



# A novel tuning strategy for multivariable model predictive control

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## Abstract

Model predictive control (MPC) has established itself as the most popular form of advanced multivariable control in the chemical process industry. However, the benefits of this technology cannot be realized unless the controller can be operated with desirable performance for an extended period of time. The objective of this work is to present an easy-to-use and reliable tuning strategy that enables the control practitioner to maintain MPC at peak performance with minimal effort. A novel analytical expression that computes the move suppression coefficients, guidelines to select the additional adjustable parameters, and their demonstration in an overall tuning strategy are some of the significant contributions of this work. The compact form for the analytical expression that computes the move suppression coefficients is derived as a function of a first order plus dead time (FOPDT) model approximation of the process dynamics. With tuning parameters computed, MPC is then implemented in the classical fashion using an internal model formulated from step response coefficients of the actual process. Just as a FOPDT model approximation has proved a valuable tool in tuning rules such as Cohen–Coon, ITAE and IAE for PID implementations, the tuning strategy presented here is significant because it offers an analogous approach for multivariable MPC. © 1998 Published by Elsevier Science Ltd.

*Keywords:* MPC; DMC; GPC; Model predictive control; Dynamic matrix; Performance based tuning; Controller design; Move suppression coefficient

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## 1. Introduction

Over the past decade, model predictive control (MPC) has become the dominant form of advanced multivariable control in the process industry. Among the different MPC formulations available in the market today, Dynamic Matrix Control (DMC) [1,2] is probably the most popular and represents the chemical process industry's standard for MPC [3]. Because of DMC's immense popularity, this work focuses on developing an easy-to-use and reliable tuning strategy (Table 1) that will enable the control practitioner

to operate DMC at peak performance with minimal effort.

Tuning of unconstrained and constrained DMC for single-input single-output (SISO) and multivariable systems has been addressed by an array of researchers. In the past, systematic trial and error tuning procedures have been proposed [1,4]. Marchetti *et al.* [5] presented a detailed sensitivity analysis of adjustable parameters and their effects on DMC performance. The method of principal component selection was presented by Maurath *et al.* [6] as a method to compute an appropriate prediction horizon and a move suppression coefficient [7]. To simplify DMC tuning, Maurath *et al.* [8] also proposed the ' $M = 1$ ' controller configuration of DMC.

Other tuning strategies have concentrated on specific aspects such as tuning for stability [9–12],

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Table 1  
DMC tuning strategy

1. Approximate the process dynamics of all controller output-process variable pairs with first order plus dead time (FOPDT)

$$\text{models: } \frac{y_j(s)}{u_i(s)} = \frac{K_{ij} e^{-P_{ij}s}}{\tau_{ij}s + 1} \quad (i = 1, 2, \dots, S; j = 1, 2, \dots, R)$$

2. Select the sample time as close as possible to:  $T = \text{Min}[\text{Max}(0.1\tau_{ij}, 0.5\theta_{ij}); i = 1, 2, \dots, S; j = 1, 2, \dots, R]$

3. Compute the prediction horizon,  $P$ , and the model horizon,  $N$ , as the process settling time in samples (rounded to the next integer):  $P = N = \text{Max}(\frac{5\tau_{ij}}{T} + k_{ij})$  where  $k_{ij} = (\frac{\theta_{ij}}{T} + 1)$  ( $i = 1, 2, \dots, S; j = 1, 2, \dots, R$ )

4. Select the control horizon,  $M$ , as an integer (usually in the range 1 to 6).

5. Select the controlled variable weights,  $\gamma_j^2$ , to scale measurements to similar magnitudes.

6. Compute the move suppression coefficients,  $\lambda_i^2$ :  $\lambda_i^2 = \frac{M}{500} \sum_{j=1}^R \gamma_j^2$

$$[\gamma_i^2 K_{ij}^2 (P - k_{ij} - \frac{3}{2} \frac{\tau_{ij}}{T} + 2 - \frac{(M-1)}{2})] (i = 1, 2, \dots, S)$$

7. Implement DMC using the traditional step response matrix of the actual process and the parameters computed in steps 1 to 6: sample time,  $T$ ; control horizon (number of moves),  $M$ ; prediction horizon (optimization horizon),  $P$ ; controlled variable weights,  $\gamma_j^2$ ; model horizon (process settling time in samples),  $N$ ; move suppression coefficients,  $\lambda_i^2$ .

robustness [13,14] and performance [15–17]. Although some of the above methods provide a complete design for DMC, they also require fairly sophisticated analysis tools and an advanced knowledge of control concepts for their implementation. Hence, there still exists a need for easy-to-use tuning strategies for DMC.

## 2. Background

The mathematical framework of DMC and the plethora of tuning parameters it offers for adjusting closed loop performance have been discussed extensively by past researchers, e.g. Refs. [1,2,5,18,19]. Therefore, only a brief discussion of multivariable DMC is given below.

### 2.1. Multivariable dynamic matrix control

For a system with  $S$  controller outputs and  $R$  process variables, the multivariable DMC performance objective (cost function) has the form [20]:

$$\begin{aligned} \text{Min } J = & [\bar{e} - A \Delta \bar{u}]^T \Gamma^T \Gamma [\bar{e} - A \Delta \bar{u}] \\ & + [\Delta \bar{u}]^T \Lambda^T \Lambda [\Delta \bar{u}] \end{aligned} \quad (1)$$

The closed form solution to Eq. (1) gives the unconstrained multivariable DMC control law:

$$\Delta \bar{u} = (A^T \Gamma^T \Gamma A + \Lambda^T \Lambda)^{-1} A^T \Gamma^T \Gamma \bar{e} \quad (2)$$

Here,  $A$  is the multivariable dynamic matrix formed from the unit step response coefficients of each controller output-process variable pair.  $\bar{e}$  is the vector of predicted errors for the  $R$  process variables over the next  $P$  sampling instants (prediction horizon).  $\Delta \bar{u}$  is the vector of controller output changes for the  $S$  controller outputs computed for the next  $M$  sampling instants (control horizon).  $\Gamma^T \Gamma$  is the matrix of controlled variable weights and  $\Lambda^T \Lambda$  is the matrix of move suppression coefficients.

$\Lambda^T \Lambda$  is a square diagonal matrix of dimensions  $(M \cdot S \times M \cdot S)$ . The leading diagonal elements of the  $i$ th  $(M \times M)$  matrix block along the diagonal  $\Lambda^T \Lambda$  are  $\lambda_i^2$ . All off diagonal elements are zero. Hence, in the multivariable DMC control law (Eq. (2)), the move suppression coefficients that are added to the leading diagonal of the system matrix,  $(A^T \Gamma^T \Gamma A)$ , are  $\lambda_i^2$  ( $i = 1, 2, \dots, S$ ). Similarly, the  $(P \cdot R \times P \cdot R)$  matrix of controlled variable weights,  $\Gamma^T \Gamma$ , has the leading diagonal elements as  $\gamma_j^2$  ( $j = 1, 2, \dots, R$ ). Again, all off-diagonal elements are zero.

### 2.2. Adjustable parameters for tuning DMC

Over the past decade, detailed studies of the DMC parameters have provided a wealth of information about their qualitative effects on closed loop perfor-

mance, e.g. Refs. [5,8,20,21]. The adjustable parameters that affect closed loop performance of DMC include the prediction horizon,  $P$ , control horizon,  $M$ , model horizon,  $N$ , sample time,  $T$ , controlled variable weights,  $\gamma_i^2$ , and move suppression coefficients,  $\lambda_i^2$ .

Practical limitations often restrict the availability of sample time,  $T$ , as a tuning parameter, e.g. Refs. [22,23]. Model horizon,  $N$ , is also not an appropriate tuning parameter since truncation of the model horizon misrepresents the effect of past moves in the predicted output and leads to unpredictable closed loop performance [24].

The choice of prediction horizon,  $P$ , is dependent on the sample time,  $T$ . Past researchers [9,11,12,25] have shown that although a large  $P$  does not significantly improve performance, it does improve nominal stability of the closed loop. For this reason,  $P$  should be selected such that it includes the steady state effect of all past computed controller output moves, i.e. it should be fixed as the open loop settling time of the process. Hence,  $P$  should not be used as the primary DMC tuning parameter.

Increasing the control horizon,  $M$ , from 2 to 6 and beyond does not alter closed loop performance significantly, especially in the presence of move suppression. Also, Rawlings and Muske [12] have shown that a necessary condition to ensure nominal stability of infinite horizon MPC is that  $M$  should be greater than or equal to the number of unstable modes of the system. Therefore,  $M$  is also not well suited to be the primary DMC tuning parameter.

The controlled variable weights,  $\gamma_i^2$ , serve a dual purpose in multivariable DMC. These weights can be appropriately chosen by the user to scale measurements of the  $R$  process variables to comparable units. Also, it is possible to achieve tighter control of a particular process variable by selectively increasing the relative weight of the corresponding least square residual. Hence, controlled variable weights are usually specified by the user for a certain application and should not be employed as the primary tuning parameters for multivariable DMC.

For a control horizon ( $M$ ) of 1, the setpoint step response is sluggish and move suppression coefficients,  $\lambda_i^2$ , greater than 0 will only further slow the process response. With  $M > 1$ , the lack of move suppression results in aggressive control effort and a

significantly underdamped process variable response. An intermediate response can be achieved by an appropriate choice of  $\lambda_i^2$ . However, further increasing  $\lambda_i^2$  can lead to an undesirable sluggish response for most applications.

Consequently,  $\lambda_i^2$  are continuous parameters that have a significant impact on closed loop performance. Furthermore, their choice is critical to the performance achieved by DMC. Therefore, the candidate best suited as the primary multivariable DMC tuning parameters are the move suppression coefficients,  $\lambda_i^2$ .

### 2.3. The multivariable DMC tuning strategy

Past researchers, e.g. Ref. [26], have indicated that the move suppression coefficients,  $\lambda_i^2$ , serve a dual purpose in the multivariable DMC control law. Their primary role is to suppress otherwise aggressive controller action when  $M > 1$ . Additionally, the  $\lambda_i^2$  serve to improve the conditioning of the system matrix,  $(A^T \Gamma^T \Gamma A)$ , by rendering it more positive definite.

The premise underlying this work is that both these effects are interrelated. When  $\lambda_i^2$  is increased, the corresponding controller output move sizes decrease, as does the condition number of the  $i$ th diagonal block of the system matrix,  $(A^T \Gamma^T \Gamma A)$ . If the effect of  $\lambda_i^2$  on the condition number of the  $i$ th diagonal block of the  $(A^T \Gamma^T \Gamma A)$  matrix can be expressed analytically, then this condition number can be maintained within specified bounds by an appropriate choice of  $\lambda_i^2$ . An upper bound on the condition number of the  $i$ th diagonal block would, in turn, prevent the  $i$ th controller output move sizes from becoming excessive.

### 2.4. Derivation of analytical expressions for $\lambda_i^2$

The key to exploiting this strong correlation between condition numbers of the  $i$ th diagonal block of the system matrix and the resultant move sizes of the  $i$ th controller output, is to obtain an analytical expression for the condition number of the  $i$ th diagonal block in terms of all the adjustable DMC parameters and the process model. Derivation of such an analyti-

cal expression for  $\lambda_i^2$  is made possible by using a first order plus dead time (FOPDT) approximation of the process dynamics.

It must be emphasized that the use of this FOPDT approximation is employed only in the derivation of the analytical expression for  $\lambda_i^2$ . Both examples presented later in this work use the traditional DMC step response matrix of the actual process upon implementation.

The primary benefit of a FOPDT model approximation is that it permits derivation of a compact analytical expression for computing  $\lambda_i^2$ . Although a FOPDT model approximation does not capture all the features of some higher order processes, it often reasonably describes the process gain, overall time constant and effective dead time of such processes [27].

The procedure followed in deriving analytical expressions for  $\lambda_i^2$  is outlined below. An approximation of the multivariable DMC system matrix,  $(A^T \Gamma^T \Gamma A)$ , is obtained using a FOPDT model approximation of the process. This  $(M \cdot S \times M \cdot S)$  matrix is comprised of  $S^2$  matrix blocks, each of dimensions  $(M \times M)$ . Interestingly, all the diagonal blocks are Hankel matrices with the additional feature that the elements of each row decrease from left to right by a constant quantity. Hence, the  $i$ th diagonal block in  $(A^T \Gamma^T \Gamma A)$  has the form:

$i$ th Diagonal Block in  $A^T \Gamma^T \Gamma A$

$$= \begin{bmatrix} \beta_i & \beta_i - \alpha_i & \beta_i - 2\alpha_i & \dots \\ \beta_i - \alpha_i & \beta_i - 2\alpha_i & \beta_i - 3\alpha_i & \dots \\ \beta_i - 2\alpha_i & \beta_i - 3\alpha_i & \beta_i - 4\alpha_i & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}_{M \times M} \quad (3)$$

$$\beta_i = \sum_{j=1}^R \gamma_j^2 K_{ij} \left( P - k_{ij} - \frac{3}{2} \frac{\tau_{ij}}{T} + 2 \right) \quad (4)$$

$$\alpha_i = \frac{\sum_{j=1}^R \gamma_j^2 K_{ij}}{2} \quad (5)$$

QR factorization of the  $i$ th diagonal block matrix (Eq. (3)) provides formulae for the maximum and

minimum eigenvalues. An expression for the condition number is then obtained as the ratio of maximum to minimum eigenvalues. Finally, by factoring out  $\lambda_i^2$ , an analytical expression that computes the move suppression coefficient is obtained as:

$$\lambda_i^2 = \frac{M}{c_i} \sum_{j=1}^R \left[ \gamma_j^2 K_{ij}^2 \left( P - k_{ij} - \frac{3}{2} \frac{\tau_{ij}}{T} + 2 - \frac{(M-1)}{2} \right) \right] \quad (6)$$

where  $c_i$  is the desired condition number of the  $i$ th diagonal block.

The choice of a condition number  $c_i$ , and hence, the upper allowable limit of ill-conditioning in the  $i$ th diagonal block of the system matrix, lies with the individual designer. The choice of a condition number of 500 in Table 1 was motivated by the rule of thumb that controller output move sizes for a change in set point should not exceed 2–3 times the steady-state change in that controller output [6,7]. However, if a faster or slower closed-loop response is more desirable, a larger or smaller condition number, respectively, can be used instead.

The analytical expression in Eq. (6), with guidelines for selection of the other adjustable parameters, provides the overall tuning strategy for multivariable DMC (Table 1).

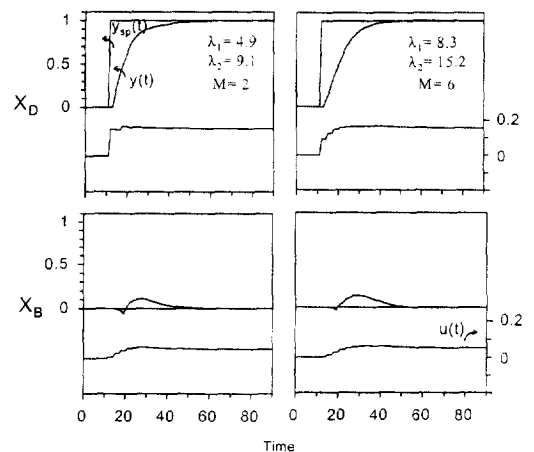


Fig. 1. Effectiveness of the DMC tuning strategy demonstrated for the 2x2 Wood and Berry column for different choices of the control horizon ( $T = 3$ ,  $T/\tau^p = 0.1$ ,  $P = N = 35$ ).

### 2.5. Implementation of the DMC tuning strategy

The proposed DMC tuning strategy, which includes the analytical expression for the move suppression coefficient,  $\lambda_i^2$ , is presented in Table 1. This tuning strategy can be applied to unconstrained multivariable DMC in closed loop with a broad class of MIMO processes that are open loop stable, including those with challenging control characteristics such as high process order, large dead time and nonminimum phase behavior.

Step 1 (Table 1) involves the identification of a first order plus dead time (FOPDT) model approximation of the process. A reasonable identification of the FOPDT model parameters ( $K_{ij}$ ,  $\tau_{ij}$  and  $\theta_{ij}$ ) is essential to the success of this tuning strategy.

Step 2 involves the selection of an appropriate sample time,  $T$ . The estimated FOPDT model parameters provides a convenient way to select  $T$ . If the designer does not have complete freedom to select a sample time exactly equal to the value computed in Table 1, then the sample time should be picked as close as possible to this recommended value.

Step 3 computes a model horizon,  $N$ , and a prediction horizon,  $P$ , from  $\tau_{ij}$ ,  $\theta_{ij}$  and  $T$ . Note that both  $N$  and  $P$  cannot be selected independent of the sample time,  $T$ . Also, it is imperative that  $N$  be equal to the open loop process settling time in samples to avoid truncation error in the model prediction [24]. In an industrial setting, the multivariable DMC architecture sometimes requires that a single  $P$  and a single  $N$  be selected for all controller output and process variable pairs.

Step 4 requires the specification of a control horizon,  $M$ . Recommended values of  $M$  range from 1 through 6. Selecting  $M > 1$  can be very useful to the practitioner since this provides advance knowledge of the impending controller output moves [26].

Step 5 requires the selection of controlled variable weights,  $\gamma_i^2$ , to achieve tighter control of a particular process variable and to scale measurements of the  $R$  process variables to similar magnitudes.

Step 6 involves computation of appropriate move suppression coefficients,  $\lambda_i^2$ . With  $M = 1$ , the need for a move suppression is obviated and  $\lambda_i^2$  is set equal to zero. However, if  $M > 1$ , the move suppression coefficients are computed from the analytical expression derived in Eq. (6).

Step 7 summarizes the DMC tuning parameters. For certain applications, more specific or stringent performance criteria regarding the controller output move sizes or the nature of the process variable response may apply. For such cases, it may be necessary to fine tune DMC for desired performance by altering  $\lambda_i^2$  from the starting values given by the tuning strategy. The recommended approach is to increase  $\lambda_i^2$  for smaller corresponding controller output move sizes and slower process variable response.

### 3. Validation of the DMC tuning strategy

Two example processes are considered for validation of the proposed tuning strategy. These simulation examples assume a negligible plant-model mismatch. In the past, several researchers have investigated the effects of plant-model mismatch on controller performance [13,14,24,28–30]. Hence, this work focuses strictly on the capabilities of the tuning strategy in providing desirable closed loop performance.

The first example applies the tuning strategy to multivariable DMC control of a  $2 \times 2$  distillation column. This methanol–water column model [31] has been used for several controller studies, including the application of model predictive controllers, e.g. Refs [11,12,21], and can be written as:

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R_F(s) \\ Q_V(s) \end{bmatrix} \quad (7)$$

Here,  $X_D$  represents the mole fraction of methanol in the distillate,  $X_B$  the mole fraction of methanol in the bottoms,  $R_F$  the reflux flow and  $Q_V$  the vapor boil-up rate. All the times are given in minutes.

A sample time,  $T$ , of 3 minutes was used. Equal controlled variable weights were employed for both process variables, i.e.  $\Gamma^T \Gamma = 1$ . The move suppression coefficients,  $\Lambda^T \Lambda$  (or  $\lambda_i^2$ ), were computed from the analytical expressions in Eq. (7) for two choices of the control horizon,  $M = 2$  and  $M = 6$ .

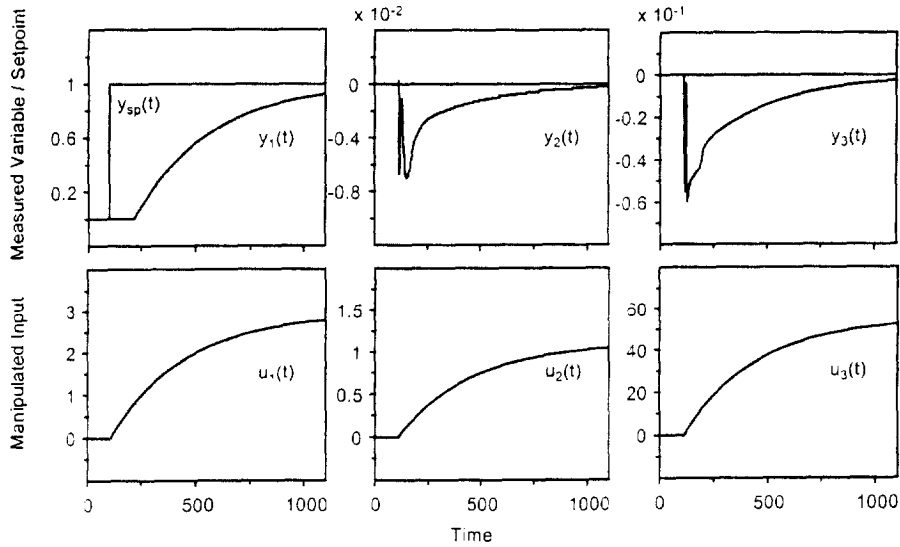


Fig. 2. Effectiveness of the DMC tuning strategy demonstrated for a  $3 \times 3$  pilot plant distillation column by Ogunnaike *et al.* [32] ( $T = 1$ ,  $P = N = 64$ ,  $M = 6$ ,  $\lambda_1 = 23.7$ ,  $\lambda_2 = 30.9$ ,  $\lambda_3 = 0.65$ ).

Figure 1 illustrates the response of the two process variables and the corresponding controller output moves for a step change in the  $X_D$  setpoint. The results in Fig. 1 show that for a user specified control horizon of  $M = 2$  or  $M = 6$ , the analytical expression for multivariable DMC computes move suppression coefficients that result in desirable closed loop performance. In either case, the output response tracks the setpoint rapidly without exhibiting over-

shoot and the corresponding controller output moves are modest in size. Despite the interactions inherent in the moderately ill-conditioned Wood and Berry column, the effect of interactions on the process variable,  $X_B$ , are dampened effectively.

The second example used for validation of the multivariable DMC tuning strategy is a  $3 \times 3$  ethanol–water distillation column. A transfer function model for this pilot plant scale column is [32]:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.012e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (8)$$

Here,  $y_1$  is the overhead ethanol mole fraction,  $y_2$  is the side stream ethanol mole fraction,  $y_3$  is the temperature on tray 19,  $u_1$  is the overhead reflux flow rate,  $u_2$  is the side stream draw-off rate and  $u_3$  is the reboiler steam pressure.

A sample time,  $T$ , of 1 minute was used. Again, equal controlled variable weights were employed. The move suppression coefficients,  $\Lambda^T \Lambda$ , were computed from the analytical expression in Eq. (7) for a control horizon,  $M$ , equal to 6. To implement the

multivariable tuning strategy, an FOPDT model fit of the higher order  $y_3 - u_3$  process transfer function was obtained with parameters  $K_{33} = 0.86$ ,  $\tau_{33} = 9.89$ ,  $\theta_{33} = 0.0$ .

Figure 2 shows the response of the three process variables and controller outputs for a setpoint step in  $y_1$ . Although the model in Eq. (8) indicates significant steady-state interactions inherent in the column dynamics, the proposed multivariable tuning strategy achieves good closed loop performance. This demonstration confirms that the tuning strategy is capable of tuning challenging applications of multivariable DMC.

#### 4. Conclusions

A tuning strategy for multivariable DMC, with a novel expression that computes the move suppression coefficients,  $\lambda_i^2$ , was presented. The application of this easy-to-use and reliable tuning strategy was demonstrated for both SISO and MIMO processes. Though not demonstrated here, the tuning strategy is also directly applicable to parametric controllers like Generalized Predictive Control (GPC) [33]. This is because GPC reduces to the DMC algorithm for a specific choice of tuning parameters. This tuning strategy can also be directly applied to constrained MPC and to systems that are non-square.

#### References

- [1] Cutler, C. R., Dynamic Matrix Control, an optimal multivariable control algorithm with constraints. PhD Dissertation, University of Houston, TX, 1983.
- [2] Cutler, C. R. and Ramaker, D. L., Dynamic Matrix Control — A computer control algorithm. *Proc. JACC*, San Francisco, CA, 1980.
- [3] Qin, J. S. and Badgwell, T. A., An overview of industrial model predictive control technology, Preprints. *Proceedings of the Chemical Process Control Conference, CPCV*, Tahoe City, CA, 1996.
- [4] Ricker, N. L., Model Predictive Control: State of the Art. In *Proceedings of the Fourth International Conference on Chemical Process Control*, eds Y. Arkun and W. H. Ray. Elsevier, Amsterdam, 1991.
- [5] Marchetti, J. L., Mellichamp, D. A. and Seborg, D. E., Predictive control based on discrete convolution models. *Ind. Eng. Chem. Process Des. Dev.*, 1983, **22**, 488.
- [6] Maurath, P. R., Laub, A. J., Mellichamp, D. A. and Seborg, D. E., Predictive controller design by principal components analysis. *Ind. Eng. Chem. Res.*, 1988b, **27**, 1204.
- [7] Callaghan, P. J. and Lee, P. L., An experimental investigation of predictive controller design by principal component analysis. *Chem. Eng. Res. Des.*, 1988, **66**, 345.
- [8] Maurath, P. R., Mellichamp, D. A. and Seborg, D. E., Predictive controller design for single input/single output systems. *Ind. Eng. Chem. Res.*, 1988a, **17**, 956.
- [9] García, C. E. and Morari, M., Internal model control: I. A unifying review and some new results. *Ind. Eng. Chem. Process Des. Dev.*, 1982, **21**, 308.
- [10] McIntosh, A. R., Performance and tuning of adaptive generalized predictive control. MSc Thesis, University of Alberta, Canada, 1988.
- [11] Clarke, D. W. and Scattolini, R., Constrained receding horizon predictive control. *Proceedings of the IEE*, 1991, **138**(4), 347.
- [12] Rawlings, J. and Muske, K., The stability of constrained receding horizon control. *IEEE Transactions on Automatic Control*, 1993, **38**, 1512.
- [13] Ohshima, M., Hashimoto, I., Takamatsu, T. and Ohno, H., Robust stability of Model Predictive Control (MPC). *Int. Chem. Eng.*, 1991, **31**(1), 119.
- [14] Lee, J. H. and Yu, Z. H., Tuning of model predictive controllers for robust performance. *Comp. Chem. Eng.*, 1994, **18**(1), 15.
- [15] McIntosh, A. R., Fisher, D. G. and Shah, S. L., Performance adaptive control: General structure and a case study. *J. Proc. Cont.*, 1992, **2**(4), 213.
- [16] Hinde, R. F. and Cooper, D. J., Using pattern recognition in controller adaptation and performance evaluation. *Proceedings of the American Control Conference*, IEEE Publications, 1993, p. 74.
- [17] Hinde, R. F. and Cooper, D. J., A unified excitation and performance diagnostic adaptive control framework. *AIChE J.*, 1995, **41**, 110.
- [18] Shridhar, R. and Cooper, D. J., A tuning strategy for unconstrained SISO model predictive control. *Ind. Eng. Chem. Res.*, 1997, **36**, 729.
- [19] Shridhar, R. and Cooper, D. J., Selection of move suppression coefficients in tuning dynamic matrix control, 1997, in press.
- [20] Garcia, C. E. and Morshedi, A. M., Quadratic programming solution of dynamic matrix control (QDMC). *Chem. Eng. Commun.*, 1986, **46**, 73.
- [21] Downs, J. J., Vogel, E. F. and Williams, V. J., Tuning and sensitivity study for model predictive control. *Am. Cont. Conf.*, 1988, **3**, 2357.
- [22] Franklin, G. F. and Powell, J. D., *Digital Control of Dynamic Systems*. Addison-Wesley Publishing, MA, 1980.
- [23] Åström, K. J. and Wittenmark, B., In *Computer Controlled Systems, Theory and Design*, ed. T. Kailath. Prentice-Hall Information and System Science Series, 1984.
- [24] Lundström, P., Lee, J. H., Morari, M. and Skogestad, S., Limitations of dynamic matrix control. *Comp. Chem. Eng.*, 1995, **19**(4), 409.
- [25] Scoekaert, P. and Clarke, D. W., *Advances in Model Based*

- Predictive Control, Chapter: Stability and Feasibility in Constrained Predictive Control*. Oxford University Press, Oxford, 1994.
- [26] Ogunnaike, B. A., Dynamic matrix control: A nonstochastic, industrial process control technique with parallels in applied statistics. *Ind. Eng. Chem. Fundam.*, 1986, **25**, 712.
- [27] Cohen, G. H. and Coon, G. A., Theoretical considerations of retarded control. *Transactions of the ASME*, 1953, **75**, 827.
- [28] Morari, M. and Zafiriou, E., *Robust Process Control*. Prentice-Hall, Englewood Cliffs, NJ, 1989.
- [29] Zafiriou, E., The robustness of model predictive controllers. *Proceedings of CPC IV* San Padre Island, TX, 1991.
- [30] Gupta, Y. P., Characteristic equations and robust stability of a simplified predictive control algorithm. *Can. J. Chem. Eng.*, 1993, **71**, 617.
- [31] Wood, R. K. and Berry, M. W., Terminal composition control of a binary distillation column. *Chem. Eng. Sci.*, 1973, **28**, 1707.
- [32] Ogunnaike, B. A., Lemaire, J. B., Morari, M. and Ray, W. H., Advanced multivariable control of a pilot plant distillation column. *AIChE J.*, 1983, **29**, 632.
- [33] Clarke, D. W., Mohtadi, C. and Tuffs, P. S., Generalized predictive control: I. The basic algorithm. *Automatica*, 1987a, **23**, 137; Generalized predictive control: II. Extensions and interpretations. *Automatica*, 1987b, **23**, 149.