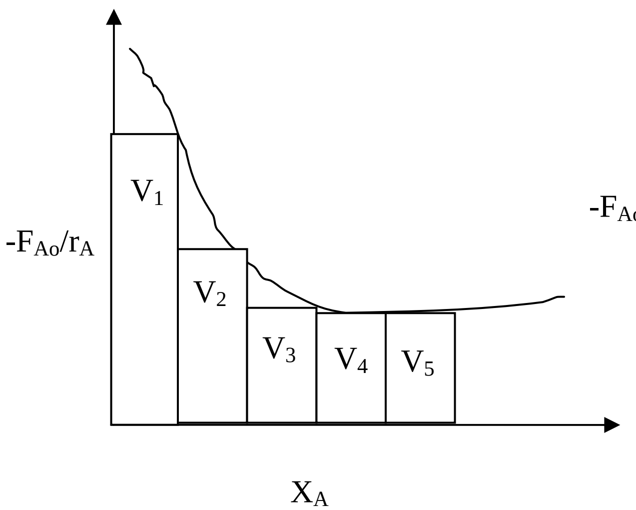
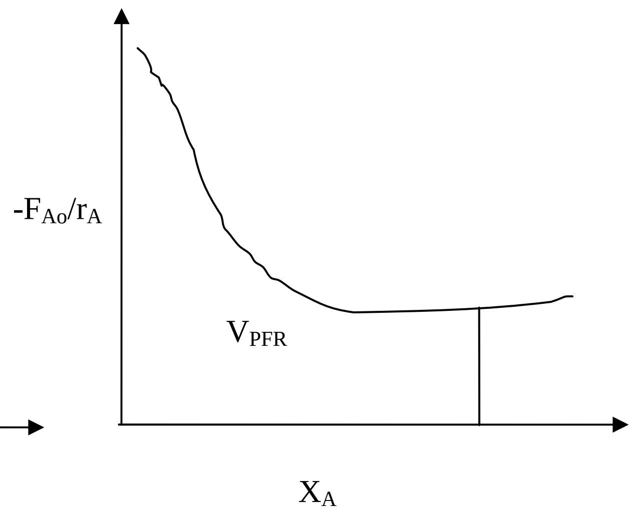
CHEG 251: Homework Assignment #2, Honors Supplement Due: January 27th, 2006

CSTR's In Series Solution

A poorly-mixed CSTR can be modeled as a series of n individual CSTRs in series. Likewise, a PFR with axial mixing can be modeled in the same way. If n = 1, we have modeled one CSTR with perfect mixing. If n → infinity, we have modeled one PFR with no axial mixing. In between, we have an approximation of a non-ideal reactor. This is easily shown using a Levenspiel plot, as follows:





The challenging part is to show this mathematically.

(a) DERIVE the outlet concentration of A for ONE CSTR (C_A), as a function of C_{AO} (inlet concentration), Q (volumetric feed rate), and V_r (volume of the one reactor), and k (from first-order kinetics, A→B, $-r_A = kC_A$).

(b) Now, lets assume we have TWO CSTRs, each of equal volume, i.e. $V_{R,1} = V_{R,2}$. In order to maintain a fair comparison between one reactor and two in series, we also insist that V_r (from part a) = $V_{R,1} + V_{R,2}$. DERIVE the oulet concentration of A for the second CSTR, in terms of C_{AO} (inlet concentration), Q (volumetric feed rate), and $V_{r,1}$ and $V_{R,2}$ (volumes of each reactor), and k (from first-order kinetics, A→B, $-r_A = kC_A$).

(c) Using your solutions from parts (a) and (b), SHOW that for N CSTRs in series, the outlet concentration of the Nth reactor is:

$$\frac{C_A}{C_{AO}} = \frac{1}{\left(1 + \frac{kV_R}{QN}\right)^N}$$

(d) SHOW that as we approach the limit of N→ infinity, this solution becomes the design equation for a PFR, i.e.

$$\lim_{N->\infty} \frac{1}{\left(1+\frac{kV_R}{QN}\right)^N} = \exp\left[-\frac{kV_R}{Q}\right]$$

HINT 1: substituting $N = \frac{1}{\in}$ into the equation should give a slightly more recognizable form…

HINT 2: remember the series expansion, $(1+x)^N = 1 + Nx + \frac{N(N-1)^2}{2!}x^2 + ...$

HINT 3: remember the series expansion, $e^x = 1 + x + \frac{x^2}{2!} + ...$

Congratulations! You've now shown how N CSTRs in series can model either a PFR, a CSTR, or intermediate mixing in a continuous reactor, all as a function of N!