A Practical Joint Network-Channel Coding Scheme for Reliable Communication in Wireless Networks

Zheng Guo*, Jie Huang† , Bing Wang*, Jun-Hong Cui*, Shengli Zhou† and Peter Willett†
*Department of Computer Science & Engineering
†Department of Electrical & Computer Engineering
University of Connecticut
Storrs, CT, 06269, U.S.A.
{guozheng, jhuang, bing, jcui, shengli, willett}@engr.uconn.edu

ABSTRACT

In this paper, we propose a practical scheme, called Non-Binary Joint Network-Channel Decoding (NB-JNCD) for reliable communication in wireless networks. It seamlessly couples channel coding and network coding, and can effectively combat the detrimental effect of fading of wireless channels, especially in large networks. On a high order Galois field, NB-JNCD combines non-binary LDPC channel coding and random linear network coding through iterative joint decoding, which helps fully exploit the spatial diversity and redundancy residing in both codes. Furthermore, the scheme can unify non-binary source coding and high order modulation without the need of any bit-to-symbol conversion and its inverse. Through analysis and simulation, we demonstrate the significant performance improvement of NB-JNCD against other schemes.

Categories and Subject Descriptors

H.1.1 [**Models and Principles**]: Systems and Information Theory—*value of information*

General Terms

Design Reliability

Keywords

Non-binary joint network-channel coding, large wireless network

1. INTRODUCTION

Compared to wire line communication, wireless communication suffers from high and time-varying packet loss due to the detrimental effect of fading of wireless channels. One method to provide reliable communication is using redundant information, which can be added either inside a packet (bit/symbol level or physical layer) or across multiple packets (packet level or network layer). The former is called *error-correction* and the latter is referred to as *erasure-correction*.

MobiHoc'09, May 18–21, 2009, New Orleans, Louisiana, USA.

Channel coding is a conventional error-correction technique adopted for point-to-point communication in wireless environments. It is implemented at the physical layer to recover erroneous bits/symbols through redundant parity check bits/symbols appending to a packet. The error recovery capability depends on the specific coding strategy and the amount of redundant bits/symbols. Channel coding has been widely employed in practical wired and wireless systems. In [24], Lin *et al.* described many sophisticated channel coding schemes, such as Reed-Solomon code, convolution code, Turbo code and low-density parity-check (LDPC) code. Many schemes have sound performance and can approach the channel capacity of non-fading channels. However, when a channel experiences slow and deep fading, the performance of channel coding degrades dramatically. In such a case, the communication through the channel cannot continue and packet loss will occur.

Erasure-correction is another technique for reliable communication through extra protection from redundant packets. This technique operates on packet level at the network layer. Generally, if there are K original packets, then more than K packets will be generated by an erasure-correction coding scheme and transmitted to the destination. The destination can recover all the original packets with any K independent successfully received packets. In this way, erasure-correction codes establish relationship across packets to provide additional protection. There are two types of erasure-correction methods: Forward Error Correction (FEC) and network coding. In a FEC method, the source generates all original and redundant packets, while in network coding, temporal and spatial diversities are exploited through multiple channels by allowing the intermediate nodes to generate redundant network-coded packets in a distributed manner.

Network coding was first introduced to achieve the multicast capacity in wired lossless networks [1]. The studies in [18, 19, 30] extended network coding to wireless networks, again focusing on lossless channels. Later, Guo *et al.* proposed an efficient error recovery scheme using network coding for lossy erasure channels in underwater sensor networks [9, 10]. They showed that random linear network coding is simple and efficient in large multi-hop wireless networks. Recently, Chen *et al.* applied network coding to user cooperation in one hop systems with single common destination, where multiple users can relay packets for each other [7]. Through analysis and simulation, these approaches have shown that network coding can reduce system outage probability significantly.

Obviously, error-correction coding and erasure-correction coding can be implemented simultaneously at physical layer and network layer respectively. However, conventional methods treat them separately, which introduce lots of waste: erasure-correction de-

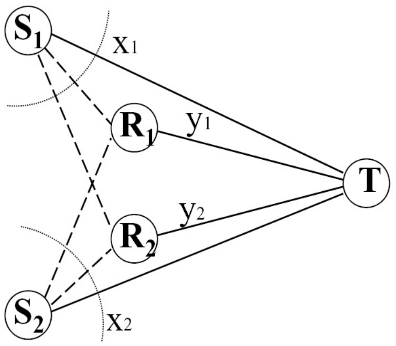


Figure 1: A simple topology with two sources, two relays and one sink.

coding cannot take advantage of the redundant information in the packets that fail channel decoding and hence are discarded at the physical layer, while error-correction decoding cannot take advantages of the network layer collaborations. Recently a number of research efforts have tried to unify the two types of coding schemes [5, 14, 12, 2, 32, 17, 27, 26, 25]. These studies use a simple topology with only one relay, binary channel coding, binary XOR network coding or unpractical physical layer network coding for the sake of easing theoretic analysis. In this paper, we propose a practical joint network-channel coding scheme, called *Non-Binary Joint Network-Channel Decoding (NB-JNCD)* for large wireless networks. NB-JNCD seamlessly couples non-binary LDPC channel coding and non-binary random linear network coding. We will first present the proposed scheme using a simple topology with two sources and two relays. After some fundamental theoretical analysis and performance evaluation, we will describe how to extend the scheme to large wireless networks. Compared with other schemes, NB-JNCD will be shown to achieve significant performance gain.

The rest of the paper is organized as follows. We first present a two-source two-relay topology and some preliminaries for the proposed NB-JNCD scheme in Section 2. We then describe the NB-JNCD coding and decoding procedures in Section 3. Theoretic analysis and simulation evaluation are presented in Sections 4 and 5 respectively. The application of NB-JNCD in large multi-hop wireless networks is studied in Section 6. Finally, related work is briefed in Section 7 followed by conclusions of the paper and future work in Section 8.

2. TOPOLOGY AND PRELIMINARIES

2.1 Topology

We first consider a simple two-source two-relay topology as shown in Fig. 1. In this topology, two sources, S_1 and S_2, transmit two independent packets, $\mathbf{x}_1$ and $\mathbf{x}_2$, to a common sink, T, with the help of two relays, R_1 and R_2. To focus on the joint decoding procedure at the sink node, we assume the channels between the sources and the relays are lossless (Our scheme does not have such a requirement, see Section 6). This simple topology is used to demonstrate the benefit of the proposed scheme; extensions to large complex topologies with all lossy channels will be described in Section 6. Upon receiving $\mathbf{x}_1$ and $\mathbf{x}_2$, the two relays, R_1 and R_2, will forward redundant packets, $\mathbf{y}_1$ and $\mathbf{y}_2$ (whose contents determine the level of collaborations), to the sink respectively. In this way, the sink node will see four packets, $\mathbf{x}_1$ from source S_1, $\mathbf{x}_2$ from source S_2, $\mathbf{y}_1$ from relay R_1 and $\mathbf{y}_2$ from relay R_2.

2.2 Channel Model

We assume that all lossy channels suffer from slow fading: fading keeps constant across one packet and varies from packet to packet independently (a.k.a block fading). We model the channel as Rayleigh fading with additive white Gaussian noise:

$$y = hx + w, \tag{1}$$

where $y \in \mathbb{C}$, $x \in \mathbb{C}$ and $w \in \mathbb{C}$ denote the received signal, the transmitted signal and the additive noise respectively, and $h \in \mathbb{C}$ denotes the fading coefficient. Since $|h|$ follows the Rayleigh distribution, $|h|^2$ follows an exponential distribution with mean $1/\lambda$. Thus the probability density function (pdf) of $|h|^2$ can be written as:

$$p(z) = \lambda e^{-\lambda z} \qquad (z = |h|^2). \tag{2}$$

Moreover, w is modeled as a zero-mean complex Gaussian random variable with two-dimensional variance N_0. Then the transmit signal to noise ratio (SNR) can be defined as $\gamma = E_s/N_0$, where $E_s = E\{|x|^2\}$. Thus the instantaneous receive signal to noise ratio is $\gamma|h|^2 = E_s|h|^2/N_0$ and the average receive signal to noise ratio is $\gamma E\{|h|^2\} = E_s/\lambda N_0$. Here $E\{.\}$ denotes the expectation operation.

2.3 Channel Coding

In the proposed NB-JNCD, we choose non-binary irregular low-density parity check (LDPC) codes presented in [16] as the channel coding scheme. The rationales behind this are: 1) LDPC code can be graphically represented using factor graph; 2) the channel coding/decoding on non-binary Galois field can be seamlessly combined with the network coding/decoding; 3) the LDPC codes presented in [16] can approach the channel capacity of non-fading channels. A non-binary LDPC is specified by a parity check matrix $\mathbf{H}$ of size $m \times n$ and a generator matrix $\mathbf{G}$ of size $k \times n$, which satisfy the relationship $\mathbf{H}\mathbf{G}^T = \mathbf{0}$. Both $\mathbf{H}$ and $\mathbf{G}$ have elements taken from GF(2^q).

2.4 Network Coding

In the proposed NB-JNCD, we choose non-binary random linear network coding as the network coding scheme. Firstly, previous studies in [9, 10, 15, 21, 23, 33] have showed that random linear network coding is efficient and sufficient. Secondly, network coding performing non-binary operations on a high order Galois field can provide independent network codes with high probability. Thirdly, the randomness of such network coding scheme renders itself applicable to large networks as it allows distributed operation on each node without interrupting others. Lastly, when non-binary random linear network coding is combined with non-binary LDPC codes, the encoding and decoding procedures can be significantly simplified: bit-to-symbol conversion and its inverse is not needed and non-binary source coding and high order modulation can be unified without any conversion.

3. NB-JNCD: CODING AND DECODING

In this section, we present the coding and decoding procedures of the proposed NB-JNCD using the topology shown in Fig. 1. We assume that all packets and operations are based on symbols (with q bits each) and Galois field GF(2^q).

3.1 Code Construction

Referring to Fig. 1, we assume that source S_1 generates a packet $\mathbf{u}_1$ with k symbols from Galois field GF(2^q), then encodes it into

$\mathbf{x}_1$ using a non-binary LDPC encoder specified by a generator matrix $\mathbf{G}_1$ of size $k \times n$ as:

$$\mathbf{x}_1 = \mathbf{u}_1 \mathbf{G}_1, \quad (3)$$

where $\mathbf{x}_1$ and $\mathbf{u}_1$ are row vectors of length n and k respectively. Thus the channel code rate is $r_c = k/n$. Similarly, the packet generated at source S_2 can be obtained as $\mathbf{x}_2 = \mathbf{u}_2\mathbf{G}_2$, where $\mathbf{G}_2$ is the code generator matrix. For simplicity, we assume that the size of $\mathbf{G}_2$ is also $k \times n$. Thus the channel code rate is the same as that for source S_1.

Assume two packets $\mathbf{x}_1$ and $\mathbf{x}_2$ are broadcasted respectively to the relays and the sink uses orthogonal channels (at different time slots or via different frequencies). After receiving packets from the sources (recall that the channels between the sources and the relays are lossless), the relays first decode and obtain the original packets, then generate packets using network coding and non-binary LDPC channel coding. The two network codes at relays R_1 and R_2 are represented as

$$\begin{aligned} \mathbf{y}_1 &= \alpha_{11}\mathbf{u}_1\mathbf{G}_{11} + \alpha_{12}\mathbf{u}_2\mathbf{G}_{12}, \\ \mathbf{y}_2 &= \alpha_{21}\mathbf{u}_1\mathbf{G}_{21} + \alpha_{22}\mathbf{u}_2\mathbf{G}_{22}, \end{aligned} \quad (4)$$

where the network coding coefficients α_{ij} $(i, j = 1, 2)$ are drawn randomly from GF(2^q) and the generator matrices $\mathbf{G}_{ij}$ $(i, j = 1, 2)$ are assumed to be of size $k \times n$. Packets $\mathbf{y}_1$ and $\mathbf{y}_2$ will be sent to the sink from R_1 and R_2 respectively.

At the sink node, four packets, $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{y}_1$ and $\mathbf{y}_2$ will be received. The sink node forms a longer code as follows:

$$[\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{y}_1\ \mathbf{y}_2] = [\mathbf{u}_1\ \mathbf{u}_2] \begin{bmatrix} \mathbf{G}_1 & 0 & \alpha_{11}\mathbf{G}_{11} & \alpha_{21}\mathbf{G}_{21} \\ 0 & \mathbf{G}_2 & \alpha_{12}\mathbf{G}_{12} & \alpha_{22}\mathbf{G}_{22} \end{bmatrix}. \quad (5)$$

Here we assume that the network coding coefficients can be conveyed to the sink without error. The code in (5) can be viewed as an integrated channel code with packets $[\mathbf{u}_1\ \mathbf{u}_2]$ and generator matrix $\mathbf{G}'$ which is specified by

$$\mathbf{G}' = \begin{bmatrix} \mathbf{G}_1 & 0 & \alpha_{11}\mathbf{G}_{11} & \alpha_{21}\mathbf{G}_{21} \\ 0 & \mathbf{G}_2 & \alpha_{12}\mathbf{G}_{12} & \alpha_{22}\mathbf{G}_{22} \end{bmatrix}. \quad (6)$$

If we define the network code rate r_n as the percentage of direct received packets over all received packets at the sink, we will have $r_n = 2/4$ for the communication scenario discussed above. Thus, the integrated code is of rate $r = r_c \times r_n$.

Given the generator matrix $\mathbf{G}'$, one can apply Gaussian elimination algorithm to obtain the corresponding parity check matrix $\mathbf{H}'$, which satisfies $\mathbf{H}'\mathbf{G}'^T = \mathbf{0}$. One option for decoding is to adopt some variants of belief propagation operating on $\mathbf{H}'$. However, it is usually hard and sometimes infeasible to perform this kind of decoding. This is because: 1) the integrated belief propagation decoding is too complicated; and 2) such decoding scheme will not provide good performance because $\mathbf{H}'$ is not sparse in general. Thus, we propose a simple iterative joint decoding algorithm, which will be described next.

3.2 Iterative Joint Decoding

In this section, we present a two-tier iterative joint network-channel decoding scheme, which implements soft decoding and allows information exchange inside and across packets.

For simplicity, we assume that all the generator matrices $\mathbf{G}_i$ $(i = 1, 2)$ and $\mathbf{G}_{ij}$ $(i, j = 1, 2)$ are the same, denoted as $\mathbf{G}$. Now we can revise (5) as

$$\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix} \mathbf{G} = \mathbf{M} \times \mathbf{U} \times \mathbf{G}. \quad (7)$$

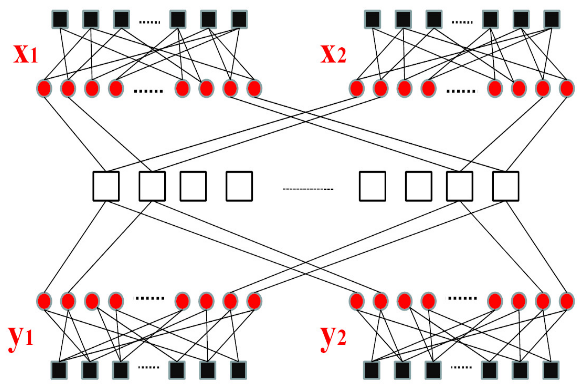


Figure 2: Factor graph representation of the integrated code.

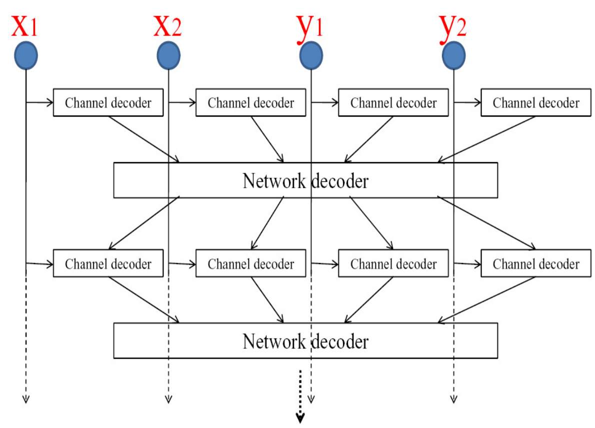


Figure 3: The procedure of iterative joint network-channel decoding.

The left hand of (7) forms a matrix of size $4 \times n$, in which each row captures the relationship among symbols inside a packet defined by channel coding, $\mathbf{G}$, and each column captures the relationship among symbols on corresponding position of different packets defined by network coding, $\mathbf{M}$.

The integrated code can be represented by a factor graph as illustrated in Fig. 2. In this figure, circles and solid rectangles represent symbol nodes and parity check nodes of the channel coding respectively, and blank rectangles represent constraint nodes of the network coding. Each symbol node (circle) is connected to one constraint node (blank rectangle) and several parity check nodes (solid rectangles). In this way, connections among all the symbols are established, allowing information to be exchanged through the links (connections) and be jointly exploited for error recovery.

The proposed joint network-channel decoding relies on iterative message exchange between two processing components, channel decoding with $\mathbf{G}$ and network decoding with $\mathbf{M}$. The type of messages exchanged can be a probability mass function (pmf) over the Galois field or its log domain version (See [16] and the references therein). The channel decoding component, $\mathbf{G}$, can be decoded through some variants of belief propagation, thanks to the sparsity of the code parity check matrix $\mathbf{H}$. The channel decoding procedure works in an iterative manner. It is comprised of symbol node updating and parity check node updating. Let us take the channel decoding of packet $\mathbf{x}_1$ as an example. Channel decoding of $\mathbf{x}_1$ is performed using information from the channel and possible *a priori* information from the network decoding component. After a maximum of L iterations of channel decoding, each symbol node of $\mathbf{x}_1$ exports extrinsic information to be used by the network decoding component, and the number of unsatisfied parity checks can be calculated via $\mathbf{c}_1 = \mathbf{H}\hat{\mathbf{x}}_1$ based on a tentative decision $\hat{\mathbf{x}}_1$. The smaller the number of nonzero entries in $\mathbf{c}_1$, the more trustable the packet $\mathbf{x}_1$ and the sooner packet $\mathbf{x}_1$ can be decoded. Therefore, the number of unsatisfied parity checks in $\mathbf{c}_1$ can be used as a metric of the goodness of packet $\mathbf{x}_1$. In particular, $\mathbf{c}_1 = \mathbf{0}$ means decoded.

As for the network decoding component, we use a *selection* updating rule. Suppose that any two rows of $\mathbf{M}$ are linearly independent (this assumption holds for NB-JNCD with high probability over a high order Galois field), then any row can be represented as a linear combination of any two other rows. Thus, we can update the *a priori* information of symbols in a packet using extrinsic information of symbols in two other best packets (a packet is said to be better if it has smaller number of unsatisfied parity checks). Let us take the network updating of packet $\mathbf{x}_1$ as an example. $\mathbf{x}_1$ can be represented as a linear combination of $\mathbf{x}_2$ and $\mathbf{y}_1$, $\mathbf{x}_2$ and $\mathbf{y}_2$, or $\mathbf{y}_1$ and $\mathbf{y}_2$. If packets $\mathbf{x}_2$ and $\mathbf{y}_1$ are better than packet $\mathbf{y}_2$, then we will use $\mathbf{x}_2$ and $\mathbf{y}_1$ to update $\mathbf{x}_1$. This *selection* updating rule can be easily generalized to any $\mathbf{M}$.

The whole decoding process can start with either channel decoding or network decoding. Fig. 3 illustrates one possible schedule which starts with channel decoding. L iterations of channel decoding and one iteration of network decoding is called a round. The decoding procedure continues round by round until all packets are correctly decoded or the maximum number of rounds is reached with a failure claimed. Except for the first round, a symbol node could combine the channel information and the updated *a priori* information from the network decoding component to perform channel decoding.

4. ANALYSIS

In this section, we analyze the diversity and capacity gains of NB-JNCD compared to three reference schemes as described below.

- *Direct Transmissions without Relays*

  In this scheme, the two sources S_1 and S_2 directly transmit packets to the sink. If we use the same form as (7) to represent this scheme, the network coding matrix $\mathbf{M}$ can be written as $\mathbf{M}^T = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}$. This scheme will be used as a baseline for comparison. To make the comparison fair, we set the transmission power at the sources in this scheme twice as that in other schemes.

- *Direct Transmissions with Relays*

  In this scheme, in addition to the direct transmissions to the sink, each source has one relay forwarding information for it. Thus, the network coding matrix $\mathbf{M}$ for this scheme becomes $\mathbf{M}^T = \begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{pmatrix}$.

- *Binary Joint Network-Channel Decoding (Binary JNCD)*

  As most existing research (See Section 7) generates network codes applying binary XOR operation, we abstract these approaches as a binary joint network-channel decoding scheme. In Binary JNCD, the channel codes are the same as those used in NB-JNCD, while the network coding coefficients $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 1$. Thus the network coding matrix $\mathbf{M}$ can be correspondingly presented as $\mathbf{M}^T = \begin{pmatrix}1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\end{pmatrix}$.

4.1 Diversity Analysis

We treat the two packets, $\mathbf{u}_1$ and $\mathbf{u}_2$, as a *generation* of size 2. Then the redundancy in network coding allows the sink to recover the whole generation from a subset of received packets. This is due to the fact that packets transmitted through independent channels can provide *spatial diversity*. To ease our analysis, we assume that all lossy links in Fig. 1 are identical and independent, with link outage probability P_e. We define the probability when at least one packet in a generation cannot be recovered as generation error rate (GER). Further, we define the exponential power of the PER/GER expression as *diversity order*. In the following, we calculate PER (packet error rate) and GER for all the four schemes discussed before.

4.1.1 Direct Transmissions without Relays

Recall that in this scheme, the sources transmit with power doubled. Thus the link outage probability P'_e for this scheme is less than P_e. We have

$$\begin{aligned}&\mathrm{PER}_D = P'_e.\\ &\mathrm{GER}_D = 1-(1-P'_e)^2 = 2P'_e - P'^2_e = 2P'_e + o(P'_e).\end{aligned} \tag{8}$$

4.1.2 Direct Transmissions with Relays

In this scheme, by exploiting spatial diversity from two independent channels, the probability that both direct transmissions and relay transmissions are corrupted will be significantly reduced as

$$\mathrm{PER}_{DR} = P_e^2. \tag{9}$$

Since no collaboration exists between the two packets in a generation, the GER can be obtained as

$$\mathrm{GER}_{DR} = 1-(1-\mathrm{PER}_{DR})^2 = 2P_e^2 + o(P_e^2). \tag{10}$$

4.1.3 Binary JNCD

In this scheme, the contents of the two network codes, $\mathbf{y}_1$ and $\mathbf{y}_2$, are identical. Thus the network codes can be treated as one code, say $\mathbf{y}$. Then the outage probability $P_{\mathbf{y}} = \mathrm{PER}_{DR} = P_e^2$. Without loss of generality, we take packet $\mathbf{x}_1$ as an example for the analysis of PER. Packet $\mathbf{x}_1$ cannot be recovered only when $\mathbf{x}_1$ and at least one of $\mathbf{x}_2$ and $\mathbf{y}$ are corrupted. Thus we have

$$\mathrm{PER}_B = P_e(1-(1-P_{\mathbf{y}})(1-P_e)) = P_e^2 + o(P_e^2). \tag{11}$$

Similarly, a generation error will happen when more than two packets among $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{y}$ are corrupted. Thus we can get

$$\mathrm{GER}_B = P_e^2 P_{\mathbf{y}} + 2P_e P_{\mathbf{y}}(1-P_e) + P_e^2(1-P_{\mathbf{y}}) = P_e^2 + o(P_e^2) \tag{12}$$

4.1.4 NB-JNCD

In the presence of more than one relay, binary XOR operation is not enough. This is because the successful reception of network codes, $\mathbf{y}_1$ and $\mathbf{y}_2$, alone cannot recover the original packets at the sink. However, this drawback can be eliminated by extending coding operations to a high order Galois field as in NB-JNCD. When the size of the Galois field increases, any two rows of the corresponding matrix $\mathbf{M}$ will become linearly independent with a high probability, which renders that any two packets can recover all others. Thus we can easily obtain PER and GER for NB-JNCD as follows:

$$\text{PER}_{NB} = P_e(P_e^3 + \binom{3}{2}P_e^2(1-P_e)) = 3P_e^3 + o(P_e^3).$$
$$\text{GER}_{NB} = P_e^4 + \binom{4}{3}P_e^3(1-P_e) = 4P_e^3 + o(P_e^3). \tag{13}$$

In short, the diversity order of GER equals to *the minimum number* that renders the residue matrix of $\mathbf{M}$ to be not full column-rank after deleting that amount of rows. The diversity order of PER for source S_1 is equal to *the minimum number* that renders the row space of the residue matrix of $\mathbf{M}$ to not contain the subspace expanded by $(1, 0)$ after deleting that amount of rows. This principle can be generalized to any generation size K.

Discussions: As we see, in Direct Transmissions without Relays, the error rate decreases linearly with the link outage probability P'_e. While both Direct Transmissions with Relays and Binary JNCD can reach diversity order of 2, NB-JNCD achieves diversity order of 3, which is optimal in the two-source two-relay topology from the diversity point of view. This indicates that the error rates in NB-JNCD decrease much faster than those in other schemes.

So far, our analysis has not considered joint decoding across packets, which accounts for coding gain. Next, we present our analysis from the capacity point of view.

4.2 Capacity Analysis

Recall that we assume all lossy links in Fig. 1 are independent and obey the same fading distribution with parameter λ (See Section 2). The instantaneous receive signal to noise ratio is $\gamma|h_{i,j}|^2$, where $\gamma = E_s/N_0$, and $h_{i,j}$ is the fading coefficient of the channel from sender i to receiver j. The maximum amount of information carried per channel can be expressed as $I_{i,j} = \log_2(1 + \gamma|h_{i,j}|^2)$.

The capacity analysis for point to point channels and single relay channels have been well studied in the literature. According to Shannon's coding theory, we can directly obtain the necessary conditions for successful recovery of both packets in a generation for Direct Transmissions without Relays as:

$$\begin{cases} I_{\mathbf{x}_1} = \log_2(1 + 2\gamma|h_{S_1,T}|^2) > r_c \\ I_{\mathbf{x}_2} = \log_2(1 + 2\gamma|h_{S_2,T}|^2) > r_c \end{cases} \tag{14}$$

where 2γ accounts for the twice transmission power in the scheme. The corresponding GER bound can be expressed as

$$\overline{\text{GER}}_{direct} = 1 - P(I_{\mathbf{x}_1} > r_c)P(I_{\mathbf{x}_2} > r_c). \tag{15}$$

Assisted by relays, the capacity of the system can be significantly increased. Since the network coded packets, $\mathbf{y}_1$ and $\mathbf{y}_2$, carry partial information of the original packets $\mathbf{x}_1$ and $\mathbf{x}_2$, the necessary conditions for successful recovery of a generation can be represented as ([8])

$$\begin{cases} \frac{1}{4}(I_{S_1,T} + I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r \\ (I_{S_1,T} + \xi_{11}I_{R_1,T} + \xi_{21}I_{R_2,T}) > r_c \\ (I_{S_2,T} + \xi_{12}I_{R_1,T} + \xi_{22}I_{R_2,T}) > r_c \end{cases} \tag{16}$$

where $\xi_{11} + \xi_{12} = 1$ and $\xi_{21} + \xi_{22} = 1$ ($\xi_{11}, \xi_{12}, \xi_{21}, \xi_{22} \in [0, 1]$). The first inequity accounts for the necessary condition to recover both original packets, and the other two represent the conditions to recover each packet with the help of the two relays respectively. The coefficients $\xi_{11}, \xi_{12}, \xi_{21}, \xi_{22}$ are referred to as *information splitters*, because the two original packets share the redundant information in the network codes. The exact values are difficult to decide and they may vary under different channel conditions. Thus, we weaken the conditions to obtain a loose upper bound of the system capacity as

$$\begin{cases} I'_{(\mathbf{x}_1,\mathbf{x}_2)} = \frac{1}{4}(I_{S_1,T} + I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r \\ I'_{\mathbf{x}_1} = (I_{S_1,T} + I_{R_1,T} + I_{R_2,T}) > r_c \\ I'_{\mathbf{x}_2} = (I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r_c \end{cases} \tag{17}$$

and the corresponding GER upper bound can be expressed as

$$\overline{\text{GER}}_{relay} = 1 - P(I'_{(\mathbf{x}_1,\mathbf{x}_2)} > r)P(I'_{\mathbf{x}_1} > r_c)P(I'_{\mathbf{x}_2} > r_c) \tag{18}$$

This upper bound is obtained under the assumption of using perfect channel codes and network codes. In next section, through simulation we will show that the performance of the proposed NB-JNCD scheme approaches this upper bound with acceptable performance loss.

5. PERFORMANCE EVALUATION

We now present extensive simulation results to demonstrate the benefits of the proposed NB-JNCD scheme. Comparison with other schemes and the capacity bounds are also included.

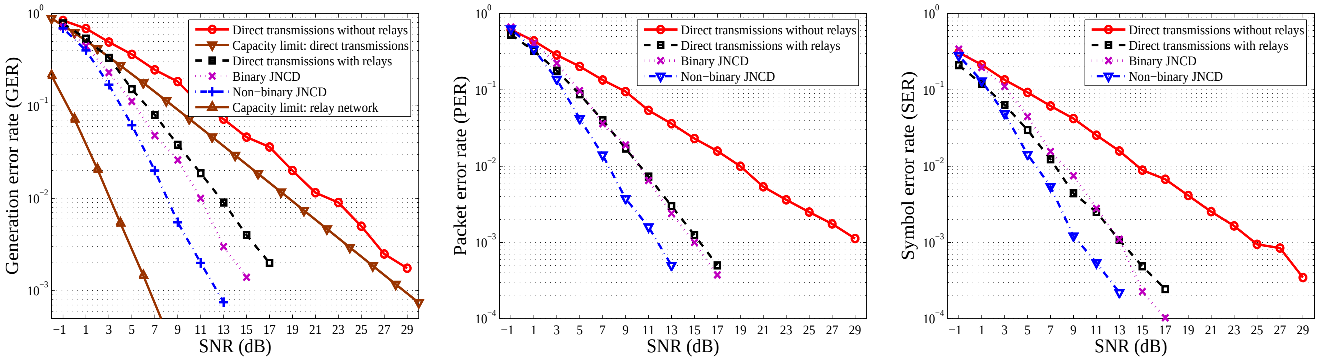
5.1 Simulation Setup

Two sources S_1 and S_2 generate packets of length $k = 800$ symbols over GF(2^4), where each symbol corresponds to 4 bits. Using a non-binary irregular LDPC code of rate $r_c = 0.8$ over GF(2^4), whose average column weight is 2.8 [16], the original packets $\mathbf{u}_1$ and $\mathbf{u}_2$ are encoded into $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively, each of length $n = 1000$ symbols. After receiving the packets$\mathbf{x}_1$ and $\mathbf{x}_2$, the relays R_1 and R_2 perform random linear network coding. Although the coefficients for NB-JNCD network coding can be randomly drawn from GF(2^4), which renders any two rows of the obtained matrix $\mathbf{M}$ linearly independent with high probability [21], we here set the coefficients as $\alpha_{11} = \alpha_{12} = 7$, $\alpha_{21} = 12$, and $\alpha_{22} = 13$ for simplicity. All lossy links in Fig. 1 are assumed to be independent and have the same fading distribution. We adopt BPSK modulation in all simulations other than those in Sections 5.6 and 6 where 16QAM modulation is employed to achieve high bandwidth efficiency. Using the metrics of generation/packet/symbol error rates, we compare the following four schemes: direct transmissions without relays, direct transmissions with relays, binary JNCD, and NB-JNCD. For a fair comparison, the transmission power in the scheme of direct transmissions without relays is doubled as in other schemes, which renders the total energy consumption the same.

5.2 Overall Performance Comparison

Fig. 4 shows the performance comparison of different schemes under various average receive SNR, where simulation is conducted until GER is below 10^{-3}. We observe that NB-JNCD outperforms other schemes under all metrics, especially at high SNR. Specifically, at GER of 10^{-3}, NB-JNCD outperforms direct transmissions without relays by about 20 dB, while outperforms the other two schemes by 3 to 5 dB.

The performance curves in Fig. 4 agree with the diversity analysis in Section 4 very well. We can see that direct transmissions



(a) (b) (c)

Figure 4: Performance comparison of NB-JNCD with other schemes.

without relays exibits a diversity order of 1, due to the lack of collaboration. Both direct transmissions with relays and binary JNCD schemes achieve diversity order of 2, thanks to the spatial diversity provided by the the relays Rendering any two rows of the network coding matrix $\mathbf{M}$ linearly independent by drawing elements from a high-order Galois field, NB-JNCD achieves a diversity order of 3, which is optimal in this particular case.

It is also interesting to note that although binary JNCD uses network coding to exploit the cooperation between packets, it does not outperform the direct transmissions with relays on PER and symbol error rate (SER). However, it achieved a performance gain about 3 dB at GER around 10^{-3}. This observation can be verified from (9)-(12). In direct transmissions with relays, the two packets $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent, whereas in binary JNCD an coopeation has been built between them through network coding. Hence, both packets in binary JNCD will be either recoverable at the same time or unrecoverable. This property may be desirable in scenarios, e.g., packet splitting and in-network processing, where only the reception of the whole generation is meaningful to the sink.

In Fig. 4(a), we also plot two curves corresponding to the capacity bounds in (15) and (18), which are obtained through numerical integration. The capacity limit on direct transmissions indicates the best performance achievable by channel coding alone. We observe that the used non-binary LDPC code can approach the outage capacity limit of the block fading channel within 3 dB; it approaches the Shannon limit of BPSK modulation over AWGN channel within 1.2 dB at PER of 10^{-4} [16]. The capacity limit on relay network represents an upper bound of the relay network under consideration, which holds for any scheme utilizing relays. We see that NB-JNCD is the closest to this upper bound with about 6 dB performance loss while sharing the same diversity order with the upper bound.

5.3 Joint Decoding Gain

One advantage of NB-JNCD is that the iterative joint decoding process allows information to be exchanged not only within each packet, but also across the packets, which further improves the decoding performance. Separate network-channel decoding (SNCD) schmes treat channel codes and network codes separately, where channel decoding is followed by network decoding with no iteration. For SNCD, the soft information in those packets with channel decoding failures will be wasted. Fig. 5 demonstrates the advantage of NB-JNCD over NB-SNCD. Although both schemes have the same diversity order, JNCD can outperform SNCD by about 2 dB on average.

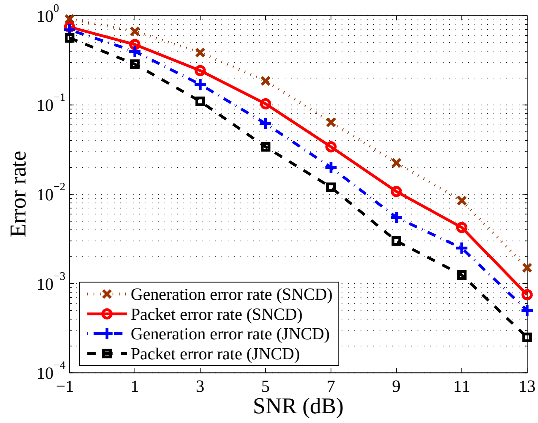
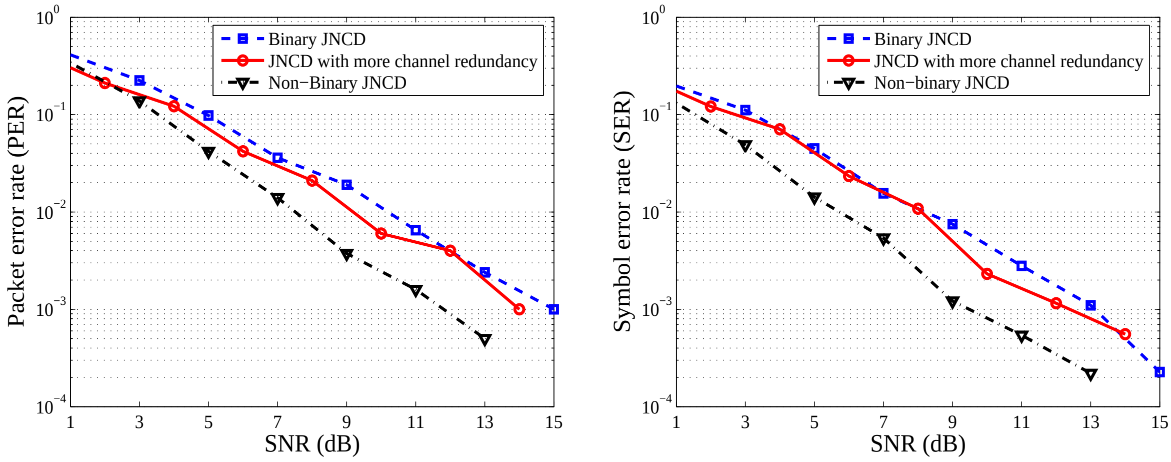


Figure 5: The gain of joint decoding over separate decoding.

5.4 Impact of Joint Decoding Procedure

One round of joint network-channel decoding procedure includes L iterations of channel decoding and one network decoding. One interesting problem is how the choice of L will impact the performance and complexity.

Note that the average row weight of the used LDPC code's parity check matrix is $2.8/(1-r_c) = 14$, that is, the average degree of the filled rectangle node in Fig. 2 is 14, while the degree of the blank rectangle nodes in Fig. 2 is only 4. Hence, check node updating of in channel decoding is much more complex than that in network decoding. For ease of comparison, we ignore the complexity of the network decoding process. We vary the number of iterations L in each round from 6 to 1. Intuitively, the smaller L is, the more frequently information is exchanged across packets through network decoding. We observe that different L values lead to similar performance, but with different decoding complexity. Fig. 6 depicts the average number of channel decoding iterations over recovered generations with different L ; note that the number of iterations of unrecovered generations are not counted in. The average number of channel decoding iterations for every packet is less than 1 at high SNR is due to the fact that some packets can be decoded without channel decoding iterations. It can be seen that NB-JNCD requires less number of iterations when L decreases. This reveals the fact that frequent information exchange through network decoding can speed up the channel decoding process. It is better to dynamically



(a) (b)

Figure 7: Different schemes under the same overall code rate.

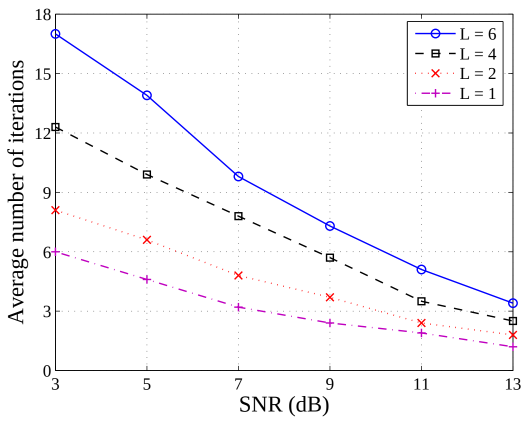


Figure 6: Average number of channel decoding iterations for decoded generations.

tune the value of L adaptively according to the channel conditions in real scenario.

5.5 Where to Put the Redundancy?

NB-JNCD exploits the redundancy in both channel codes and network codes. An interesting question is: with a fixed total code rate $r = r_c \times r_n$, how shall we split redundancy in the channel codes network codes? Berger *et al.* optimized the code rate assignment in joint error- and erasure-correction coding and claimed that more redundancy across packets is desirable in severe fading channels [6]. We have used $r_c = 0.8$ and $r_n = 0.5$ in our simulations. By removing one relay which leads to $r_n = 2/3$, we can put more redundancy in channel coding with rate $r_c = 0.6$ while keeping the total code rate unchanged. The performance comparison is shown in Fig. 7. Having only one relay, the new scheme can achieve a diversity order of at most 2. We observe that the performance of the new scheme is slightly better than that of binary JNCD with two relays, but much worse than that of NB-JNCD with two relays. Since there will be multiple relays available in large networks, it is beneficial to exploit network cooperations rather than put too much effort on protecting a single channel. The potential benefits of NB-JNCD in large wireless networks will be illustrated in Section 6.

5.6 Using High Order Modulation

Practical communication systems often employ high order modulation to increase the spectral efficiency. In NB-JNCD, all operations are executed on symbols in a high order Galois field. Using modulations having the same size as the Galois field, each coded symbol can be directly mapped onto a constellation point. All encoding/decoding and modulation/demodulation processes can be unified without the need of bit-to-symbol conversion and its inverse.

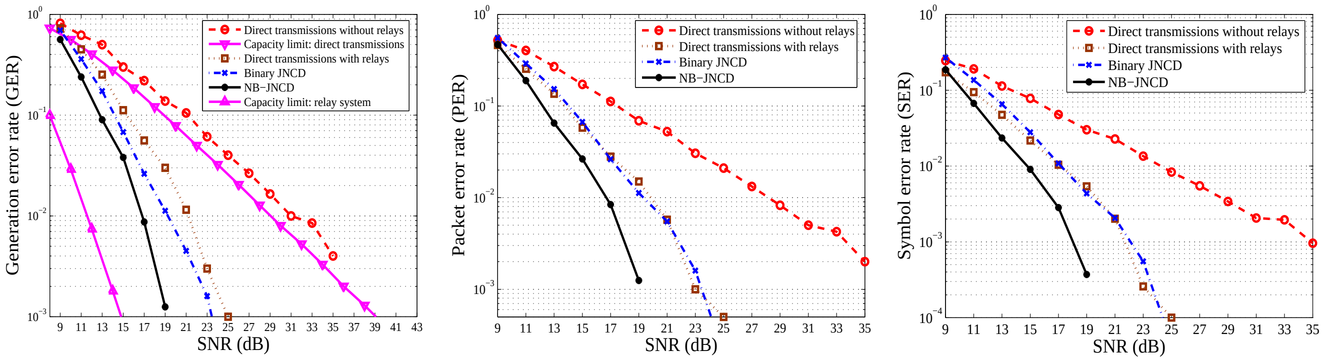
Fig. 8 presents the results with 16QAM modulation, which leads to a spectral efficiency of $4 \times r = 1.6$ bits/s/Hz compared with $r = 0.4$ bits/s/Hz for BPSK modulation. The two capacity bounds in Fig. 8 are obtained by augmenting the right hand sides of (14), (16), and (17) by four times since each 16QAM symbol carries four bits. Accouting for the fact that each bit transmission only gets $1/4$ of the symbol power, the results in Fig. 8 are comparable with those in Fig. 4 and all conclusions for BPSK hold true for 16QAM modulation. The gap between NB-JNCD and the capacity upper bound with 16QAM is about 4 dB which shows a gain about 2 dB over the case with BPSK, because 16QAM is more bandwidth efficient than BPSK.

6. EXTENSION TO LARGE NETWORKS

Using a simple topology with two sources, two relays and one sink, we have demonstrated the benefits of our proposed NB-JNCD scheme through both analysis and simulation. In this section, we describe how NB-JNCD can be applied in large, multi-hop networks.

6.1 NB-JNCD in General Wireless Networks

In a large, multi-hop wireless network, every link may suffer from the detrimental effect of fading. When a link is in deep fading, high packet loss happens and the communication through the link cannot continue. Thus, traditional unicast routing protocols designed for wire line networks, which usually take the shortest path for information delivery, are vulnerable to individual link loss and cannot provide reliable service for end-to-end communication. One alternative way is to rely on multi-path routing where more than one



(a) (b) (c)

Figure 8: Performance comparison with 16QAM modulation.

routes exist for the source to reach the destination. When there exist several independent messages to be transmitted to the same destination in the network, instead of performing simple store-and-forward operations at the intermediate nodes, network coding, which allows the intermediate nodes to encode the incoming packets before forwarding, can be employed to achieve better performance [9].

Without loss of generality, assume that a source, or K separate sources, generates K packets represented as $\mathbf{u}_i$, $(i = 1, \ldots, K)$ with each packet containing k symbols from GF(2^q). These K packets, expanding a packet space of dimension K, will be encoded using non-binary LDPC coding followed by random linear network coding before they are injected to the network. For simplicity, we assume that all nodes use the same channel coding generator matrix $\mathbf{G}$. For an intermediate node, suppose that it receives M packets, $\mathbf{x}_1, \ldots, \mathbf{x}_M$. which are already network encoded and can be expressed as $\mathbf{x}_i = \sum_{j=1}^{K} \alpha_{ij}\mathbf{u}_j\mathbf{G}$. Thus, we have

$$\begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_M \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \ldots & \alpha_{1K} \\ \vdots & \vdots & \vdots \\ \alpha_{M1} & \ldots & \alpha_{MK} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_K \end{pmatrix} \mathbf{G} \tag{19}$$

The network coding matrix $\mathbf{M} = [\alpha_{ij}]$ is of size $M \times K$. Assume that the row space of $\mathbf{M}$ is of dimension K' ($K' \leq \min\{K, M\}$). Then the intermediate node can recover at most K' independent packets. If $K' = M$, i.e., when an independent packet arrives, there is no redundant packet, then channel decoding is applied to each packet separately. If $K' < M$, i.e., when a dependant packet arrives, there exist redundant packets and the proposed iterative joint network-channel decoding can be applied to exploit the redundancy for error recovery. Assume that M' packets are successfully decoded as $\mathbf{x}'_i = \sum_{j=1}^{K} \alpha'_{ij}\mathbf{u}_j\mathbf{G}$. Then the relay can regenerate arbitrary N (depending on the specific schemes) network codes using random linear network coding as $\mathbf{y}_i = \sum_{j=1}^{M'} \beta_{ij}\mathbf{x}'_j = \sum_{k=1}^{K} \gamma_{ik}\mathbf{u}_k\mathbf{G}$ $(1 \leq i \leq N)$, where β_{ij} is randomly drawn from GF(2^q) and $\gamma_{ik} = \sum_{j=1}^{M'} \beta_{ij}\alpha'_{jk}$. In this way, nodes in the network can work in a distributed manner, where later received packets can be directly added into the decoding process without any interruption.

At the destination node, to recover the whole generation, the number of received packets should be no less than K and the corresponding matrix $\mathbf{M}$ should be full-column rank. Thus, an appropriate routing algorithm should guarantee the destination to receive enough packets. Otherwise, retransmission should be triggered.

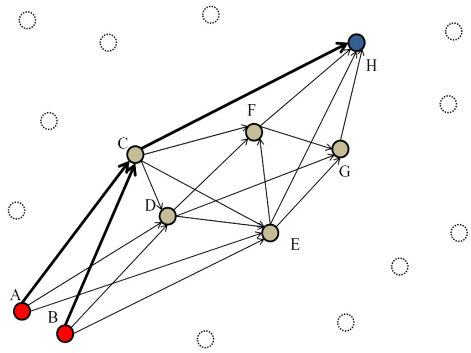


Figure 9: A wireless network with A and B communicating with H.

6.2 Performance in Large, Multi-hop Wireless Networks

In this section, we evaluate the performance of NB-JNCD in a large multi-hop network, illustrated in Fig. 9. In this network, two sources, A and B, intend to send a sequence of generations to H, and each generation contains two packets having one packet from each source. Due to the broadcasting nature of wireless transmission, signals sent from a particular node (e.g., A) can be overheard by several nodes (e.g., C, D, E) for free. In Fig. 9, these links are indicated by directed solid lines. Traditional unicast takes the shortest path $A(B) \rightarrow C \rightarrow H$ (Bold solid lines in Fig. 9) for information delivery. Multi-path routing, on the other hand, can take advantage of this broadcasting feature to facilitate information delivery. Suppose that the whole transmission in Fig. 9 operates in an upstream-to-downstream fashion, that is, all nodes process according to this schedule $(A{,}B)\rightarrow C\rightarrow D\rightarrow E\rightarrow F\rightarrow G\rightarrow H$. The real schedule can be different according to the underlying routing algorithm, but it will not affect the process and performance. All intermediate nodes which receive at least one packet successfully are responsible for forwarding one network encoded packet. We consider three schemes: unicast, multi-path routing with NB-JNCD, and multi-path routing with Binary JNCD. For the scheme with NB-JNCD, all network coding operations are performed on Galois field, whereas for the scheme with Binary JNCD, all network coding operations are binary XOR. Same as in Section 2, all the channels are assumed to be i.i.d. and obey the same fading model.

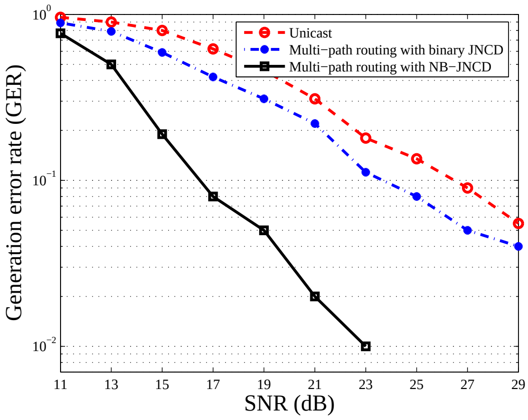


Figure 10: Performance in a large, multi-hop wireless network.

We compare the performance of the three schemes under the same setting as in Section 5 with 16QAM modulation. The results are plotted in Fig. 10. From this figure, we can observe that the multi-path routing with NB-JNCD experiences much sharper GER performance than the multi-path routing with Binary JNCD and the traditional unicast. Specifically, at GER of 10^{-1}, the performance gain of the multi-path routing with NB-JNCD against the multi-path routing with Binary JNCD (unicast, resp.) is about 6 dB (10 dB, resp.). Thus, we can conclude that NB-JNCD is very promising in large, multi-hop wireless networks.

Another interesting metric in large networks is throughput. Although multi-path routing with network coding should transmit some redundant packets, since the interference of one transmission is limited inside the one-hop neighborhood, we believe the proposed NB-JNCD can achieve a higher throughput than others under high SNRs. Comprehensive comparison on throughput will be conducted in future work.

7. RELATED WORK

Based on the code construction and decoding procedure, existing researches on unifying channel coding and network coding for better performance can be roughly classified into three categories: *separate network channel coding*, *distributed channel coding* and *joint network-channel coding*.

Separate network-channel coding is straightforward. Larsson *et al.* and Tran *et al.* utilized network coding to implement the first type Hybrid ARQ for one-source, multi-sink one hop networks in [22] and [29] respectively. They constructed network codes based on feedback from sinks. Berger *et al.* theoretically analyzed the optimization problem in joint erasure-correction and error-correction coding schemes [6]. All these studies treat two levels of codes separately and do not fully exploit the precious redundant information.

In the literature, several approaches distribute the procedure of channel coding to different nodes in a network and they can be classified as distributed channel coding. Bao and Li proposed Adaptive Network Coded Cooperation (ANCC) for multiple transmitters sending data to a common receiver [5]. Their contribution is to match code-on-graph with network-on-graph to dynamically construct low-density generator matrix (LDGM) codes. This scheme requires thousands of transmitters to form an entire code through network coding and stringent inter-user synchronization at the bit/baud level. Later, in [4, 3], the same authors analyzed the outage properties of ANCC. In general, ANCC is not practical as it is infeasible to have such large number of nodes in real networks.

The design principle in joint network-channel coding is that the redundancy in both channel codes and network codes should be jointly exploited to support the decoding of each other. Recently, there are many active studies in this direction. The second type hybrid-ARQ [28] and the nested codes [31, 20] utilized retransmitted network codes to decode erroneous packets received previously. Hausl *et al.* first presented iterative network and channel decoding on a tanner graph in [14]. They also proposed joint network-channel coding for both multi-access relay channel and two-way relay channel [12, 11, 13]. Bao and Li extended ANCC [5] to GANCC on packet level [2] and presented the general framework that unifies channel coding and network coding. Yang *et al.* and Kang *et al.* further proposed iterative network and channel decoding when the relays cannot perfectly recover packets in [32] and [17] respectively. Nazer *et al.* and Narayanan *et al.* even applied lattice code on relays considering the multi-access property of wireless networks to approach the capacity in [27, 26, 25]. Overall, these joint network-channel coding schemes are designed for small wireless networks with specific topologies, using binary operations through additive Gaussian channels, sending analogy values or depending on unpractical physical layer network coding. Moreover, the transmissions of each node is required to be well scheduled and the joint network-channel codes to be well designed. Hence, they cannot be easily applied to large multi-hop wireless networks. In addition, the network codes in these schemes are usually generated with XOR operation, which is not sufficient in large networks to provide enough independent packets in a distributed way.

8. CONCLUSIONS AND FUTURE WORK

In this paper, we presented practical non-binary joint network-channel decoding (NB-JNCD) for reliable communication in wireless networks. The proposed NB-JNCD seamlessly combines non-binary channel coding and random linear network coding and can be directly coupled with high order modulation to provide high bandwidth efficiency. An iterative two-tier decoding scheme was proposed to jointly exploit redundancy inside packets and across packets for error recovery. Both theoretic analysis and simulation have demonstrated the significant benefits of NB-JNCD. Compared to other schemes, NB-JNCD can fully exploit the spatial diversity and can approach the capacity upper bound with acceptable performance loss.

Future Work: We plan out future work in the following directions: 1) derive a tighter capacity upper bound for the two-source two-relay topology; 2) perform diversity and capacity analysis for large networks; 3) study the throughput of NB-JNCD compared with other schemes in large networks; and 4) design an optimal joint network-channel code.

Acknowledgment

We acknowledge the support from NSF CAREER award 0746841, NSF CAREER Grant No. 0644190, Grant No. 0709005, Grant No. 0721834, Grant No. 0821597, ONR YIP Grant No. N000140810864 and ONR grant N00014-07-1-0429. We thank Tiffany Jing Li in Lehigh University for helpful comments.

9. REFERENCES

[1] R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung. Network Information Flow. *IEEE Transactions on Information Theory*, 46(4), July 2000.

[2] X. Bao and J. Li. A Unified Channel-Network Coding Treatment for User Cooperation in Wireless Ad-Hoc

Networks. In *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, Seattle, WA, USA, July 2006.

[3] X. Bao and J. Li. An Information Theoretic Analysis for Adaptive-Network-Coded-Cooperation (ANCC) in Wireless Relay Networks. In *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, Seattle, WA, USA, July 2006.

[4] X. Bao and J. Li. On the Outage Properties of Adaptive Network Coded Cooperation (ANCC) in Large Wireless Networks. In *Proceedings of IEEE International Conference on Accoustic, Speech, and Signal Processing (ICASSP)*, Toulouse, France, May 2006.

[5] X. Bao and J. Li. Adaptive Network Coded Cooperation (ANCC) for Wireless Relay Networks: Matching Code-on-graph with Network-on-graph . *IEEE Transactions on Wireless Communications*, 7(2):574–583, Febuary 2008.

[6] C. R. Berger, S. Zhou, Y. Wen, P. Willett, and K. Pattipati. Optimizing Joint Erasure- and Error-Correction Coding for Wireless Packet Transmissions. *IEEE Transactions on Wireless Communications*, July 2008.

[7] Y. Chen, S. Kishore, and J. Li. Wireless Diversity through Network Coding. In *Proceedings of IEEE Wireless Communications and Networking Conference*, Las Vegas, NV, USA, September 2006.

[8] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. WILEY, second edition, 2006 July.

[9] Z. Guo, B. Wang, and J.-H. Cui. Efficient Error Recovery Using Network Coding in Underwater Sensor Networks. In *Proceedings of IFIP Networking*, pages 227–238, Atlanta, Georgia, USA, May 2007.

[10] Z. Guo, B. Wang, P. Xie, W. Zeng, and J.-H. Cui. Efficient Error Recovery with Network Coding in Underwater Sensor Networks. *Ad Hoc Networks*, 7(4):791–802, 2009.

[11] C. Hausl. Improved Rate-Compatible Joint Network-Channel Code for the Two-Way Relay Channel. In *Proceedings of 15th Joint Conference on Communications and Coding (JCCC)*, Solden, Austria, March 2006.

[12] C. Hausl and P. Dupraz. Joint Network-Channel Coding for the Multiple-Access Relay Channel. In *Proceedings of International Workshop on Wireless Ad-hoc and Sensor Networks (IWWAN)*, New York, NY, JUN 2006.

[13] C. Hausl and J. Hagenauer. Iterative Network and Channel Decoding for the Two-Way Relay Channel. In *Proceedings of IEEE International Conference on Communications*, Istanbul, June 2006.

[14] C. Hausl, F. Schreckenbach, and I. Oikonomidis. Iterative Network and Channel Decoding on a Tanner Graph. In *Proceedings of the 43rd Annual Allerton Conference on Communication, Control, and Computing*, Monticello, USA, September 2005.

[15] T. Ho, M. Medard, R. Koetter, D. Karger, M. Effros, J. Shi, and B. Leong. A Random Linear Network Coding Approach to Multicast. *IEEE Transaction on Information Theory*, 52(10), October 2006.

[16] J. Huang, S. Zhou, and P. Willett. Nonbinary LDPC Coding for Multicarrier Underwater Acoustic Communication. *IEEE J. Selected Areas in Communications Special Issue on Underwater Wireless Communications and Networks*, 26(9):1684–1696, 2008.

[17] J. Kang, B. Zhou, Z. Ding, and S. Lin. LDPC Coding Schemes for Error Control in a Multicast Network. In *Proceedings of IEEE International Symposium on Information Theory*, Toronto, ON, Canada, July 6-11 2008.

[18] S. Katti, D. Katabi, W. Hu, H. Rahul, and M. Medard. The Importance of Being Opportunistic: Practical Network Coding For Wireless Environments. In *Proceedings of Allerton conference*, September 2005.

[19] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft. XORs in The Air: Practical Wireless Network Coding. In *Proceedings of SIGCOMM*, Pisa, Italy, September 2006.

[20] J. Kliewer, T. Dikaliotis, and T. Ho. On the Performance of Joint and Separate Channel and Network Coding in Wireless Fading Networks. In *Proceedings of IEEE Information Theory Workshop on Information Theory for Wireless Networks*, Solstrand, July 2007.

[21] R. Koetter and M. Medard. An Algebraic Approach to Network Coding. *IEEE/ACM Transactions on Networking*, 11(5):782–795, October 2003.

[22] P. Larsson and N. Johannson. Multi-User ARQ. In *Proceedings of IEEE 63rd Vehicular Technology Conference*, pages 2052–2057, Melbourne, Vic., May 2006.

[23] S.-Y. R. Li, R. W. Yeung, and N. Cai. Linear Network Coding. *IEEE Transactions on Information Theory*, 49(2):371–381, February 2003.

[24] S. Lin and D. J. Costello. *Error Control Coding*. Prentice Hall, second edition, 2004.

[25] K. Narayanan, M. P. Wilson, and A. Sprintson. Joint Physical Layer Coding and Network Coding for Bi-Directional Relaying. In *Proceedings of Allerton Conference on Communication, Control and Computing*, Monticello, IL, USA, 2007.

[26] B. Nazer and M. Gastpar. Compute-and-Forward: Error-Correcting Codes for Wireless Network Coding on the Physical Layer. In *Proceedings of SECON Workshops*, pages 1–5, San Francisco, CA, U.S.A., June 16-20 2008.

[27] B. Nazer and M. Gastpar. Compute-and-forward: Harnessing Interference with Structured Codes. In *Proceedings of IEEE International Symposium on Information Theory*, Toronto, ON, Canada, July 6-11 2008.

[28] R. Thobaben. Joint Network/Channel Coding for Multi-user Hybrid-ARQ. In *Proceedings of Int. ITG Conf. on Source and Channel Coding*, Ulm, Germany, January 2008.

[29] T. Tran, T. Nguyen, and B. Bose. A Joint Network-Channel Coding Technique for Single-Hop Wireless Networks. In *Proceedings of IEEE NETCOD*, HongKong, China, 2008.

[30] Y. Wu, P. A. Chou, and S.-Y. Kung. Information Exchange in Wireless Networks with Network Coding and Physical-layer Broadcast. In *Proceedings of 39th Annual Conference on Information Sciences and Systemes (CISS)*, Baltimore, MD, USA, March 2005.

[31] L. Xiao, T. E. Fuja, J. Kliewer, and J. D. J. Costello. Nested Codes with Multiple Interpretations. In *Proceedings of The 40th Annual Conference on Information Sciences and Systems*, NJ, USA, March 2006.

[32] S. Yang and R. Koetter. Network Coding over a Noisy Relay : a Belief Propagation Approach. In *Proceedings of IEEE International Symposium on Information Theory*, Nice, France, June 24-29 2007.

[33] X. Zhang, G. Neglia, J. Kurose, and D. Towsley. On the Benefits of Random Linear Coding for Unicast Applications in Disruption Tolerant Networks. In *Proceedings of IEEE NETCOD*, 2006.