① Solution is $f(s) = B_0 \cdot \dfrac{m_+ \cdot \exp[m_- \cdot s + m_+] - m_- \cdot \exp[m_+ \cdot s + m_-]}{m_+^2 \cdot \exp[m_+] - m_-^2 \cdot \exp[m_-]}$

$$m_\pm = \frac{B_0 \pm \sqrt{B_0^2 + 4B_0 \cdot Da}}{2} = \frac{1}{2}\left[B_0 \pm \sqrt{B_0^2 + 4B_0 \cdot Da}\right]$$

$m_\pm = \frac{1}{2} \cdot B_0 \left[1 \pm \left(1 + \frac{2Da}{B_0}\right)^{1/2}\right]$. now, use expansion,

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \cdots$, so

$m_\pm = \frac{1}{2} \cdot B_0 \cdot \left[1 \pm \left(1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2} + \cdots\right)\right]$, or

$$m_+ = \frac{1}{2} B_0 \left[1 + 1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2}\right] = B_0 + Da - \frac{Da^2}{B_0}.$$

$$m_- = \frac{1}{2} B_0 \left[1 - 1 - \frac{2Da}{B_0} + \frac{2Da^2}{B_0^2}\right] = -Da + \frac{Da^2}{B_0}.$$

$(m_+ + m_-) = B_0$, and $(m_+ - m_-) = B_0\left(1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2}\right)$.

which is good to know, since we're looking at outlet conc, $f(s=1)$, or.

$$f(1) = B_0 \frac{(m_+ - m_-)}{m_+^2 \cdot \exp[m_+] - m_-^2 \cdot \exp[m_-]} \cdot \exp(m_+ + m_-).$$

(i) if $B_0 \to \infty$. first, lets rewrite this to be if $\frac{1}{B_0} \to 0$ (easier for maths).

$$\lim_{1/B_0 \to 0} f(1) = \lim_{1/B_0 \to 0} B_0 \cdot \frac{\left(B_0 + 2Da - 2\frac{Da^2}{B_0}\right) \cdot \exp[B_0]}{\left(B_0 + Da - \frac{Da^2}{B_0}\right)^2 \cdot \exp\left[B_0 + Da - \frac{Da^2}{B_0}\right] - \left(-Da + \frac{Da^2}{B_0}\right)^2 \cdot \exp\left[-Da + \frac{Da^2}{B_0}\right]}$$

ⓐ $\div \exp[B_0]$,

$$= B_0 \frac{\left(B_0 + 2Da - 2\frac{Da^2}{B_0}\right)}{\left(B_0 + Da - \frac{Da^2}{B_0}\right)^2 \exp\left[Da - \frac{Da^2}{B_0}\right] - \left(-Da + \frac{Da^2}{B_0}\right)^2 \cdot \exp\left[-B_0 - Da + \frac{Da^2}{B_0}\right]}$$

ⓑ extract B_0 from terms,

$$= \frac{B_0^2\left(1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2}\right)}{B_0^2\left(1 + \frac{Da}{B_0} - \frac{Da^2}{B_0^2}\right)\cdot \exp\left[Da - \frac{Da^2}{B_0}\right] - \left(Da + \frac{Da^2}{B_0}\right)^2 \cdot \exp\left[-B_0 - Da + \frac{Da^2}{B_0}\right]}$$

© cancel out B_0^2; or, multiply by $\frac{1/B_0^2}{1/B_0^2}$.

$$= \frac{1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2}}{\left(1 + \frac{Da}{B_0} - \frac{Da^2}{B_0^2}\right)\cdot \exp\left[Da - \frac{Da^2}{B_0}\right] - \cancel{B_0^2}\cdot\left(Da + \frac{Da^2}{B_0}\right)^2 \cdot \exp\left[-B_0 - Da + \frac{Da^2}{B_0}\right]} \text{, or}$$

$$= \frac{1 + \frac{2Da}{B_0} - \frac{2Da^2}{B_0^2}}{\left(1 + \frac{Da}{B_0} - \frac{Da^2}{B_0^2}\right)\cdot \exp\left[Da - \frac{Da^2}{B_0}\right] - \left(\frac{Da}{B_0} + \frac{Da^2}{B_0^2}\right)^2 \cdot \exp\left[-B_0 - Da + \frac{Da^2}{B_0}\right]}.$$

now, recognize $\frac{1}{B_0} \to 0$, $\frac{1}{B_0^2} \to 0$.

$$\lim_{1/B_0 \to 0} f(1) = \frac{1 + 0 - 0}{\exp[Da] - 0} = \underline{\underline{\exp^{-Da}}}. \quad \Leftarrow \text{PFR soln.}$$

(iii) limit as $1/B_0 \to \infty$, or $B_0 \to 0$. Let's rewrite our definitions...

$$M_\pm = \frac{1}{2}\left[B_0 \pm \left(B_0^2 + 4B_0\cdot Da\right)^{1/2}\right], \text{ or}$$

$$= \frac{1}{2}\left[B_0 \pm \left((4B_0\cdot Da)\left(\frac{B_0^2}{4B_0 Da} + 1\right)\right)^{1/2}\right], \text{ or}$$

$$= \frac{1}{2}\left[B_0 \pm \sqrt{2^2 B_0 Da}\cdot\left(\frac{B_0}{4Da} + 1\right)^{1/2}\right], \text{ or}$$

$$= \frac{1}{2}\left[B_0 \pm \sqrt{2^2 B_0 Da}\left(1 + \frac{B_0}{8Da} - \frac{B_0^2}{124 Da^2}\right)\right], \text{ and}$$

$$\cancel{M_+ = \frac{1}{2}B_0 + \sqrt{B_0 Da}, \quad M_- = \frac{1}{2}B_0 -}$$

$$M_+ = \frac{1}{2}B_0 + \sqrt{B_0 Da} + \sqrt{B_0 Da}\cdot\frac{B_0}{8Da} - \sqrt{B_0 Da}\cdot\frac{B_0^2}{124 Da^2}\ldots$$

$$M_- = \frac{1}{2}B_0 - \sqrt{B_0 Da} - \sqrt{B_0 Da}\cdot\frac{B_0}{8Da} + \sqrt{B_0 Da}\cdot\frac{B_0^2}{124 Da^2}\ldots$$

and $M_+ + M_- = \frac{1}{2}Bo + \frac{1}{2}Bo = \underline{Bo}$.

$$M_+ - M_- = 2\sqrt{Bo\,Da} + 2\frac{Bo}{8Da}\cdot\sqrt{Bo\,Da} + 2\sqrt{Bo\,Da}\cdot\frac{Bo^2}{124\,Da^2} + \cdots$$

now,

$$\lim_{Bo\to 0} f(1) = \lim_{Bo\to 0} Bo\,\frac{2\sqrt{Bo\,Da}\left(1+\frac{Bo}{8Da}+\frac{Bo^2}{124\,Da^2}+\cdots\right)\cdot\exp[Bo]}{\left(\frac{1}{2}Bo+\sqrt{Bo\,Da}+\sqrt{Bo\,Da}\cdot\frac{Bo}{8Da}+\cdots\right)^2\cdot\exp\left[\frac{1}{2}Bo+\sqrt{Bo\,Da}+\cdots\right]-\left(\frac{1}{2}Bo-\sqrt{Bo\,Da}+\cdots\right)^2\cdot\exp[\cdots]}$$

@ bring out Bo #'s for canceling ...

$$\lim_{Bo\to 0} f(1) = \lim_{Bo\to 0} \frac{2Bo\sqrt{Bo\,Da}\left(1+\frac{Bo}{8Da}+\frac{Bo^2}{124\,Da^2}+\cdots\right)\exp[Bo]}{\left(Bo\,Da+\frac{1}{2}Bo\sqrt{Bo\,Da}+\frac{1}{4}Bo^2+\cdots\right)\exp\left[\frac{1}{2}Bo+\sqrt{Bo\,Da}+\cdots\right]-\left(Bo\,Da-\frac{1}{2}Bo\sqrt{Bo\,Da}+\frac{1}{4}Bo^2\right)\cdot\exp[\cdots]}$$

discard 2nd order and higher terms of Bo ...

$$\lim_{Bo\to 0} f(1) = \lim_{Bo\to 0} \frac{2Bo\sqrt{Bo\,Da}\left(1+\frac{Bo}{8Da}\right)\exp[Bo]}{Bo\left(Da+\frac{1}{2}\sqrt{Bo\,Da}\right)\exp\left[\frac{1}{2}Bo+\sqrt{Bo\,Da}+\cdots\right]-Bo\left(Da-\frac{1}{2}\sqrt{Bo\,Da}+\cdots\right)\exp\left[\frac{1}{2}Bo-\sqrt{Bo\,Da}+\cdots\right]}$$

cancel Bo, use expansion: $e^x = 1 + x + \cdots$, and

$$\lim_{Bo\to 0} f(1) = \lim_{Bo\to 0} \frac{\left(2\sqrt{Bo\,Da}\right)\left(1+Bo+\cdots\right)}{\left(Da+\frac{1}{2}\sqrt{Bo\,Da}\right)\left(1+\frac{1}{2}Bo+\sqrt{Bo\,Da}+\cdots\right)-\left(Da-\frac{1}{2}\sqrt{Bo\,Da}\right)\cdot\left(1+\frac{1}{2}Bo-\sqrt{Bo\,Da}+\cdots\right)}$$

now, lowest-order term in Bo is $Bo^{1/2}$, so discard all higher order terms ...

$$\lim_{Bo\to 0} f(1) = \lim_{Bo\to 0} \frac{2\sqrt{Bo\,Da}}{\left(Da+\frac{1}{2}\sqrt{Bo\,Da}\right)\left(1+\sqrt{Bo\,Da}\right)-\left(Da-\frac{1}{2}\sqrt{Bo\,Da}\right)\left(1-\sqrt{Bo\,Da}\right)},$$

$$=\lim_{Bo\to 0} \frac{2\sqrt{Bo\,Da}}{\cancel{Da}+\frac{1}{2}\sqrt{Bo\,Da}+Da\sqrt{Bo\,Da}+\cancel{\frac{1}{2}Bo\,Da}-\cancel{Da}+Da\sqrt{Bo\,Da}+\frac{1}{2}\sqrt{Bo\,Da}-\cancel{\frac{1}{2}Bo\,Da}}$$

$$=\lim_{Bo\to 0} \frac{\cancel{2\sqrt{Bo\,Da}}}{\cancel{2\sqrt{Bo\,Da}}\,(1+Da)} = \underline{\underline{\frac{1}{1+Da}}} \Leftarrow \text{CSTR soln.}$$

#3: Using Chart #19, we see that estimated Schmidt #'s,

a) for liquid $\equiv \dfrac{1\times10^{-3}}{10^{3}\cdot10^{8}} \approx 10^{8}$, which is off the chart.

b) for gas $\equiv \dfrac{1\times10^{-5}}{10^{0}\cdot10^{-6}} \approx 10$, and $Re\cdot Sc = 10^{6}$ which is just off the chart.

problem is, that chart is for laminar flow; what we should have used is Figure 20 from Levenspiel, which directly correlates the laminar and turbulent theory/data . . .

a) Looking up, $\dfrac{D}{\mu d_t} = 0.5$, and

$$\frac{D}{u} = (0.5)(3cm) = 1.5cm, \text{ and } Bo = \frac{uL}{D} = \frac{300cm}{1.5cm} = \boxed{200}$$

b) Looking up, $\dfrac{D}{\mu d_t} = 0.24$, and

$$\frac{D}{u} = (0.24)(3cm) = 0.72cm, \text{ and } Bo = \frac{uL}{D} = \frac{300cm}{0.72cm} = \boxed{416.7}$$

c) $Re_p = 200$; from chart, $\dfrac{D\epsilon}{u d_p} = 0.5$.

$$\frac{D}{u} = \frac{(0.5)(4mm)}{0.4} = 5mm. \qquad Pe = \frac{3\times10^{3}mm}{5mm} = \boxed{600}$$

using Radial Dispersion Model,

$$\frac{vL}{D} = Pe, \quad Da = k_1\theta.$$

i) PFR; $f(1) = e^{-Da}$, if $X = 0.99$, then $f(1) = 0.01$ and

$\ln[0.01] = -Da$, and $\theta = 1$ sec so $\boxed{k_1 = 4.605 \text{ sec}^{-1}}$

ii) PFR + Dispersion.

$$m_{\pm} = \left(Pe \pm \sqrt{Pe^2 + 4 Pe\, Da}\right)\frac{1}{2} \Rightarrow \frac{D\epsilon}{u \cdot D_p} = 1, \text{ so } Pe = 38.4, \text{ and}$$

iteratively solve using GoalSeek:

$$f(1) = 0.01 = \left(\frac{m_+ e^{(m_- + m_+)} - m_- e^{(m_+ + m_-)}}{m_+^2 \cdot e^{m_+} - m_-^2 \cdot e^{m_-}}\right) Pe,$$

and find $Da = 5.143$, or $\boxed{k_1 = 5.143 \text{ sec}^{-1}}$

iii) PFR + Recycle:

$$f(1) = \frac{e^{(-5.143)/(R+1)}}{(R+1) - R \cdot e^{(-5.143)/(R+1)}} = 0.01.$$

satisfied by $R = 0.16328$.

iv) $$\text{error} = \frac{|(4.605 - 5.143)|}{5.143} = \boxed{10.4\%}$$