

ME 343: Mechanical Design-3

Design of Shaft (continue)

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Objectives

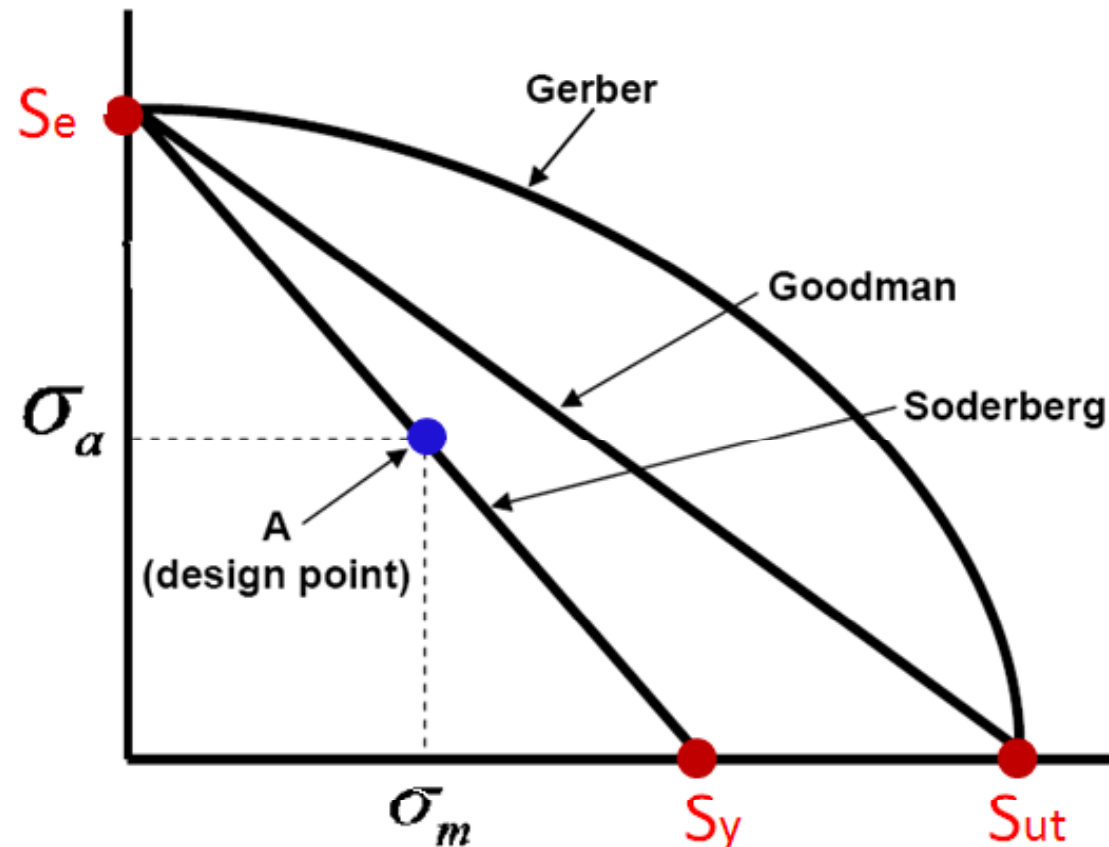
At the end of this lesson, we should be able to

- Understand design method for variable load
- Define equivalent stresses on shaft
- Design shaft based on stiffness and torsional rigidity
- Understand critical speed of shafts

Shaft design based on fatigue

- Any *rotating shaft* loaded by stationary bending and torsional moments will be stressed by *completely reversed bending stress* while the torsional stress will remain steady (i.e., $M_a = M$; $M_m = 0$; $T_a = 0$; $T_m = T$).
- A design method for variable load (fatigue), like Soderberg, Goodman or Gerber criteria can be followed.

Shaft design based on fatigue



Design under variable normal load (fatigue)

Shaft design based on fatigue

A is the design point for which the stress amplitude is σ_a and the mean stress is σ_m . In the Soderberg criterion the mean stress material property is the yield point S_y , whereas in the Goodman and Gerber criteria the material property is the ultimate strength S_{ut} . For the fatigue loading, material property is the endurance limit S_e in reverse bending.

Shaft design based on fatigue

Soderberg criterion

$$\frac{FS \sigma_a}{S_e} + \frac{FS \sigma_m}{S_y} = 1$$

mod-Goodman criterion

$$\frac{FS \sigma_a}{S_e} + \frac{FS \sigma_m}{S_{ut}} = 1$$

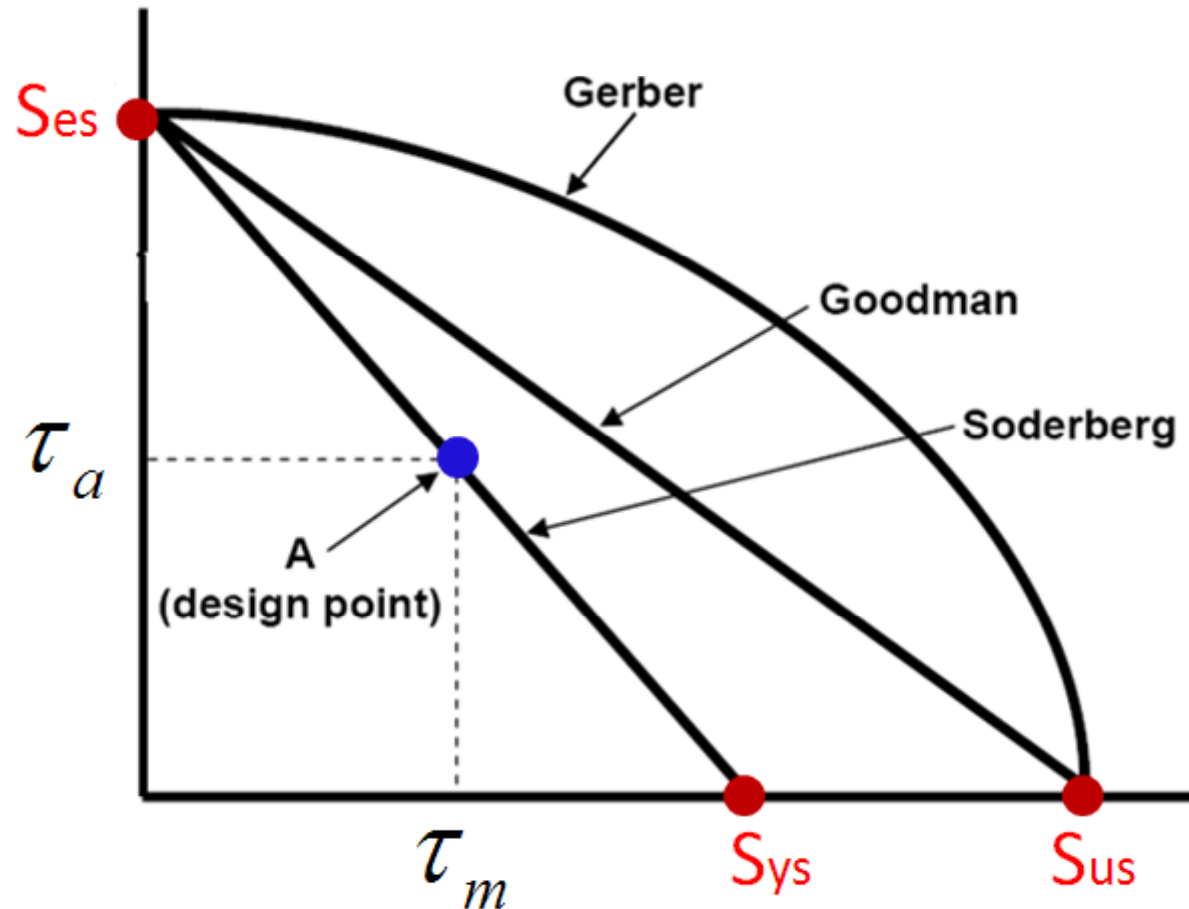
Gerber criterion

$$\frac{FS \sigma_a}{S_e} + \left(\frac{FS \sigma_m}{S_{ut}} \right)^2 = 1$$

where

σ_a = stress amplitude (alternating stress); S_e = endurance limit (fatigue limit for completely reversed loading); σ_m = mean stress; S_y = yield strength; S_{ut} = ultimate tensile strength and FS = factor of safety.

Shaft design based on fatigue



Design under variable shear load (fatigue)

Shaft design based on fatigue

- It is most common to use the Soderberg criterion.

$$\frac{FS K_f \sigma_a}{S_e} + \frac{FS \sigma_m}{S_y} = 1$$

$$\frac{S_y K_f \sigma_a}{S_e} + \sigma_m = \frac{S_y}{FS}$$

$$\frac{S_y K_f \sigma_a}{S_e} + \sigma_m = \sigma_{eq}$$

Normal Stress Equation

K_f = fatigue stress concentration factor

$$\frac{S_{ys} K_{fs} \tau_a}{S_{es}} + \tau_m = \tau_{eq}$$

Shear Stress Equation

Shaft design based on fatigue

$$\frac{S_y K_f \sigma_a}{S_e} + \sigma_m = \sigma_{eq}$$

$$\frac{S_{ys} K_{fs} \tau_a}{S_{es}} + \tau_m = \tau_{eq}$$

- σ_{eq} and τ_{eq} are equivalent to allowable stresses (S_y/FS) and (S_{ys}/FS), respectively.
- Effect of variable stress has been effectively defined as an equivalent static stress.
- Conventional failure theories can be used to complete the design.

Shaft design based on fatigue

- *Max. shear stress theory + Soderberg line (Westinghouse Code Formula)*

$$\tau_{\max} = \tau_{\text{allowable}} = \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2}$$

$$\frac{S_y}{2 FS} = \sqrt{\left(\frac{S_y K_f \sigma_a}{S_e} + \sigma_m\right)^2 + \left(\frac{S_y K_{fs} \tau_a}{S_e} + \tau_m\right)^2}$$

$$d^3 = \frac{32 FS}{\pi} \sqrt{\left(\frac{K_f M_a}{S_e} + \frac{M_m}{S_y}\right)^2 + \left(\frac{K_{fs} T_a}{S_e} + \frac{T_m}{S_y}\right)^2}$$

Shaft design based on rigidity

- Deflection is often the more demanding constraint. Many shafts are well within specification for stress but would exhibit too much deflection to be appropriate.
- Deflection analysis at even a single point of interest requires complete geometry information for the entire shaft.

Shaft design based on rigidity

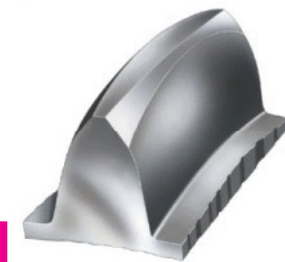
- It is desirable to design the dimensions at critical locations to handle the stresses, and fill in reasonable estimates for all other dimensions, before performing a deflection analysis.
- Deflection of the shaft, both linear and angular, should be checked at *gears* and *bearings*.

Shaft design based on rigidity

- Slopes, lateral deflection of the shaft, and/or angle of twist of the shaft should be within some prescribed limits.

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	< 0.0005 rad

Transverse deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in



Crowned tooth

Diametral pitch, $P = \text{number of teeth/pitch diameter}$. 1 in = 25.4 mm.

Shaft design based on rigidity

- In case of sleeve bearings, shaft deflection across the bearing length should be less than the *oil-film thickness*.

Twist angle:
$$\theta (rad) = \frac{T (N.mm) L (mm)}{G (N / mm^2) J (mm^4)}$$

G : shear modulus; J : polar moment of inertia

- The limiting value of θ varies from 0.3 deg/m to 3 deg/m for machine tool shaft to line shaft respectively.

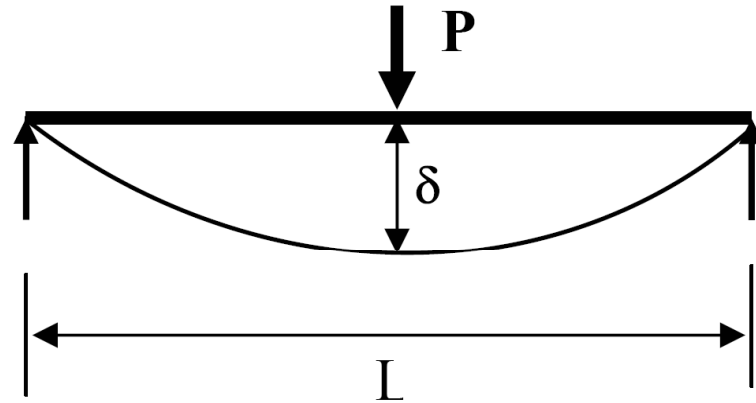
Shaft design based on rigidity

Lateral deflection:

- *Double integration*
- *Moment-area*
- *Energy (Castigliano Theorem)*

$\delta = f$ (applied load, material property, moment of inertia and given dimension of the beam).

From the expression of moment of inertia, and known design parameters, including δ , shaft dimension may be obtained.



Double Integration Method

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} \quad \theta(x) = \frac{dy}{dx} = \int \frac{M(x)}{EI(x)} dx$$

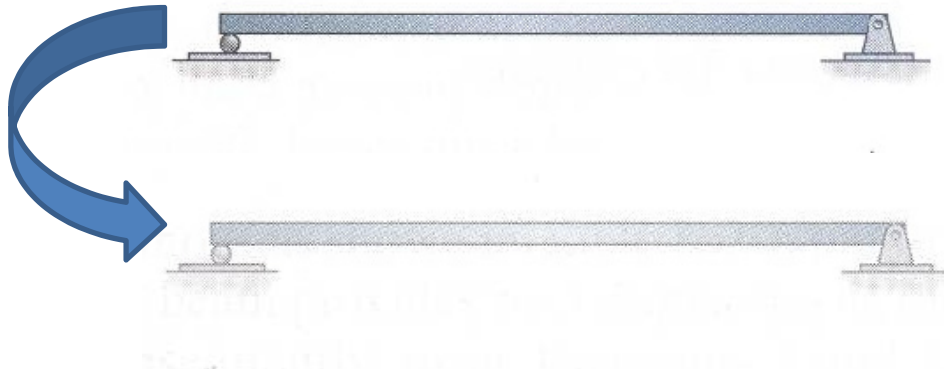
$$y(x) = \iint \frac{M(x)}{EI(x)} dx^2$$

Use boundary conditions to obtain integration constants

Conjugate beam method

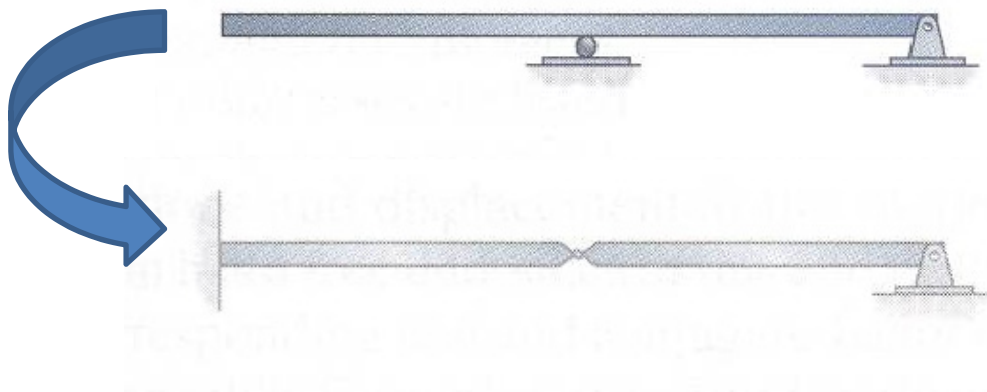
- Was developed by Otto Mohr in 1860
- Slope (real beam) = Shear (conj. beam)
- Deflection (real beam) = Moment (conj. beam)
- Length of conj. beam = Length of real beam
- The load on the conjugate beam is the M/EI diagram of the loads on the actual beam

Conjugate beam method



Real beam

Conjugate beam



Real beam

Conjugate beam

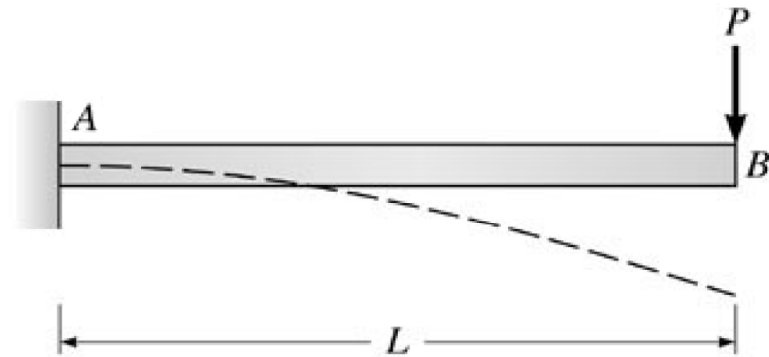
Example 1

Determine the slope and deflection at the tip of a cantilever using the conj. beam method.

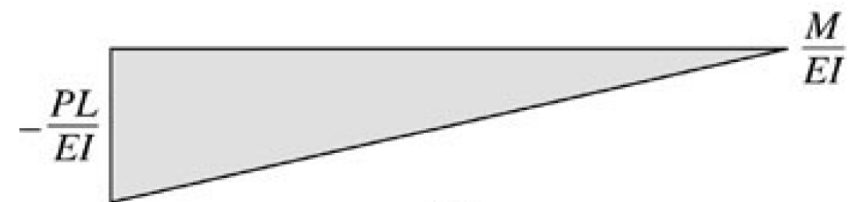
Sol:

$$\theta_B = \frac{PL^2}{2EI}$$

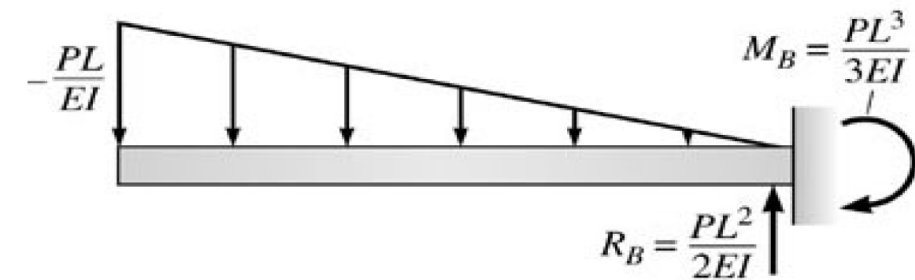
$$y_B = \frac{PL^3}{3EI}$$



(a)



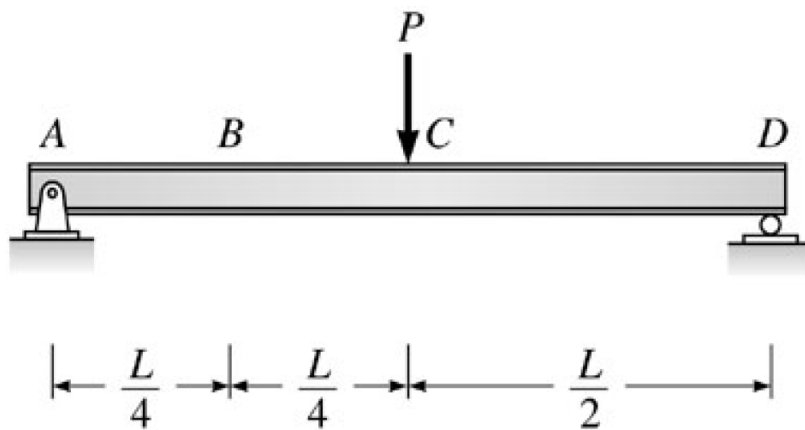
(b)



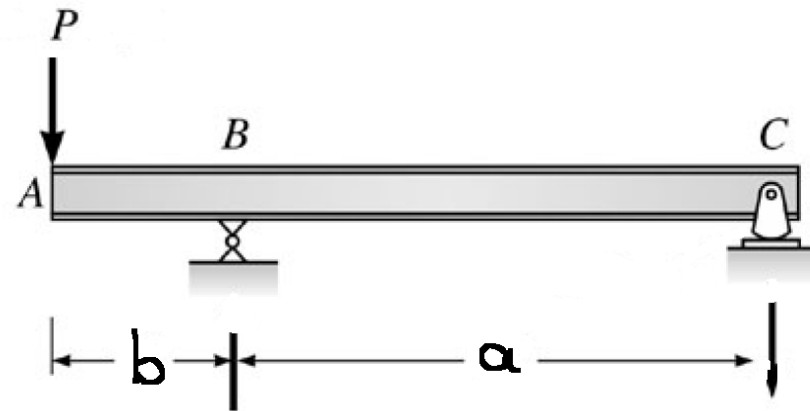
(d)

Example 2

Compute the maximum deflection and the slopes at the bearings. EI is constant.



(a)



(b)

Critical speed of rotating shaft

- Critical speed of a rotating shaft is the speed where it becomes dynamically unstable.
- The frequency of free vibration of a non-rotating shaft is same as its critical speed.

$$N_{critical} (RPM) = \frac{60}{2\pi} \sqrt{g \frac{(w_1 \delta_1 + w_2 \delta_2 + \dots + w_n \delta_n)}{(w_1 \delta_1^2 + w_2 \delta_2^2 + \dots + w_n \delta_n^2)}}$$

Critical speed of rotating shaft

- W_1, W_2, \dots : weights of the rotating bodies (N)
- $\delta_1, \delta_2, \dots$: deflections of the respective bodies (m)
- For a simply supported shaft, half of its weight may be lumped at the center for better accuracy.
- For a cantilever shaft, quarter of its weight may be lumped at the free end.

Shaft design: general considerations

- Axial thrust loads should be taken to ground through a single thrust bearing per load direction.
- Do not split axial loads between thrust bearings as thermal expansion of the shaft can overload the bearings.
- Shaft length should be kept as short as possible, to minimize both deflections and stresses.

Shaft design: general considerations

- A cantilever beam will have a larger deflection than a simply supported (straddle mounted) one for the same length, load, and cross section.
- Hollow shafts have better stiffness/mass ratio and higher natural frequencies than solid shafts, but will be more expensive and larger in diameter.

Shaft design: general considerations

- Slopes, lateral deflection of the shaft, and/or angle of twist of the shaft should be within some prescribed limits.
- First natural frequency of the shaft should be at least three times the highest forcing frequency. (A factor of ten times or more is preferred, but this is often difficult to achieve).

Example 3

Determine the diameter of a shaft of length $L = 1\text{m}$, carrying a load of 5 kN at the center if the maximum allowable shaft deflection is 1mm . What is the value of the slope at the bearings. Calculate the critical speed of this shaft if a disc weighting 45 kg is placed at the center. $E = 209\text{ GPa}$. $\rho_{\text{st}} = 8740\text{ kg/m}^3$.

Example 3: solution

Maximum deflection:

$$y = \frac{PL^3}{48EI}$$

$$1 = \frac{5000 \times (1000)^3}{48 \times (209 \times 10^3) \times \frac{\pi}{64} d^4}$$

$$\therefore d = 56.45 \text{ mm}$$

From a standard shaft size, $d = 58 \text{ mm}$

Example 3: solution

Slope at bearings:

$$\theta_1 = \theta_2 = \frac{PL^2}{16EI} = \frac{5000 \times (1000)^2}{16 \times (209 \times 10^3) \times \frac{\pi}{64} \times (58)^4}$$
$$= 0.0027 \text{ rad}$$

which is much for tapered and cylindrical roller bearings. However, this value may be acceptable for deep-groove ball bearing.

Example 3: solution

Critical speed:

Shaft weight = $\rho \times A \times L = 7840 \times A \times 1 = 20.7 \text{ kg}$

$$\delta = \frac{PL^3}{48EI} = \frac{(0.5 \times 20.7 + 45) \times 9.807 \times (1000)^3}{48 \times (209 \times 10^3) \times \frac{\pi}{64} \times (58)^4} = 0.097 \text{ mm}$$

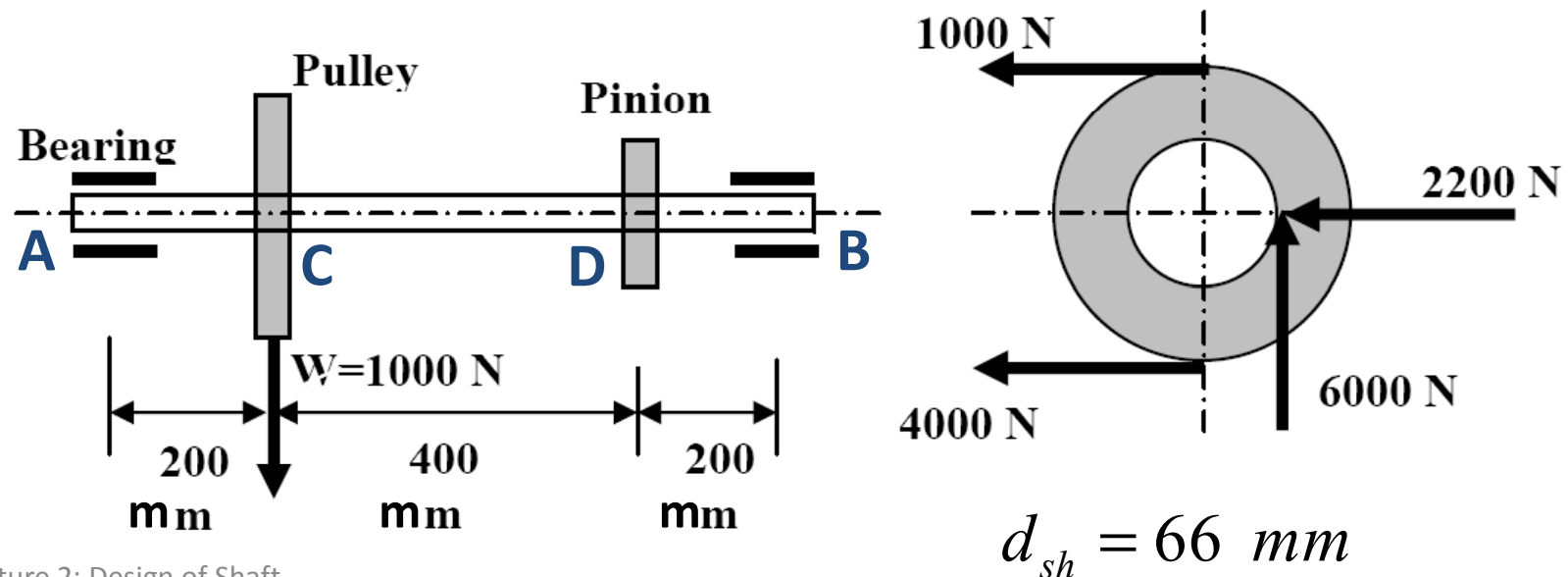
$$N_{critical} = \frac{60}{2\pi} \sqrt{\frac{9.807}{0.097 \times 10^{-3}}} = 3030 \text{ RPM}$$

The operating speed of the shaft should be well below this value (say less than 1000 rpm).

Example 4

For the same problem solved in the previous lesson, determine the following:

1. Lateral deflection at D (using Conj. Beam method)
2. Slope at bearings A and B (using Conj. Beam method)
3. Critical speed



Shaft design: summary

- Shaft design means material selection, geometric layout, stress and strength (static and fatigue), deflection and slope at bearings.
- Conjugate beam method for slope and deflection calculations.
- Some design considerations: (axial load, shaft length, support layout, hollow shafts, slopes and deflection, operating speed)

Report

- Write a short report on *material selection* for shaft manufacturing. Talk briefly about *heat treatment processes* that may be considered.
- Deadline: **Thursday, April 7, 2011**
- How to submit:
 - Send attachment to mosaadaly2000@yahoo.com
Please give a **title** to the email as your student number followed by your name, e.g.
999 Name Surname
 - Print out: not recommended

Slides and sheets

Available for download at:

<http://www.engr.uconn.edu/~aly/Courses/ME343/>