Adaptive Control with Unknown Parameters in Reference Input

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Abstract—In this paper, Model Reference Adaptive Control problem is considered with reference input dependent upon the unknown parameters of the system. Due to the uncertainty in the reference input, conventional linear-in-parameters adaptive controller cannot achieve the control objective without enforcing persistent excitation (PE). A new technique is presented for introducing an excitation signal and regulating its magnitude, dependent upon the convergence of the output tracking and parameter errors. Intelligent excitation ensures parameter convergence similar to conventional PE, but initiates only when necessary. As a result, the regulated output tracks the unknown reference input. Simulations illustrate the theoretical findings.

I. INTRODUCTION

Conventional Model Reference Adaptive Controller (MRAC) is designed to track a given reference trajectory. However, in realistic applications, very often the reference trajectory is not specified apriori and depends upon the unknown parameters of system dynamics. One such example is the target tracking problem with visual sensors, where the characteristic length of the target is unknown and the relative range is not observable from available measurements [1]. Another example is the autopilot design problem for a trailing aircraft that follows a lead aircraft in a closed-coupled formation, where the trailing aircraft must constantly seek an optimal position relative to the leader to minimize the unknown drag effects introduced by the wing tip vortices of the lead aircraft [2].

Motivated by practical applications, in this paper we generalize the conventional model reference adaptive control framework for a reference input, dependent upon the unknown parameters of the system dynamics. Following the methodologies in [3], [4], one can enforce PE condition to ensure parameter convergence, however a constant PE deteriorates the output tracking and the control objective cannot be met. A new technique, named intelligent excitation, is presented to solve the output tracking problem while ensuring parameter convergence. The main feature of it is that it initiates excitation only when necessary, and it vanishes as the parameter error converges to a neighborhood of the origin.

The paper is organized as follows. Section II presents the problem formulation for the adaptive control with reference input dependent upon the unknown parameters of the system. Intelligent excitation is introduced in Section III. The convergence of the regulated output to the desired value is shown in Section IV. In Section V, we illustrate the simulation results, and Section VI concludes the paper.

II. PROBLEM FORMULATION

Consider the following single-input single-output system dynamics:

\[
\begin{align*}
\dot{x}(t) &= (A)x(t) + (\lambda)u(t) + (\epsilon) \\
\end{align*}
\]

where \( x \in \mathbb{R}^n \) is the system state vector (measurable), \( A \in \mathbb{R}^{n \times n} \) is the control signal, \( \lambda \in \mathbb{R}^n \) is a known constant vector, \( r \in \mathbb{R}^n \) is a known constant vector, \( \lambda \in \mathbb{R}^n \) is an unknown constant with known sign, \( r \in \mathbb{R}^n \) is the regulated output. The control objective is to enforce the output \( x(t) \) to track \( r(t) \), where \( r \) is a known map : \( \mathbb{R}^{n \times n} \times \mathbb{R} \rightarrow \mathbb{R} \), dependent upon unknown parameters of the system and .

As in classical model reference adaptive control (MRAC), we introduce the following assumption:

Assumption 1: There exist a Hurwitz matrix \( m \in \mathbb{R}^{n \times n} \), a column vector \( m \in \mathbb{R}^n \), ideal constant parameters \( x \in \mathbb{R}^n \), \( r \in \mathbb{R}^n \) such that

\[
\begin{align*}
\bullet & \quad (m_m) \text{ is controllable} \\
\bullet & \quad m = (\mathbf{a}^T) \\
\bullet & \quad r = m
\end{align*}
\]

In addition, we assume that \( r \) belongs to a compact set: \( \Theta_r \), where \( \min_r \) and \( \max_r \) have the same sign. Hence, the region, where the unknown parameter \( r \) lies, can be characterized as:

\[
\Theta_{r_{\min}} = \frac{m}{\max_r} \quad \Theta_{r_{\max}} = \frac{m}{\min_r}
\]

In case of a known reference signal \( r \), conventional MRAC provides a solution to the above problem formulation. The challenge here is that the reference signal \( r \) depends upon unknown parameters and is therefore unknown, even if the map \( r \) is given.
III. ADAPTIVE CONTROLLER USING INTELLIGENT EXCITATION

In this section, we will propose a new intelligent excitation technique and incorporate it into conventional MRAC to solve the tracking problem in the presence of unknown \( \theta \). In section III-A, we will recall the traditional MRAC for the system in (1). In section III-B, we will introduce the concept of intelligent excitation to achieve the control objective.

A. Model Reference Adaptive Controller

In a traditional MRAC scheme, for any bounded reference input signal \( \hat{\gamma}(t) \), a reference model is given by:

\[
\dot{m}(t) = m(m(t) + m g \hat{\gamma}(t)) \\
m(t) = \tau(m(t))
\]

(3)

where \( m \) and \( \dot{m} \) are defined in Assumption 1, is the same as in (1) and \( g \in \mathbb{R} \) is a design gain, defined by

\[
g = \lim_{s \to 0} \frac{1}{\tau(-m)^{-1} m}
\]

(4)

In case if \( \hat{\gamma}(t) = \hat{\gamma} \), where \( \hat{\gamma} \) is a constant, one can substitute (4) into the transfer function between \( \hat{\gamma} \) and \( m \)

\[
\frac{m(t)}{\hat{\gamma}(t)} = g \tau(-m)^{-1} m
\]

and apply the final value theorem to arrive at

\[
\lim_{t \to \infty} m(t) = \hat{\gamma}
\]

(5)

Direct adaptive model reference feedback/feedforward control signal is defined as

\[
\gamma(t) = \frac{m(t)}{\hat{\gamma}(t)} + r(t) g \hat{\gamma}(t)
\]

(6)

where \( x(t) \in \mathbb{R}^{n} \) and \( r(t) \in \mathbb{R} \) are the adaptive gains. Substituting (6) into (1), the closed-loop system dynamics are:

\[
\dot{x}(t) = (m - (x)^T + r(t) g \hat{\gamma}(t))
\]

(7)

It follows from Assumption 1 that the system in (7) can be rewritten:

\[
\dot{x}(t) = (m - (x)^T + r(t) g \hat{\gamma}(t))
\]

(8)

and the reference model in (3) can be rewritten:

\[
\dot{m}(t) = m(m(t) + \frac{\dot{\gamma}(t)}{r(t)})
\]

(9)

Let \( \gamma(t) = (t - m(t) \) be the tracking error. Then the tracking error dynamics are:

\[
\dot{\gamma}(t) = m(t) + ((x)^T + r(t) g \hat{\gamma}(t))
\]

(10)

where \( \dot{x}(t) = x(t) - x_{\ast} \) and \( \dot{r}(t) = r(t) - r_{\ast} \) denote parameter errors. Let \( \tau(t) \) be the solution of the algebraic equation \( \tau m + m = -r(t) \) for arbitrary 0. Consider the following adaptive laws, by invoking the projection operator [7]:

\[
\dot{x}(t) = -v_{x} \tau(t) \text{ sgn}(t) \\
\dot{r}(t) = \text{Proj}(-v_{r} \tau(t) \text{ sgn}(t) + r(t))
\]

(11)

where \( v_{x} \) and \( v_{r} \) are positive design gains, \( \Theta_{r_{\ast}} \) and \( \Theta_{r_{\ast}} \) are defined in (2). Define the Lyapunov function candidate

\[
V(t) = \tau(t) + \gamma(t)^T x(t) + \gamma(t)^T r(t)
\]

(12)

It can be verified easily that

\[
\dot{V}(t) \leq -\tau(t) \gamma(t) \leq 0
\]

(13)

Hence all error signals are bounded. Application of the Barbalat’s lemma further implies asymptotic convergence of the tracking error to zero, i.e.,

\[
\lim_{t \to \infty} \gamma(t) = 0
\]

(14)

If \( \hat{\gamma}(t) = \hat{\gamma} = \gamma \), it follows from (5) and (14) that

\[
\lim_{t \to \infty} \gamma(t) = 0
\]

(15)

However, nothing can be claimed for the convergence of parameter errors to zero. Since \( \hat{\gamma}(t) \) are unknown, then the reference input \( \gamma(t) \) is not available. We can use \( \hat{\gamma}(t) \), the estimates of \( \gamma(t) \), to construct \( \hat{\gamma}(t) \). However without parameter convergence, there is no guarantee of \( \gamma(t) \rightarrow 0 \) as \( t \rightarrow \infty \). Next, we augment \( \hat{\gamma}(t) \) with an intelligent excitation signal to redefine the reference input \( \hat{\gamma}(t) \) to meet the control objective.

B. Adaptive Controller with Intelligent Excitation

Let

\[
\hat{\gamma}(t) = \frac{m(t)}{\hat{\gamma}(t)} + \hat{\gamma}(t) + \hat{\gamma}(t)^T x(t)
\]

(16)

If

\[
\lim_{t \to \infty} \hat{\gamma}(t) = 0 \quad \lim_{t \to \infty} \hat{\gamma}(t) = 0
\]

(17)

then it can be verified easily that

\[
\lim_{t \to \infty} \hat{\gamma}(t) = \lim_{t \to \infty} \hat{\gamma}(t) = 0
\]

(18)

Let the reference signal \( \hat{\gamma}(t) \) be

\[
\hat{\gamma}(t) = \hat{\gamma}(t) + x(t)
\]

(19)

where \( \hat{\gamma}(t) = \hat{\gamma}(t) + \hat{\gamma}(t) \) denote parameter errors. Let \( \tau(t) \) be the solution of the algebraic equation \( \tau m + m = -r(t) \) for arbitrary 0. Consider the following adaptive laws, by invoking the projection operator [7]:

\[
x(t) = x(t) \sum_{i=1}^{m} \sin(i)
\]

(20)

where \( x_{i} \), \( x_{i} \), \( x_{i} \), \( x_{i} \), and \( i = 1 \) are positive design gains, \( \Theta_{r_{\ast}} \) and \( \Theta_{r_{\ast}} \) are defined in (2). Define the Lyapunov function candidate

\[
V(t) = \tau(t) + \gamma(t)^T x(t) + \gamma(t)^T r(t)
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It can be verified easily that

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However, nothing can be claimed for the convergence of parameter errors to zero. Since \( \hat{\gamma}(t) \) are unknown, then the reference input \( \gamma(t) \) is not available. We can use \( \hat{\gamma}(t) \), the estimates of \( \gamma(t) \), to construct \( \hat{\gamma}(t) \). However without parameter convergence, there is no guarantee of \( \gamma(t) \rightarrow 0 \) as \( t \rightarrow \infty \). Next, we augment \( \hat{\gamma}(t) \) with an intelligent excitation signal to redefine the reference input \( \hat{\gamma}(t) \) to meet the control objective.
It follows from (3) that the transfer function from \( \hat{y}(\cdot) \) to \( m(\cdot) \) is:
\[
\frac{m(\cdot)}{\hat{y}(\cdot)} = ( - m)^{-1} m g \tag{21}
\]

Let
\[
\det( - m)^{-1} m g = \frac{n(s)}{d(s)} \tag{22}
\]

It follows from (21) that
\[
(\cdot) = [1 \ (( - m)^{-1} m g)]^T \tag{23}
\]

We define a \((+1)\times 2\) matrix \( \Omega \) with its \(\tau\)th row, \(\theta\)th column element as
\[
\Omega_{pq} = \left\{ \begin{array}{ll}
\text{Re}(p(\cdot)) & \text{if is odd} \\
\text{Im}(p(\cdot)) & \text{if is even}
\end{array} \right. \tag{24}
\]

where \(\begin{bmatrix} \frac{n}{2} \end{bmatrix}\) denotes the smallest integer that is greater than \(\frac{n}{2}\) and \(\bar{p}(\cdot)\) is the \(\tau\)th element of \((\cdot)\) defined in (23). We choose \(1 \leq \tau \leq m\) to ensure that \(\Omega\) has full row rank. The design gain \(x_2\) is chosen as
\[
x_2 = \frac{2}{\min_{i=1,\ldots,m} i} \tag{25}
\]

Details on how to choose the constants \(x_1, x_3, x_4\), will be discussed in section IV.

The transfer function \(( - m)^{-1} m g\) can be expressed as:
\[
( - m)^{-1} m g = \frac{n(s)}{d(s)} \tag{24}
\]

where \(\frac{n(s)}{d(s)}\) is a \(\tau\)th order polynomial, and \((\cdot)\) is a \(1 \times 1\) vector with its \(\tau\)th element being a polynomial function:
\[
(\cdot) = \sum_{j=1}^{n} i_j j^{-1} \tag{26}
\]

**Lemma 1:** If \((m, m)\) is controllable, the matrix of the entries \(i_j\) defined in (26) is full rank.

**Proof of Lemma 1:** Controllability of \((m, m)\) for the time-invariant linear system in (3) implies that for arbitrary \(\hat{y} \in \mathbb{R}^n\), \(m(0) = 0\) and arbitrary \(1\), there exists \(\cdot \in [0, 1]\) such that
\[
m(\cdot) = \hat{y} \tag{27}
\]

If \(m(0)\) is not full rank, then there exists \(\neq 0 \in \mathbb{R}^n\) such that
\[
\tau (\cdot) = 0 \tag{28}
\]

If \(m(0) = 0\), for any \((\cdot) \neq 0\), it follows from (28) that
\[
\tau m(\cdot) = 0 \quad \forall \quad 0 \tag{29}
\]

This contradicts (27), in which \(\hat{y} \in \mathbb{R}^n\) is assumed to be an arbitrary point. Therefore, \(\cdot\) must be full rank.

**Remark 1:** The existence of \(\cdot\) and \(1 \leq \tau \leq m\) can be verified easily. It follows from Assumption 1 and Lemma 1 that \(\cdot\) is full rank, and hence \((\cdot)\) contains linearly independent polynomials. We can rewrite the transfer function in (23) as:
\[
(\cdot) = \frac{1}{(\cdot)} [1 (\cdot) \ldots m(\cdot)]^T \tag{30}
\]

Since \((\cdot)\) is a \(\tau\)th order polynomial, and \(i(\cdot)\) are \((-1)^\tau\)th order polynomials, then \((\cdot)\) and \(i(\cdot)\) are linearly independent. Hence, \((\cdot)\) contains \(+1\) linearly independent functions of \(\cdot\), and therefore there exist
\[
\cdot \leq \cdots \leq m, \text{ which ensure that } \Omega \text{ has full row rank.}
\]

**IV. CONVERGENCE RESULT OF ADAPTIVE CONTROLLER WITH INTELLIGENT EXCITATION**

**Theorem 1:** For the system in (1) and the adaptive controller with intelligent excitation in (3), (6), (11), (20), we have
\[
\lim_{t \to \infty} |(\cdot) - (\cdot)| \leq (x_4) \tag{31}
\]

where
\[
(x_4) = x_4 \sum_{i=1}^{m} |(\cdot) i| \tag{32}
\]

and
\[
(\cdot) = \tau (\cdot) g \tag{33}
\]

**Proof of Theorem 1:** We prove the theorem in two steps. At first, we prove the asymptotic convergence of parameter errors \(\hat{x}(\cdot)\) and \(\tilde{r}(\cdot)\) to zero. Then, we prove the convergence of \((\cdot)\) to the desired reference signal \((\cdot)\) with error bounds proportional to \(x_4\).

**Step 1:** In this step, we will prove
\[
\lim_{t \to \infty} \hat{x}(\cdot) = 0 \quad \lim_{t \to \infty} \tilde{r}(\cdot) = 0 \tag{34}
\]

It follows from equation (14) that
\[
\lim_{t \to \infty} (\cdot) = 0 \forall \in [0, x_2] \tag{35}
\]

and hence
\[
\lim_{t \to \infty} \int_{t}^{t+k_{2}} \tau (\cdot) d\tau (\cdot) = 0 \tag{36}
\]

which combined with (20) implies that
\[
\lim_{t \to \infty} x(\cdot) = x_4 \tag{37}
\]

It follows from (35) and the adaptive law (11) that for \(\cdot \in [0, x_2]\) there exist \(\hat{x} \in \mathbb{R}^n\) and \(\tilde{r} \in \mathbb{R}\) such that
\[
\lim_{t \to \infty} x(\cdot) = \hat{x} \quad \lim_{t \to \infty} r(\cdot) = \tilde{r} \tag{38}
\]

Equation (38) further implies that
\[
\lim_{t \to \infty} \hat{0}(\cdot) = \hat{0}(\cdot) + \hat{r}(\cdot) = \hat{r} \tag{39}
\]

while equation (37) implies that for \(\cdot \in [0, x_2]\)
\[
\lim_{t \to \infty} (\cdot) = x_4 \sum_{i=1}^{m} \sin(i(\cdot)) \tag{40}
\]

From (22) it follows that \(\hat{r}(\cdot)\) is the output response of the linear system, defined by \((\cdot)\), to \(\hat{x}(\cdot) = \hat{0}(\cdot) + \hat{r}(\cdot)\)
where \( x_1 \) is defined in (40) and \( \bar{x}_1 \) is the \( i^{th} \) element of \( \bar{x} \). Therefore, \( \bar{x}(\tau) \) can be expressed as the sum of three components:

\[
\bar{x}(\tau) = \bar{x}_1(\tau) + \bar{x}_2(\tau) + \bar{x}_3(\tau)
\]

(41)

for \( \tau \in [0, x_2] \) where \( \bar{x}_1(\tau) \) and \( \bar{x}_2(\tau) \) are the particular solutions of the linear system (22) corresponding to \( x(\tau) \) and \( \dot{x}(\tau) \) respectively, while \( \bar{x}_3(\tau) \) is the general solution of the linear system (22) for the initial value at \( \tau = 0 \) and can therefore be expressed by a set of exponential functions of \( \tau \). It follows from (39) that

\[
\lim_{t \to \infty} \bar{x}_2(\tau) = 0 \quad \text{for} \quad \tau \in [0, x_2]
\]

(42)

Next we use Fourier transformation to construct the particular solutions of the linear system (22) in response to \( x(\tau) \) defined in (40). If the input signal to the linear system (4) is:

\[
x_1 \sum_{i=1}^{m} \sin(i(\omega))
\]

(43)

with its Fourier transformation

\[
E_2(\omega) = \sum_{i=1}^{m} \sqrt{\frac{1}{2}} \left[ (\omega + i) - (\omega - i) \right]
\]

(44)

the Fourier transformation of the particular solution, corresponding to the input (43), is:

\[
x_1 \sum_{i=1}^{m} \sin(i(\omega))
\]

(45)

Hence, the particular solution corresponding to the input signal in (43) will be

\[
\bar{x}_1(\tau) = \sum_{i=1}^{m} \left( \text{Re}(\omega(\tau)) \sin(\omega) + \text{Im}(\omega(\tau)) \cos(\omega) \right)
\]

(46)

Therefore, the particular solution corresponding to \( x(\tau) \) defined in (40) satisfies

\[
\lim_{t \to \infty} \bar{x}_1(\tau) = \sum_{i=1}^{m} \left( \text{Re}(\omega(\tau)) \sin(\omega(\tau)) + \text{Im}(\omega(\tau)) \cos(\omega(\tau)) \right)
\]

(47)

Equation (47) can be rewritten as

\[
\bar{x}_1(\tau) = \Omega(\tau) \bar{x}_0(\tau)
\]

(48)

where \( \Omega \) is defined in (24) and \( \bar{x}_0(\tau) \) is a \( 2 \times 1 \) vector with its \( i^{th} \) element being:

\[
\bar{x}_0(\tau) = \left\{ \begin{array}{ll}
\sin(\omega(\tau)) & \text{if } i \text{ is odd} \\
\cos(\omega(\tau)) & \text{if } i \text{ is even}
\end{array} \right.
\]

(49)

where \( \tau \in [0, x_2] \). Since \( \Omega \) has full row rank and the elements of \( \bar{x}_0(\tau) \) are linearly independent functions, then the elements of \( \bar{x}_1(\tau) \) are linearly independent over \( \tau \in [0, x_2] \). Notice that as \( \tau \to \infty \), \( \bar{x}_2(\tau) \) tends to a constant (see eq.(42)). The general solution \( \bar{x}_3(\tau) \) is a sum of exponential functions. Therefore \( \bar{x}_3(\tau) \) cannot be expressed by a linear combination of sinusoidal functions \( \bar{x}_1(\tau) \) over \( \tau \in [0, x_2] \). Hence, all the elements of \( \bar{x}(\tau) \) are linearly independent over the time interval \( [0, x_2] \) as \( \tau \to \infty \).

For a nonzero \( \tau \), since the elements of \( \bar{x}(\tau) \) are linearly independent over the interval \( [0, x_2] \), there exists some \( \tau \in [0, x_2] \) such that

\[
\bar{x}(\tau) = \sum_{i=1}^{m} \left( \text{Re}(\omega(\tau)) \sin(\omega(\tau)) + \text{Im}(\omega(\tau)) \cos(\omega(\tau)) \right)
\]

(50)

Since \( \bar{x}(\tau) \) is Lipschitz continuous, it follows from (50) that there exist \( 1, \omega \in [0, x_2] \), where \( 1, \omega \) such that

\[
\lim_{t \to \infty} \int_{t+\tau}^{t+\tau} \bar{x}(\tau) = 0
\]

(51)

It follows from (10), (35) and (51) that

\[
\lim_{t \to \infty} \int_{t+\tau}^{t+\tau} \bar{x}(\tau) = 0
\]

(52)

Since (52) contradicts (35), (49) must be true, i.e.

\[
\lim_{t \to \infty} \int_{t+\tau}^{t+\tau} \bar{x}(\tau) = 0
\]

(53)

From (35) and (53), we have \( \lim_{t \to \infty} \bar{x}(\tau) = 0 \). Since \( \bar{x}(\tau) \) is non-increasing, then (34) holds.

\textbf{Proof of Step 2:} Since \( \bar{x}(\tau) \) is the response of the linear system (3) to the input signal \( \bar{x}(\tau) \), it can be presented as \( \bar{x}(\tau) = \bar{x}_0(\tau) + \bar{x}_1(\tau) \), where \( \bar{x}_0(\tau) \) and \( \bar{x}_1(\tau) \) are the responses to the input signal components \( \bar{x}_1(\tau) \) and \( \bar{x}_0(\tau) \), respectively. It follows from (18) and (34) that

\[
\lim_{t \to \infty} \bar{x}_1(\tau) = 0
\]

(54)

which along with (5) implies that

\[
\lim_{t \to \infty} \bar{x}_0(\tau) = 0
\]

(55)

Since (3) is a linear system, then a sinusoidal input \( \bar{x}_1(\tau) \) results in a sinusoidal output with the magnitude bounded by \( |\bar{x}_1(\tau)| \), where \( |\bar{x}_1(\tau)| \) is defined in (33). From (37) it follows that as \( \tau \to \infty \), the steady state response \( \bar{x}_1(\tau) \) to the input signal \( \sum_{i=1}^{m} \sin(i(\omega)) \), is bounded by (54):

\[
\lim_{t \to \infty} |\bar{x}_1(\tau)| \leq \omega \sin(\omega)
\]

(56)

It follows from (55) and (56) that

\[
\lim_{t \to \infty} |\bar{x}_1(\tau)| \leq \omega \sin(\omega)
\]

(57)

which along with (14) yields (31).
Theorem 1 demonstrates that the intelligent excitation signal ensures asymptotic convergence of parameter errors to zero. It also proves that the upper bound of the output error between ( ) and ( ) is proportional to , which defines the magnitude of the excitation signal as → ∞. To reduce the output error, we need to set extremely small. However small excitation signal can cause extremely slow parameter convergence, which is not acceptable in practice. The rationale of the intelligent excitation is its ability to keep the magnitude of the excitation signal at a reasonable value before the parameter error converges to zero is demonstrated in the following Lemma.

Lemma 2: For the system in (1) and the adaptive controller with intelligent excitation in (3), (6), (11), (20), if → ∞, we have

\[ x( ) = x_3 \] (58)

unless ( ) → 0.

Proof of Lemma 2: We will prove this by mathematical induction principle. Assume that

\[ x( ) = x_3 \] (59)

For any time instant ≥ x_2 such that ( ) 0, we will prove that

\[ x( ) = x_3 \] (60)

if → ∞. Indeed, if

\[ \int_{t-k_2}^{t} ( )^T ( ) \neq 0 \] (61)

from (20) it follows that x( ) = x_3 as → ∞. If (61) is not true, i.e.

\[ \int_{t-k_2}^{t} ( )^T ( ) = 0 \] (62)

then, since ( ) is Lipschitz continuous, we have

\[ ( ) = 0 \quad \forall \in [-x_2] \] (63)

and hence

\[ \overset{x}{=} = \overset{x}{=} \overset{r}{=} \overset{r}{=} \in [-x_2] \] (64)

for constant \( \overset{x}{=} \in \mathbb{R}^p \) and \( \overset{r}{=} \in \mathbb{R} \). Since ( ) ≥ 0, we conclude from (12) and (63) that

\[ \overset{r}{=} \left[ \begin{array}{c} \overset{r}{=} \overset{r}{=} \overset{r}{=} \end{array} \right]^T \neq 0 \] (65)

Upon the parameter error convergence to zero, for any reference input signal \( \overset{r}{=} \), no matter it has an excitation in it or not, we have

\[ \int_{t-k_2}^{t} ( )^T ( ) = 0 \] (66)

and hence, from (20) it follows that x( ) = x_4. Intelligent excitation adjusts the magnitude of the excitation signal through "a feedback mechanism" determined by the integral of the tracking error ( ) over a finite time interval.

Remark 2: For practical implementation, due to the presence of noise and transient errors, we can choose \( x_4 = 0 \) without worrying about the premature disappearance of the intelligent excitation. The constant gain \( x_4 \) is inverse proportional to the bound of the parameter tracking error, so setting it large will increase the accuracy of parameter estimates. The gain \( x_3 \) is the amplitude of the excitation signal, which controls the rate of convergence.

V. SIMULATION RESULT

We consider the SISO dynamic system in (1) with:

\[ A = \begin{bmatrix} 1 & -1 \\ -24 & -9.0711 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \quad \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \quad \begin{bmatrix} 1 & 0 \end{bmatrix}^T - \begin{bmatrix} -3 & 3333 \end{bmatrix} \] when \( t \leq 50 \) and \( \begin{bmatrix} 1 & 0 \end{bmatrix} = -16.6667 \) when \( t = 50 \). The reference signal is \( \begin{bmatrix} 1 & 0 \end{bmatrix} \) and the objective is to design control signal ( ) to let system output ( ) track the reference input ( )
We choose a reference model with the following parameters: 

\[ m = \begin{bmatrix} 0 & 1 \\ -25 & 7 \end{bmatrix}, \quad m = [25 \ 25]^\top, \quad g = 0 \ 1239. \]

Lyapunov equation is solved with the following matrix:

\[ \hat{v}_x = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad v_x = 5. \]

\[ x_1 = 2000, \quad x_2 = 2, \quad x_3 = 15, \quad x_4 = 12, \quad x_5 = 0. \]

We choose \( \Omega = 2 \) as \( \Omega = 10 \) for the chosen \( \Omega \). Simulation results are given in Figs. 1, 2, and 3. Fig. 1 plots the time histories of \( \hat{\theta}_1(t) \) with the chosen \( \hat{\theta}_1(t) \) and \( \hat{\theta}_2(t) \). Simulation results are given in Figs. 1, 2, and 3. The design gains in the intelligent excitation are set to \( \omega_0 = 2000 \). Lyapunov function is solved with the following matrix:

\[ \hat{v}_x = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad v_x = 5. \]

The trajectory of \( \hat{\theta}_1(t) \) with the chosen \( \hat{\theta}_1(t) \) and \( \hat{\theta}_2(t) \) is plotted in Fig. 2. The time histories of parameter estimates are demonstrated in Fig. 3, which shows the convergence of \( \hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t) \) to \( x_1(t), x_2(t), x_3(t) \), respectively, under intelligent excitation. Figures 1, 2, and 3 demonstrate that i) intelligent excitation vanishes as parameter convergence takes place, ii) intelligent excitation is needed in adaptive control architecture. It can also be used to increase robustness of the adaptive controller, where parameter drift may cause instability. Extension to nonlinear systems will be reported in a forthcoming publication.

VI. CONCLUSION

In this paper, we augment the traditional MRAC with an intelligent excitation signal to solve the output tracking for a reference input that depends upon the unknown parameters of the system. The main feature of the new technique is that it initiates excitation only when necessary. It is shown that the intelligent excitation is a general technique for the situations where parameter convergence is needed in adaptive control architecture. It can also be used to increase robustness of the adaptive controller, where parameter drift may cause instability. Extension to nonlinear systems will be reported in a forthcoming publication.

REFERENCES


