Example Test Two

NAME: _____________________________________

This is a “closed book/closed notes/closed calculators/phones/laptops/etc.” test.
Make sure to show your work and explain your answers.
No credit will be given for lucky guesses.

0. [1 pt] Do you understand the instructions above?

Yes    
No     

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Earned bonus points:  
GRAND TOTAL:  


1 Language Classes [25 pts]

(a) [4 pts] Define Context Free Languages (CFL) and use your definition to specify one such language.

(b) [10 pts] State the Pumping Lemma for CFLs. Prove that the following language is not a CFL:

\[ A = \{ w \mid w \in \{a,b,c\}^* \text{ and contains equal number of } a \text{'s, } b \text{'s, and } c \text{'s } \} \.

(c) [4 pts] Define Turing-Decidable Languages.

(d) [4 pts] Define Turing-Recognizable Languages.

(e) [3 pts] Define the Halting Problem as a language.
2 Languages & Automata [20 pts]

Complete and label the Venn diagram representing all of the following sets (add ovals as necessary; assume $P \neq NP$):

1. $CFL$: the set of all context-free languages
2. $NPDA$: the set of all languages recognized by Non-deterministic PDAs
3. $P$: the set of all languages decidable in polynomial time by Deterministic Turing Machines
4. $NP$: the set of all languages decidable in polynomial time by Non-deterministic Turing Machines
5. $DL$: The set of all Decidable Languages
6. $DTM$: The set of all languages that have Deterministic Turing Machine deciders
7. $TR$: The set of all Turing Recognizable languages
8. $NTM2$: The set of all languages that have Non-Deterministic 2-tape Turing Machine deciders
9. $E$: The set of all languages that have Enumerators
3 Turing Machines and Time Complexity [25 pts]

Let \( \langle V, E \rangle \) be some suitable representation of a graph, where \( V \) are the vertices and \( E \) are the edges. Let \( \text{REACH} \) be the language \( \text{REACH} = \{ \langle \langle V, E \rangle, s, t \rangle \mid \langle V, E \rangle \text{ is a graph where there is a path from vertex } s \text{ to vertex } t \} \).

Consider the following high-level description of a non-deterministic Turing Machine \( N \).

\[
N(\langle \langle V, E \rangle, s, t \rangle):
\]
1. Check the input graph for well-formedness and that \( s, t \in V \)
2. Non-deterministically choose an ordered set of vertices \( \{v_1, \ldots, v_k\} \) from \( V \)
3. If \( v_1 \neq s \) then \( \downarrow - \)
4. For \( j = 1 \) to \( k - 1 \) do
   \[
   \text{if } (v_j, v_{j+1}) \notin E \text{ then } \downarrow -
   \]
5. If \( v_k = t \) then \( \downarrow + \) else \( \downarrow - \)

(a) [5 pts] Is this Turing Machine a \textit{decider} for \( \text{REACH} \)? Explain your answer.

(b) [5 pts] Show that the time complexity (running time) of this Turing Machine is polynomial.

(c) [5 pts] Does it follow from (a), (b) that \( \text{REACH} \) is in \( P \)? \( NP \)? Both? Explain.

(d) [10 pts] Prove (without relying on your answers above) that \( \text{REACH} \) is in \( P \).
4 Turing Decidability and Recognizability [30 pts]

(a) [6 pts] Give two examples of Turing-decidable problems.
1.
2.

(b) [6 pts] Give two examples of Turing-recognizable, but undecidable problems
1.
2.

(c) [6 pts] Consider the language \( A = \{ \langle M, w, q \rangle \mid \text{Turing Machine } M \text{ on input } w \text{ enters state } q \} \)
Is \( A \) decidable [ ]? Recognizable [ ]? Both [ ]? Explain.

(d) [12 pts] If \( A \) above is decidable, describe a high-level algorithm for a decider.
If undecidable, prove that it is not decidable.