the limit of small buoyancy, the destabilizing effect of density gradients was predicted by the analysis and it was also confirmed to some extent by their experiments. Although the asymmetrical disturbances (sinuous) were found to be more unstable than axisymmetric (varicose) disturbances, they observed symmetric lumps near the nozzle.

Experiments on the oscillatory behavior of buoyant plumes of helium and helium-air mixtures are reported for a range of nozzle diameters (3.6 cm< d<20 cm), source velocities, and plume densities. Measurements include pulsation frequencies as determined from total pressure fluctuations along the plume centerline in addition to the phase resolved laser Doppler velocity measurements. These nonreacting buoyant plumes are found to exhibit periodic oscillations of plume boundaries which subsequently evolve into toroidal vortices within one-half diameter above the nozzle exit. These oscillations and vortices are similar to those observed in pool fires, although their frequency scaling is somewhat different. The frequency relationship is well represented by the expression $S=0.8Ri^{0.38}$, where the Strouhal number is $S=fd/V_0$ and the Richardson number is defined as $Ri = [(\rho_p - \rho_p)g d]/\rho_p V_0^2$. Parameters $f$, $V_0$, $p$ are frequency, source velocity, and density and subscripts $p$ and $\infty$ refer to the plume fluid and ambient, respectively. Between $Ri=100$ and 500, a transition in the frequency scaling is observed as evidenced by more turbulent and vigorously mixing plumes beyond this transition. In this region, $S=2.1Ri^{0.28}$. This change in scaling can be explained by the effect of turbulent mixing on local plume density and the resulting modification of the convection speed of the toroidal vortices. These results provide a consistent basis for the mechanism of the observed instability which is quite different than other types of flow instabilities. Additionally, the phase resolved velocity field of a pulsating buoyant plume reveals a strong buoyant acceleration along the plume centerline followed by a deceleration in the region of the toroidal vortex formation. The strong upward acceleration is also accompanied by significant radial inflow toward the plume centerline determining the entrainment characteristics of these pulsating buoyant plumes.

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exit in some experiments which later distorted to the asymmetrical mode. Large undulations were also observed in buoyant jets of a plume-like character where predictions based on small disturbance theories become inadequate.

Kyle and Sreenivasan\textsuperscript{9} concentrated on the near field of variable density, nonbuoyant jets. They observed two different types of instability in the jet near field depending on the density ratio of jet fluid to surroundings. When this ratio was greater than 0.6, shear layer instability was observed similar to constant density jets. When this ratio was less than 0.6, they found an intense oscillatory instability whose behavior was found to depend on the density ratio and the ratio of the nozzle diameter to the momentum thickness at the nozzle exit. In addition to these nonuniform density jet studies, considerable amount of research exists on the “far-field” behavior of buoyant plumes starting with the early works of Morton et al.\textsuperscript{10,11} Subsequently, the far-field characteristics of nonreacting plumes and fire plumes\textsuperscript{12,13} have been established, including scaling relationships for plume velocities, temperatures, species concentrations, and entrainment rates.

While these studies have identified many important aspects of buoyant jets and plumes, the “near field” of very low momentum, highly buoyant plumes has not been a subject of detailed investigation. This limiting regime is very relevant and interesting in fire-related problems where plumes of large sizes do occur. In connection with fire and toxic gas spread, the region of first few diameters downstream of the source is of great interest since this is where most of the entrainment and mixing occur in fires.\textsuperscript{14} Furthermore, buoyant plumes of this type exhibit a quasiperiodic oscillatory behavior resulting in formation of large toroidal vortices which strongly affect the flow field in the plume and its entrainment characteristics. For example, it is well known that pool fires exhibit quasi-periodic oscillations which primarily scale with inverse square root of pool diameter.\textsuperscript{15–18} While oscillation frequencies have been reported by many investigators, the mechanism of this instability has not been fully understood. Some earlier experiments with helium plumes and simulated pool fires\textsuperscript{16} had shown that oscillations are solely a consequence of buoyancy near the source of a plume. Our earlier measurements of pulsation frequency revealed similar scaling for nonreacting plumes and pool fires. However, this was based on a limited number of helium plume experiments in the parameter range where frequency data overlapped. Subsequently, Hamins et al.\textsuperscript{17} reported that nonreacting buoyant plumes of helium for a wider range of Froude (or Richardson) numbers revealed a somewhat different frequency scaling than that for pool fires. Utilizing the experimental data of Hamins et al.,\textsuperscript{17} Delichatsios\textsuperscript{19} has given an explanation of the different frequency scaling for nonreacting buoyant plumes based purely on dimensional analysis.

In order to fully characterize oscillations in buoyant nonreacting plumes and the flow field near plume origin, we have embarked on an experimental study of plume oscillations including oscillation frequencies of pure helium and helium-air mixture plumes. We also conducted phase resolved velocity measurements to unveil the various stages of the plume flow field development during these oscillations.

In this article, we first present plume oscillation frequency measurements utilizing a novel experimental technique, previously employed by the authors.\textsuperscript{16} Correlation of oscillation frequencies are presented for a wide range of Froude or Richardson numbers and a range of density ratios between the plume fluid and the ambient. We later present measurements of velocity field within one diameter height above the nozzle exit. These measurements, conducted in a phase-resolved manner, allow identification of various stages that the flow field exhibits within one oscillation cycle. Finally, we discuss the instability mechanism which leads to a periodic oscillatory state in this flow field. We include a simple kinematic analysis to explain the frequency scaling observed in nonreacting and reacting buoyant plumes.

II. EXPERIMENTAL

A. Experimental setup

The experimental setup was designed to conduct detailed velocity measurements by conditional laser Doppler velocimetry as well as to determine the pulsation frequencies of buoyant plumes originating from circular nozzles of 3.6, 5.2, 7.3, 10.0, 15.2, and 20 cm diam. Nozzles were constructed from either PVC or metal pipe. The outside surfaces of these nozzles were chamfered at 10° with an exit lip thickness of 1.0 mm. The length to diameter ratio ranged from 3 to 10 for all nozzles with the large values for the smaller diameter nozzles. A 64 mesh, flat stainless steel screen was placed across the nozzle exit. The main purpose of this screen was to provide a uniform flow condition at the nozzle exit which was otherwise impossible due to rapid buoyant acceleration of helium with respect to the surrounding air medium. Presence of the screen prevented back flow of air upstream of the nozzle exit and also insured a plug flow exit condition. For the large nozzles, with \( L/d = 3–4 \), a honeycomb flow straightener was also placed for flow uniformity upstream of the nozzle exit. These nozzles were placed vertically in draft-free surroundings as shown in Fig. 1. Twenty mesh window screens were formed into a circular cylinder far from the nozzles \((D = 1.0 \text{ m})\) to prevent any influence of turbulent

![FIG. 1. Schematics of the experimental setup.](image-url)
eddy structures that might be present in room air. Studied plumes were free of any forced draft above them so that no artificial downstream influence was imposed.

Characterization of plume oscillation frequencies was facilitated by a total pressure probe placed along the plume centerline at a height of one-half nozzle diameter. Selection of this location was such that large velocity oscillations in vortex formation and convection phases can be unambiguously detected by the pressure probe. The pressure probe was connected by a short Tygon tubing to the high pressure side of a differential pressure transducer of capacitance type (Septra Model 264, +0.05 in. of H₂O column) while the low pressure side was exposed to ambient pressure. Considering the total pressure \( P(t) = P_0 + \rho U^2(t)/2 \), where \( P_0 \) is the static pressure, \( \rho \) is density, and \( U(t) \) is the time varying velocity at the probe tip, the differential pressure transducer responds to variations in the square of velocity. The pressure transducer output was first amplified (Tektronix AM502) and filtered by a low-pass filter before being processed by a frequency spectrum analyzer (Analogic Data 6000). An example of the pressure trace and its fast Fourier transform (FFT) is shown in Fig. 2 for a helium plume originating from the 10-cm-diam nozzle. The frequency peak at 4.5 Hz is associated with the pulsation frequency of this helium plume. The frequency peaks below and above this fundamental frequency represent subharmonics and superharmonics of 4.5 Hz. Figure 3 shows a sequence of photographs of the same plume obtained by laser MicrO scattering from small water droplets. These photographs, obtained at 30 fps, suggest that a complete pulsation cycle (for example, frames 1 to 8) occurs at around 4.5 Hz. The probe acoustic response to pressure oscillations was estimated to be 13 ms for the probe/sensor configuration in our experiments. This value is considerably smaller than the typical period of studied plume oscillations in the range 0.07 to 0.5 s.

The velocity field of an oscillating buoyant plume of helium-air mixture was mapped using conditional laser Doppler velocimetry using the 10-cm-diam nozzle. The nozzle was surrounded by a co-flow chamber, as shown in Fig. 1, which provided the external particle seeding in the rapidly contracting region of the plume near the nozzle. The LDV system was a single component velocimeter in the backscatter mode. A continuous laser beam from a 6-W argon-ion laser (Innova 90-6) operating at 488 nm was utilized with transmitting and receiving optics as shown in Fig. 1. Laser beam was first passed through a tunable polarizer (TSI Model No. 9102-2), which polarized the light according to the measurement direction. A prismatic beam splitter (TSI 9115) was used to split the beam into two parallel paths 50 mm apart. A Bragg cell (TSI 9182-3A) shifted the frequency of one of the two beams by 1 MHz to measure the direction of the particles, as well as velocities passing through zero. A 122-mm focal length lens was used for focusing the two laser beams into the measurement location. The probe volume dimensions were 1.0 mm×3.8 mm. The receiving optics were configured in the backscatter mode and placed along the optical axis before the objective lens, which served for both transmission and collection of laser light. A photomultiplier tube (TSI 9140) collected the scattered light signal which was processed by TSI IFA 550 signal processor. TSI software, running on an IBM PC/XT computer was used to calibrate and set the filters of the IFA 550. Data collection was handled by a data acquisition system described below.

Data collection from the pressure transducer and the LDV system was handled by an IBM PC/XT computer interfaced with two Data Translation DT2801 A/D acquisition boards, running a FORTRAN controlling program. Two methods could be employed to carry out conditional data acquisition. One method is continuous acquisition of the “control” and the measured signals and postprocessing those data to obtain conditionally averaged quantities. Another method is to trigger the data acquisition only when the condition on the control signal is satisfied. The first method requires high data rates in order to reconstruct conditionally averaged velocities at all phases. However, the second method becomes desirable in situations where the data rate is dependent on uncontrollable factors such as local particle seeding density in the flow. For this reason, data were collected by this second method which was implemented in the following manner. First, the amplified pressure signal was sent to a digital oscilloscope (LeCroy Model 9852), where the external trigger was set at one of the four phases on the sinusoidal pressure signal: the maximum point (A), the negative slope zero crossing (B), the minimum point (C), and the positive slope zero crossing (D). The trigger signal from the oscilloscope was then directed to a “time window” generator. This device controlled the flow of data from the IFA 550 processor to the IBM PC/XT acquisition system. When the window controller received a trigger signal from the oscilloscope, it created a 10-ms time window during which the LDV signal was collected. At each spatial location, data acquisition continued until 100 data points were collected at each phase. Experimental uncertainty in the velocity measurement was less than 3% in the central part of the plume and up to 13% near the plume edges.

Measurements of velocity using the laser Doppler velocimetry technique require presence of small particles as light scattering media in the flow field. For LDV measurements, particles must be large enough to scatter a significant
portion of the incident light, but small enough to follow local flow accelerations. In the present study, the helium/air plume was seeded with water droplets from two different seeders. Seeding of the core helium-air flow was accomplished by a six jet droplet generator (TSI Model No. 9306) while the co-flow seeding was affected by a domestic ultrasonic humidifier. Droplets from both seeders were measured to be less than 2 μm in diameter by a phase Doppler particle sizer. The ability of these small droplets to follow local flow accelerations was assessed by calculating droplet acceleration from rest in a gas stream of constant velocity using droplet equation of motion accounting for weight and drag forces. It was determined that these droplets relaxed to 95% of gas velocity within a fraction of millisecond from the time of injection.

III. RESULTS AND DISCUSSION

A. Frequency measurements

As described in the previous section, a number of experiments were conducted to determine the relationship of plume oscillation frequency to plume parameters. The first set of experiments dealt with pure helium plumes emanating from nozzles with diameters of: 3.6, 5.2, 7.3, 10.0, 15.2, and 20.0 cm. In a second set of experiments, mixtures of helium and air were injected through the same nozzles to determine the effects of plume density on oscillatory plume behavior. The relevant parameters of this problem are nozzle exit velocity, \( V_0 \), nozzle diameter, \( d \), gravitational acceleration, \( g \), density of injected plume gas, \( \rho_p \), and the ambient density, \( \rho_a \). In case of pool fires and turbulent reacting jets, variable den-
sity effects arise locally due to chemical composition and temperature changes resulting from exothermic combustion reactions. In the absence of combustion, density variations occur due to compositional variations as a result of mixing of different density fluids. The aforementioned quantities can be formed into a number of nondimensional groups given by

- Strouhal number: \[ S = \frac{f d}{V} \]
  where \( V = V_0 \) or \( \sqrt{g d} \)
  or \( \sqrt{\Delta \rho g d / \rho} \)

- Density parameter: \[ \Pi_1 = \frac{(\rho_e - \rho_p)}{\rho_e} \]
  or \( \Pi_2 = \frac{(\rho_e - \rho_p)}{\rho_p} \)

- Richardson number: \( \text{Ri}_0 = \frac{g d V_0^2}{\rho p^\gamma} \) or \( \text{Ri}_1 = \frac{\Pi_1 \text{Ri}_0}{\rho p^\gamma} \)
  or \( \text{Ri}_2 = \frac{\Pi_2 \text{Ri}_0}{\rho p^\gamma} \)

- Froude number: \( \text{Fr}_f = \frac{R_i^j}{R_i^j - 1} \)

Strouhal number describes a nondimensional oscillation frequency. Density ratio can be constructed with either the ambient fluid or the plume density in the denominator as indicated by \( \Pi_1 \) or \( \Pi_2 \). Richardson (or inverse Froude) number is a measure of the ratio of buoyancy force to jet momentum. Combinations of these parameters, also being dimensionless, may also be constructed. While the primary dimensionless parameters have been listed in terms of the nozzle exit velocity, it can be envisioned that a more relevant velocity scale in highly buoyant flows could be taken as \( \sqrt{g d} \), or more appropriately as \( \sqrt{\Pi_1 g d} \) or \( \sqrt{\Pi_2 g d} \).

Experiments were conducted with variations in source diameter, nozzle exit velocity, and plume fluid density such that a correlation of the form \( S (R_i, \Pi) \) can be found. The plume gas density at the source was varied by mixing helium with air in different proportions. In Fig. 4, plume pulsation frequency data are plotted in the form of \( S (\text{Ri}_0) \). A reasonable correlation of data is obtained up to \( \text{Ri}_0 = 100 \). But, for \( \text{Ri}_0 > 100 \), data begin to scatter for helium-air mixture plumes as well as pure helium plumes originating from \( d = 15 \) and \( 20 \)-cm nozzles. These nondimensional parameters, also used by Hamins et al.\textsuperscript{17} to correlate diffusion flame and nonreacting plume pulsation frequencies, do not account for the density difference between the plume fluid and its surroundings which, in buoyant flows, provides the necessary driving force. While the nondimensional parameters of Fig. 4 produce a good correlation of diffusion flame and helium plume data, these parameters do not take account of differing plume fluid densities. Thus they cannot be the universal coordinates. The success of these parameters in correlating these data lies mostly in the similar densities for hydrocarbon-air diffusion flames as well as helium plumes injected into air, with density ratios, \( \rho_e / \rho_p \approx 7 \)–8. In order to introduce the effect of density, Richardson number could be modified as \( \text{Ri}_1 = \Pi_1 \text{Ri}_0 \) or \( \text{Ri}_2 = \Pi_2 \text{Ri}_0 \). Initial data were plotted using both parameters. It was found however that \( S (\text{Ri}_2) \) correlation was much poorer (not shown here) than \( S (\text{Ri}_1) \) even though \( \text{Ri}_2 \), with plume fluid density in the denominator, appears to be the more appropriate form of Richardson number at the nozzle exit. Figure 5 shows the correlation of data in the form of \( S (\Pi_1) \). It can be seen that the correlation of data improves for the helium-air plumes, especially for the 15-cm-diam nozzle, leading to the expression \( S = 0.8 \text{Ri}_1^{0.38} \). This correlation also agrees well with that developed by Hamins et al.\textsuperscript{17} The correlation coefficient for \( S = 0.8 \text{Ri}_1^{0.38} \) was found to be \( r^2 = 0.99 \) for the data with \( \text{Ri}_1 = 500 \), whereas correlation of data in Fig. 4 yielded \( r^2 = 0.96 \) for \( \text{Ri}_0 \leq 500 \). Better correlation of data with \( \text{Ri}_1 \) emphasizes the importance of the buoyancy term \( [(\rho_e - \rho_p) g d] \). The only difference between \( \text{Ri}_1 \) and \( \text{Ri}_2 \) is the density in the denominator as \( \rho_e \) or \( \rho_p \), respectively. The inability of \( \text{Ri}_2 \) to correlate the data may point to the fact that source density alone is not an important scaling parameter, rather a reference density such as that of the ambient is more appropriate.

While \( S (\text{Ri}_1) \) correlation is satisfactory for the majority of the data, helium plume data for 15- and 20-cm nozzle diameters do not follow this correlation beyond \( \text{Ri}_1 = 100 \). In the region \( \text{Ri}_1 > 100 \), frequency data appear more scattered and exhibit a different scaling as \( S \times \text{Ri}_1^{0.28} \). A closer examination of the experimental data for \( d = 15 \) and 20-cm-diam nozzles indicate that significant changes occur in plume structure between \( \text{Ri}_1 = 100 \) and 500. This transition in plume pulsation frequency had not been observed before to the best

![FIG. 4. Correlation of pulsation frequency in terms of Strouhal number and Richardson number, \( \text{Ri}_0 \). Helium plumes: \( \nabla d = 3.6 \text{ cm}, \bigcirc d = 5.2 \text{ cm}, \bigcirc d = 7.3 \text{ cm}, \Delta d = 10.2 \text{ cm}, \bigtriangledown d = 15.2 \text{ cm}. \) Helium-air mixtures: \( \blacklozenge d = 3.6 \text{ cm}, \blacklozenge d = 5.2 \text{ cm}, \blacktriangle d = 7.3 \text{ cm}, \blacklozenge d = 10.2 \text{ cm}, \blacklozenge d = 15.2 \text{ cm}. \) Experiments were conducted with variations in source diameter, nozzle exit velocity, and plume fluid density such that a correlation of the form \( S (\text{Ri}_1, \Pi) \) can be found. The plume gas density at the source was varied by mixing helium with air in different proportions. In Fig. 4, plume pulsation frequency data are plotted in the form of \( S (\text{Ri}_0) \). A reasonable correlation of data is obtained up to \( \text{Ri}_0 = 100 \). But, for \( \text{Ri}_0 > 100 \), data begin to scatter for helium-air mixture plumes as well as pure helium plumes originating from \( d = 15 \) and \( 20 \)-cm nozzles. These nondimensional parameters, also used by Hamins et al.\textsuperscript{17} to correlate diffusion flame and nonreacting plume pulsation frequencies, do not account for the density difference between the plume fluid and its surroundings which, in buoyant flows, provides the necessary driving force. While the nondimensional parameters of Fig. 4 produce a good correlation of diffusion flame and helium plume data, these parameters do not take account of differing plume fluid densities. Thus they cannot be the universal coordinates. The success of these parameters in correlating these data lies mostly in the similar densities for hydrocarbon-air diffusion flames as well as helium plumes injected into air, with density ratios, \( \rho_e / \rho_p \approx 7 \)–8. In order to introduce the effect of density, Richardson number could be modified as \( \text{Ri}_1 = \Pi_1 \text{Ri}_0 \) or \( \text{Ri}_2 = \Pi_2 \text{Ri}_0 \). Initial data were plotted using both parameters. It was found however that \( S (\text{Ri}_2) \) correlation was much poorer (not shown here) than \( S (\text{Ri}_1) \) even though \( \text{Ri}_2 \), with plume fluid density in the denominator, appears to be the more appropriate form of Richardson number at the nozzle exit. Figure 5 shows the correlation of data in the form of \( S (\Pi_1) \). It can be seen that the correlation of data improves for the helium-air plumes, especially for the 15-cm-diam nozzle, leading to the expression \( S = 0.8 \text{Ri}_1^{0.38} \). This correlation also agrees well with that developed by Hamins et al.\textsuperscript{17} The correlation coefficient for \( S = 0.8 \text{Ri}_1^{0.38} \) was found to be \( r^2 = 0.99 \) for the data with \( \text{Ri}_1 = 500 \), whereas correlation of data in Fig. 4 yielded \( r^2 = 0.96 \) for \( \text{Ri}_0 \leq 500 \). Better correlation of data with \( \text{Ri}_1 \) emphasizes the importance of the buoyancy term \( [(\rho_e - \rho_p) g d] \). The only difference between \( \text{Ri}_1 \) and \( \text{Ri}_2 \) is the density in the denominator as \( \rho_e \) or \( \rho_p \), respectively. The inability of \( \text{Ri}_2 \) to correlate the data may point to the fact that source density alone is not an important scaling parameter, rather a reference density such as that of the ambient is more appropriate.

While \( S (\text{Ri}_1) \) correlation is satisfactory for the majority of the data, helium plume data for 15- and 20-cm nozzle diameters do not follow this correlation beyond \( \text{Ri}_1 = 100 \). In the region \( \text{Ri}_1 > 100 \), frequency data appear more scattered and exhibit a different scaling as \( S \times \text{Ri}_1^{0.28} \). A closer examination of the experimental data for \( d = 15 \) and 20-cm-diam nozzles indicate that significant changes occur in plume structure between \( \text{Ri}_1 = 100 \) and 500. This transition in plume pulsation frequency had not been observed before to the best
of our knowledge. It will be the subject of further discussion later in this article.

Pulsation frequencies were nondimensionalized using the nozzle exit velocity in the correlations presented above and also by Hamins et al.\textsuperscript{17} Although the source velocity plays a role very close to the nozzle exit, it is expected that the buoyancy induced velocity rapidly exceeds the magnitude of the source velocity and therefore, a buoyant velocity scale may be more appropriate. In Fig. 6, Strouhal numbers based on two different velocity scales were plotted as a function of Ri\textsubscript{1}. Figure 6(a) shows the Strouhal number, S\textsubscript{1} based on the buoyant velocity scale of $\sqrt{gd}$. It appears that this scaling results in poor correlation of data for individual data sets as well as the large amount of scatter for some of the data. We believe however, this is not the proper velocity scale since it does not contain the density difference which is the driving force in buoyant flows. Thus, a modified velocity scale of $\sqrt{(\rho_\infty - \rho_p)gd/\rho_\infty}$ was introduced in Fig. 6(b). This velocity scale produced a correlation of similar quality to that of Fig. 5 for all data. The resulting correlation can be expressed as $S_2 = 0.8 \text{ Ri}_1^{-0.12}$ for $\text{Ri}_1 < 100$. The two correlations can be easily shown to be identical.

We now turn our attention to the transition in the pulsation frequency data occurring between $\text{Ri}_1 = 100$ and 500. In order to determine the origin of this transition, visual observations, and frequency spectra of total pressure fluctuations for the two pure helium plumes below and above this transition were compared. In Fig. 7, a sequence of three photographs are shown for the two plumes whose pulsation frequencies lie below and above the transition. The photographs on the left [Fig. 7(a)] are for a pure helium plume originating from a 10-cm-diam nozzle with $\text{Ri}_1 = 220$, $\text{Re}_d = 50$. These photographs, obtained by planar Mie scattering from a laser light sheet, indicate that the plume starts with a sharply contracting laminar interface between seeded core flow of helium (bright regions) and surrounding air (dark regions). As the column of helium rises, a vortical ring is formed with its core first appearing at a height of approximately one-half nozzle diameter above the nozzle exit. Upstream of the vortex in the columnar zone, there is an internal region which is

FIG. 5. Correlation of pulsation frequency in terms of Strouhal number and Richardson number, $\text{Ri}_1$. Data legend is same as Fig. 4.

FIG. 6. Correlation of pulsation frequency in terms of different definitions of Strouhal numbers (a) $S_1 = f \sqrt{df}$. (b) $S_2 = f \sqrt{df/(1 - \rho_d/\rho_\infty)}$ as a function of $\text{Ri}_1$. Data legend is same as Fig. 4.
completely devoid of any seeded helium flow, indicating that some of the surrounding air had penetrated into this region. It is interesting to note that this penetration extends close to the nozzle exit plane, in the form of a bubble. It is not clear at this point how this unseeded stream, presumed to be ambient air, penetrates into the core of the helium stream. Downstream of the vortex, flow evolves progressively toward a more turbulent state starting with a collection of wrinkled laminar diffusion layers.

The three photographs on the right [Fig. 7(b)] belong to a pure helium plume emanating from the 20-cm-diam nozzle with $Ri_1 = 470$ and $Re_d = 96$. This plume looks clearly quite different than the plume on the left. It is significantly more turbulent with large regions of unseeded ambient air, appearing in the core regions of the flow. Periodic pulsations and vortex rings, although less apparent visually, are still present and can be detected from the velocity fluctuations along the plume centerline. In this rapidly accelerating region of buoyant plumes, this transition to turbulence is believed to be a result of locally high shear rates between the plume fluid and the ambient. In fact, a critical value of Richardson or Froude number may be the relevant transition criterion replacing Reynolds number employed in nonbuoyant flows. The total pressure oscillations at the plume centerline along with the corresponding frequency spectra are shown in Fig. 8 for these two plumes. The pressure-time trace for the helium plume from 10-cm nozzle is more periodic than that for the 20-cm-diam nozzle. Correspondingly, the frequency spectrum for 10-cm nozzle contains a single peak associated with vortex shedding. On the contrary, frequency spectrum for the 20-cm nozzle is much more dispersed, indicating a less periodic nature of the oscillations for the turbulent plume. The change in the frequency scaling will be explained based on these observations later in this article.

B. Plume velocity field

Phase-resolved velocity measurements were performed in a helium-air mixture plume originating from a 10-cm-diam nozzle using laser Doppler velocimetry. Plume parameters were $Ri_1 = 50$, $Ri_2 = 195$, plume density ratio, $\rho_p/\rho_a = 0.26$. The co-flow velocity was 2.4 cm/s, providing minimal interference to the early part of plume development. Measurements were concentrated in a region spanning one nozzle diameter height from the nozzle centerline to the nozzle edge. Radial extent of these measurements was limited by available seeding. Although a co-flow chamber provided external seeding at low velocity outside the plume boundary, this stream was quickly ingested into the plume near the vortex formation region. Figure 9 shows the velocity measurements at four instants during one pulsation period. Phase A [Fig. 9(a)] corresponds to the maximum in the pres-
sure signal, followed by phase B [Fig. 9(b)] at the negative slope zero crossing, phase C [Fig. 9(c)] at the minimum and phase D [Fig. 9(d)] at the positive slope zero crossing. At all phases of plume motion, a strong buoyant acceleration along the plume centerline is evident. Centerline velocity increases from few tens of centimeters per second to over one meter per second over a distance of several centimeters.

At phase A, representing the instant at which plume centerline velocity at $z = d/2$ is maximum, strong axial acceleration occurs over the central 10- to 15-mm radial region. This is also the region where a toroidal vortex ring is first formed. In fact, the toroidal vortex is formed in a region centered around $z \approx d/2$ as it can be seen in Fig. 3 in frames 2, 9, and 16. The axial acceleration is also accompanied by a strong radial inflow toward the plume axis near the base. At phase B, toroidal vortex has convected downstream and continues its influence by its higher induced flow velocities behind it. The radial velocity components are somewhat smaller than those at phase A, when the vortex was closer to the nozzle. At phase C, the toroidal vortex has completely moved out of the measurement region. However, a strong centerline acceleration is still maintained at this instant with lower velocity magnitudes for $z \leq d/2$. Finally, at phase D, the velocity field becomes more uniform and vertically oriented across the plume cross section. The strong flow acceleration is reestablished between phases D and A.

In addition to the vector plots delineating the general features of the velocity field, the vertical velocity components are plotted in Fig. 10. Velocity profiles near the nozzle exit are relatively uniform in the central portion of the nozzle exit with rapid acceleration ensuing a short distance downstream. In particular, velocity profiles deviate considerably from the conventional Gaussian profiles in the region of the toroidal vortex. The velocity profiles exhibit inflection points between $0.5 < z/d < 0.8$. In this vortex formation region, superposition of viscous vortex velocity profile on its convection speed could account for these shapes of the velocity distributions. Downstream convection of the vortex can be followed in this figure as indicated by roman numerals.

In understanding plume oscillation mechanism and the underlying flow dynamics, characteristics of buoyant acceleration along the plume centerline can provide valuable information. Figure 11 shows plume centerline velocity distributions at four instants during one pulsation cycle. The most striking feature is the rapid acceleration at phase A, followed by a deceleration with the peak velocities occurring around $z/d \approx 0.6$. It is precisely in this region where the toroidal vortex formation is initiated by transfer of axial momentum.

![Fig. 9](image-url)
to radial motion. At later phases (B and C), flow velocities are lower below this location due to stagnation behind the vortex and higher above because of the induced suction behind the convecting toroidal vortex. These results imply, as suggested earlier, that toroidal vortex formation is a result of rapid buoyant acceleration of light plume fluid in heavier, more or less quiescent surroundings. Once this toroidal vortex forms, it affects the surrounding flow field as it convects upward. Then, its influence decays near the plume source, setting up a strong acceleration in the lower region of a buoyant, nonreacting plume at the start of the next cycle.

C. Characteristics of the instability

Pulsations in buoyant plumes are characterized by periodic oscillations of plume boundaries in the region of rapid contraction near its source as described earlier. These oscillations grow into toroidal ring vortices downstream evolving in the region $d/2 \leq z \leq d$. This type of instability has been observed and reported extensively (see for example the citations in Ref. 16) in pool flames of gaseous, liquid, and solid fuels. Although these investigations reported pulsation frequencies, no comprehensive effort was undertaken to understand the nature of this instability. Cetegen and Ahmed carried out experiments on simulated pool flames of propane, nonreacting hot gas plumes and helium plumes with the objective of explaining the instability mechanism by altering plume behavior, introducing a variety of physical interferences. Experiments indicated that pulsations were sensitive to perturbations around the nozzle lip and the region of rapid contraction above it. By introducing vertical and horizontal screens in this vicinity, plume oscillations could be suppressed to a great extent. It was also found that plume oscillation frequency was mildly influenced by the magnitude of source velocity. It was insensitive to changes in heat release per unit volume achieved by inert dilution of the fuel stream. It was suggested there that the pulsations are connected to disturbances introduced into the Rayleigh–Taylor unstable region of the flow characterized by a high density fluid lying on top of a lower density plume effluent. Computed shapes of buoyant plume interfaces are shown in Fig. 12 for different values of Richardson number. These interface shapes were computed for inviscid acceleration of the plume fluid with height for an assumed parabolic radial distribution of axial plume velocity, invoking the mass conservation without any mixing with the ambient air. The sharp contraction of the plume interface above the nozzle results in an unstable density configuration with the light plume fluid residing under the heavier ambient fluid. In contrast to this unstable density stratification for nonreacting plumes, diffusion flames possess a density distribution where relatively hot light fuel stream lies below the high temperature, low density diffusion flame zone bounded on top by the heavier ambient air.

FIG. 10. Vertical component of velocity vector throughout the lower part of the plume in Fig. 9.

FIG. 11. Plume centerline velocity profiles of the same plume depicted in Figs. 9 and 10.
bution are somewhat different than those for nonreacting plumes, the overall instability of the density stratification appears to produce similar effects.

The perturbations to this unstable density stratification could rapidly amplify under the action of gravity. The perturbations to this unstable layer can arise from either shear layer instability along this interface or some other flow disturbance upstream or a downstream disturbance providing a feedback mechanism, such as the induced velocity field of a toroidal vortex downstream. Any perturbations originating upstream of nozzle exit would be greatly damped by the high resistance screen placed at the exit of the nozzle and thus could be excluded. While there is some debate about the exact nature of the feedback mechanism, our earlier experiments suggest that the toroidal vortex feedback is the most likely source of excitation. Our earlier flame experiments have shown that oscillations could be suppressed to a large extent when the coherence of the downstream vortex is destroyed artificially by screens. Furthermore, scaling of oscillation frequency is consistent with the convective time scale of these vortical structures as it will be explained below.

If we accept for the moment that the oscillatory state is sustained by perturbations resulting from the induced velocity field around the toroidal vortex, we can relate the pulsation frequency to the convection velocity of these vortices. Pulsation frequency is given by

\[ f = \frac{1}{\tau} = \frac{U_c}{d}, \]

(1)

where \( f \) is the pulsation frequency, \( \tau \) is the pulsation period, \( d \) is the distance (nozzle diameter) within which the vortex formation occurs. Vortex convection velocity, \( U_c \), can be related to the buoyant gas velocity along the plume axis by \( U_c = CU \), where \( U \) is the centerline gas velocity and \( C \) is a constant of proportionality. The inviscid gas momentum balance along the plume axis can be expressed as

\[ U \frac{dU}{dz} = \frac{\Delta \rho}{\rho} g, \]

(2)

where the density difference is in general a function of the axial coordinate \( z \). Integrating Eq. (2) with respect to \( z \) results in an expression for \( U(z) \),

\[ U(z) = \left[ U^2(0) + 2g \int_0^z \frac{\Delta \rho}{\rho} (z) dz \right]^{1/2}. \]

(3)

The frequency of pulsations can be extracted from the kinematic relation,

\[ \frac{dz_v}{dT} = CU(z_v), \]

(4)

where \( z_v \) is the axial location of the vortex. Substituting for \( U(z_v) \) from Eq. (3) under the approximation that \( U(z_v) \gg U(0) \) for \( z_{min} < z_v < d \) and integrating between \( z_v = z_{min} = 0 \) and \( z_v = d \), yields the frequency or period as

\[ f = \frac{1}{\tau} = \frac{C \sqrt{2g}}{\int_0^d \left[ \int_0^{z_v} \frac{\Delta \rho}{\rho} (z) dz \right]^{1/2}} dz_v. \]

(5)

In flames, density gradients are maintained due to local heat release of combustion, constantly supplying buoyancy to the flow with increasing downstream distance within the flame height. Thus \( \Delta \rho/\rho = \Delta T/\rho c_m \) is constant locally in flames where \( \Delta T_f \) represents the temperature rise in a typical hydrocarbon diffusion flame. If the density gradient is assumed to be constant, Eq. (5) yields for flames:

\[ f = C \sqrt{\frac{\Delta T_f}{2T_m d}}. \]

(6)

This result is in good agreement with a large body of experimental data (see the references cited in Ref. 16). It is indeed remarkable that data for diffusion flames originating from a wide range of source diameter \((10^{-2} \text{ to } 10^2 \text{ m}) \) fueled by many different types of fuels follow this correlation closely. It is also interesting to note that results of Becker and Massaro for nonreacting jets also suggest a similar scaling of oscillation frequency with inverse square root of nozzle diameter, although they studied completely momentum dominated constant density jets. Because of the significant differences in the two flows, this similarity is considered coincidental.

Now, we turn our attention to nonreacting buoyant plumes with a fixed source buoyancy flux which is continuously redistributed downstream due to mixing. Local buoyancy in the flow consequently decays with downstream distance. Expressing the density deficiency as a decaying function of distance by \( \Delta \rho/\rho(z) = [\Delta \rho/\rho(z)]_0 e^{-mz} \), where \( m < 0 \), and substituting this expression into Eq. (5) and integrating,

\[ f = \frac{1}{\tau} = C (1 - m) \sqrt{\frac{g}{2(1 + m)}} \left[ \frac{\Delta \rho}{\rho} \left| \frac{1}{\rho} \right| dz_v \right]^{1/2}. \]

(7)

Depending on the decay rate of the density deficiency, present model predicts a different scaling for the buoyant plume oscillation frequency. First looking at the region below the transition where \( S = 0.8 \text{ Ri}^{0.38} \), pulsation frequency scales as \( f \propto d^{-0.62} \). Comparing this experimental result with the above expression, the value of decay exponent is found to be \( m = -0.24 \). Although we have no direct way of verifying this decay exponent, we presented some experimental

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FIG. 12. Calculated shapes of buoyant plume boundaries based on conservation of mass for different Richardson numbers.
Phase resolved velocity measurements in a periodic helium/air plume clearly show the various stages during one oscillation cycle. A rapid buoyant acceleration of plume fluid near the centerline is followed by the formation of a toroidal vortex ring around a height of one-half nozzle diameter. As this vortex ring convects downstream, it retains its influence on the upstream flow field. Finally, buoyant acceleration re-establishes and the formation of the next toroidal vortex is initiated. Velocity profiles within the plume are influenced by the presence of the toroidal vortex.

IV. CONCLUSIONS

It has been found experimentally that nonreacting buoyant plumes undergo quasiperiodic oscillations of the type observed in pool fires. Although the characteristics of this plume instability appear to be similar in both cases, frequency scaling with the source diameter is somewhat different. Furthermore, a transition in the scaling of buoyant plume pulsation frequency is observed in our experiments. In the earlier correlations of pulsation frequency of the form \( f \propto d^{0.72} \), with a value of \( m = -0.44 \). This result suggests a more rapid decay of density deficiency with downstream distance than the previous case. Thus transition to turbulence and the associated faster decay of density along the plume axis are believed to be a reasonable cause for the change in the observed frequency scaling.

Transition to turbulence in the present flow configuration is not believed to occur as a result of a change in the flow characteristics at the nozzle exit as the Reynolds number is little different between the two cases discussed earlier and depicted in Figs. 7 and 8. It is however believed to be due to the high shear rates along the curved plume boundary which are greatly amplified with increasing Richardson number. Experiments suggest a critical value of this Richardson number between 100 and 500.

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It has been found experimentally that nonreacting buoyant plumes undergo quasiperiodic oscillations of the type observed in pool fires. Although the characteristics of this plume instability appear to be similar in both cases, frequency scaling with the source diameter is somewhat different. Furthermore, a transition in the scaling of buoyant plume pulsation frequency is observed in our experiments. In the earlier correlations of pulsation frequency of the form \( f \propto d^{0.72} \), the density difference between the plume fluid and the surrounding medium was omitted. Our experimental data for plumes of helium and helium-air mixtures suggest correlations of the form \( S = \frac{f}{d/V_0} \) or \( S_2 = \frac{f}{\left( \frac{p_V}{d} \left( \frac{\rho_0 - \rho_p}{\rho_d} \right) \right)^{1/2}} \) and Richardson number is defined as \( R_i = \left( \frac{d}{\rho_0 \rho_p} \right)^{1/2} \). Our experimental data for pure helium plumes and plumes of helium-air mixtures fit well into the correlations expressed as \( S = 0.8 R_i^{0.38} \) or \( S_2 = 0.8 R_i^{0.12} \) for \( R_i < 100 \). Both of these correlations can be easily shown to be identical. Experimental correlation of pulsation frequency also suggests that a transition occurs in the scaling between \( R_i = 100 \) and 500. Beyond this transition, \( S = 2.1 R_i^{0.28} \) or \( S_2 = 2.1 R_i^{-0.22} \). This transition is found to be associated with turbulent mixing of the plume fluid with its surroundings close to the nozzle exit. We believe this transition results from locally high shear rates around the rapidly contracting regions of plume boundaries which can possibly be characterized by a critical value of Richardson number.

The mechanism of the instability leading to the periodic oscillatory state of the flow field is connected with the highly unstable (Rayleigh–Taylor) density stratification in the sharply contracting region of the flow just above the nozzle exit as perturbed by the downstream vortex-induced feedback. The proposed feedback mechanism fits well with the observed frequency scaling and the transition in the frequency scaling due to turbulence.