PROBLEM 9.53

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net energy increase of the air passing through the gas turbine combustor; (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:

![Gas turbine schematic diagram]

ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 300 K$ and $p_0 = 100 kPa$.

ANALYSIS: Data are obtained for each principal state from Table A-22:

<table>
<thead>
<tr>
<th>State</th>
<th>$T(K)$</th>
<th>$p$(kPa)</th>
<th>$h$ (kJ/kg)</th>
<th>$s^o$(kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>100</td>
<td>300.19</td>
<td>1.70208</td>
</tr>
<tr>
<td>2</td>
<td>621.1</td>
<td>1000</td>
<td>629.71</td>
<td>2.44539</td>
</tr>
<tr>
<td>3</td>
<td>1400</td>
<td>950</td>
<td>1515.42</td>
<td>3.36200</td>
</tr>
<tr>
<td>4</td>
<td>873.9</td>
<td>100</td>
<td>903.72</td>
<td>2.81558</td>
</tr>
</tbody>
</table>

The increase in flow exergy of the air passing through the heat exchanger is taken as the net input of exergy to the gas turbine.

Input: $\dot{m}(e_{33} - e_{44}) = \dot{m} \left[ (h_3 - h_2) - T_0 (s_3^o - s_2^o - R \ln(P_3/P_2)) \right]$

Evaluating $\dot{m}$:

$$\dot{m} = \frac{(Ar)_1 \dot{p}_1}{RT_1} = \frac{(5 m^3/s)(100 kPa)}{\frac{(8.314 \text{ kJ}}{28.97 \text{ kJ-K})}(300 K)} \left| \frac{1 \text{ kJ}}{1 \text{ kPa}} \right| \left| \frac{1 \text{ kJ}}{1 \text{ N.m}} \right| = 5.807 \text{ kg/s}$$

Thus

$$\dot{m}(e_{33} - e_{44}) = \frac{(5.807 \text{ kg/s}) \left[ (1515.42 - 629.71) kJ/kg \right]}{(300 K)(3.36200 - 2.44539 - \frac{8.314 \text{ kJ}}{28.97 \text{ kJ-K}}(950 \frac{\text{kJ}}{1000 \text{ kJ-K}}))} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 3524 \text{ kW (Input)}$$

Continued on next slide
Problem 9-53 continued

Energy is destroyed due to irreversibilities in the compressor and turbine. Thus, using $\dot{E}_d = T_0 S^*$, we get:

**Destructions:**

$$\dot{E}_{d,\text{comp}} = T_0 m \left( s_2^0 - s_1^0 \right) = T_0 m \left[ (s_2^0 - s_3^0) - R \ln \frac{p_2}{p_1} \right] = 143.8 \text{ kW}$$

$$\dot{E}_{d,\text{turb}} = T_0 m \left[ (s_4^0 - s_3^0) - R \ln \frac{p_4}{p_3} \right] = 173.6 \text{ kW}$$

The net power developed by the cycle represents the output of energy from the cycle, or:

$\text{Work} = m \left[ (h_3^0 - h_4) - (h_2 - h_1) \right]$

$$= 1641.5 \text{ kW (Output)}$$

Finally, the air enters the gas turbine at $p_3$ and $T_0$. Thus, the change in flow exergy from inlet to exit represents the loss due to the hot air being discharged. Thus:

**Loss:**

$$m (e_4^0 - e_3^0) = m \left[ (h_4 - h_1) - T_0 \left( s_4^0 - s_1^0 - R \ln \frac{p_4}{p_1} \right) \right]$$

$$= 1564.8 \text{ kW}$$

**Summarizing**

- **Input:** 3524 kW
- **Disposition:**
  - Net Power: 1641.5 kW (46.6%)
  - Destroyed: 173.6 kW (9.4%)
  - Loss: 1564.8 kW (44.9%)

Since the objective of the gas turbine is to produce power, an exergetic efficiency expressed as the ratio of the desired output (net power) to the exergy input is, by inspecting the summary,

$$\eta = 46.6\%$$

1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.10.