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A practical multiple model adaptive strategy for multivariable model predictive control

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Abstract

Model predictive control (MPC) has become the leading form of advanced multivariable control in the chemical process industry. The objective of this work is to introduce a multiple model adaptive control strategy for multivariable dynamic matrix control (DMC). The novelty of the strategy lies in several subtle but significant details. One contribution is that the method combines the output of multiple linear DMC controllers, each with their own step response model describing process dynamics at a specific level of operation. The final output forwarded to the controller is an interpolation of the individual controller outputs weighted based on the current value of the measured process variable. Another contribution is that the approach does not introduce additional computational complexity, but rather, relies on traditional DMC design methods. This makes it readily available to the industrial practitioner.

Keywords: Model predictive control; Dynamic matrix control; Adaptive control; Multiple models; Industrial control

1. Introduction

Model predictive control (MPC) has established itself in industry as an important form of advanced control (Townsend & Irwin, 2001). Since the advent of MPC, various model predictive controllers have evolved to address an array of control issues (García, Prett, & Morari, 1989; Froisy, 1994). Dynamic matrix control (DMC) (Cutler & Ramaker, 1980) is the most popular MPC algorithm used in the chemical process industry today (Qin & Badgwell, 1996; Townsend, Lightbody, Brown, & Irwin, 1998). A major part of DMCs appeal in industry stems from the use of a linear finite step response model of the process and a quadratic performance objective function. The objective function is minimized over a prediction horizon to compute the optimal controller output moves as a least squares problem.

When DMC is employed on nonlinear chemical processes, the application of this linear model based controller is limited to relatively small operating regions. Specifically, if the computations are based entirely on the model prediction (i.e. no constraints are active), the accuracy of the model has significant effect on the performance of the closed loop system (Gopinath, Bequette, Roy, Kaufman, & Yu, 1995). Hence, the capabilities of DMC will degrade as the operating level moves away from the original design level of operation.

To maintain the performance of the controller over a wide range of operating levels, a multiple model adaptive control (MMAC) strategy for DMC has been developed. While MMAC will not capture severe nonlinear dynamic behavior, it will provide significant benefits over linear controllers. The work focuses on a MMAC strategy for processes that are stationary in time, but nonlinear with respect to the operating level. This method is not applicable to processes where the gain of the process changes sign.

The method of approach is to construct a small set of DMC process models that span the range of expected operation. By combining the process models to form a nonlinear approximation of the plant, the true plant behavior can be reasonably achieved (Banerjee, Arkun, Ogunnaike, & Pearson, 1997). If linear process models and controllers are employed, the wealth of design and tuning strategies for the linear controllers can be used.

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Nomenclature

$a_{rs,i}$	ith unit step response coefficient for controller output s on process variable r
$\boldsymbol{A}$	dynamic matrix
d	prediction error
$\bar{\boldsymbol{e}}$	vector of predicted errors
i	index
k	discrete dead time
K	process gain
l	level of operation index
M	control horizon (number of controller output moves)
N	model horizon (process settling time in samples)
P	prediction horizon
R	number of measured process variables
S	number of controller outputs
T	sample time
u	controller output variable
$x_{l,s}$	weighting factor
$\hat{y}_r$	predicted process variable for process variable r
$y_{r,l}$	value of process variable r at level l
$y_{meas,r}$	current measurement of process variable r

Greek symbols

$\Delta u_{adap,s}$	adapted controller output move for controller output s
$\Delta\bar{\boldsymbol{u}}$	vector of controller output moves to be determined
γ_r^2	controlled variable weight (equal concern factor) in MIMO DMC
$\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}$	matrix of move suppression coefficients
λ_s^2	move suppression coefficients (controller output weight) in MIMO DMC
$\boldsymbol{\Gamma}^T\boldsymbol{\Gamma}$	matrix of controlled variable weights
θ	effective dead time of process
τ	overall process time constant
τ_{p_1}	1st process time constant
τ_{p2}	2nd process time constant

Abbreviations

DMC	Dynamic matrix control
FOPDT	First order plus dead time
MIMO	Multiple-input multiple-output
MMAC	Multiple model adaptive control
MPC	Model predictive control
PID	Proportional integral derivative
POR	Peak overshoot ratio

This is a benefit to the control practitioner since they do not have complete knowledge of the nonlinear control strategies currently available in the literature (Schott & Bequette, 1991; Townsend et al., 1998).

The accuracy of the nonlinear approximation can be increased by combining more models. However, this is expensive because each model requires the collection of plant data at a different level of operation. The number of DMC process models ultimately employed is a practical determination made by the control practitioner on a case-by-case basis. In most cases, the practitioner will balance the expense of collecting data with the need to improve the nonlinear approximation.

The novelty of this work lies in the specific details of the strategy. The technique involves designing and combining multiple linear DMC controllers. Each controller has their own step response model that describes the process dynamics at a specific level of operation. The final controller outputs forwarded to the controllers are obtained by interpolating between the individual controller outputs based on the values of the measured process variables. The tuning parameters for each controller are obtained by using already published tuning guidelines. The result is a simple and easy to use method for adapting the control performance without increasing the computational complexity of the control algorithm.

2. Background

In the past, adding an adaptive mechanism to MPC has been approached in a number of ways. Researchers have primarily focused on updating the internal process model used in the control algorithm. Several articles review various adaptive control mechanisms for controlling nonlinear processes (Seborg, Edgar, & Shah, 1986; Bequette, 1991; Di Marco, Semino, & Brambilla, 1997). In addition, Qin and Badgwell (2000) provide a good overview of nonlinear MPC applications that are currently available in industry. As illustrated by these works, the adaptive control mechanisms consider the use of a nonlinear analytical model, combinations of linear empirical models or some mixture of both.

A popular approach for adaptive MPC is to linearize the nonlinear analytical model at each sampling instance (García, 1984; Krishnan & Kosanovich, 1998; Gattu & Zafiriou, 1992, 1995; Lee & Ricker, 1994; Gopinath et al., 1995; Peterson, Hernández, Arkun, & Schork, 1992). Others have used the nonlinear analytical model to obtain linear state space models at different operating levels. These models are then weighted using a Bayesian estimator at each sampling instance to obtain an adapted internal process model (Lakshmanan & Arkun, 1999; Bodizs, Szeifert, & Chovan, 1999). Analytical models are difficult to obtain due to the underlying physics and chemistry of the process, and they are often too complex to employ directly in the optimization calculation (Morari & Lee, 1999).

Simple nonlinear output transformations have been applied to the nonlinear analytical equations in order to linearize the process model (Georgiou, Georgakis,

& Luyben, 1988). While this method improves the performance of DMC for nonlinear processes, output transformations can be challenging to design for some applications. The nonlinear analytical model can be used directly in the control algorithm by modifying the performance objective functions or process constraints (Ganguly & Saraf, 1993; Sistu, Gopinath, & Bequette, 1993; Katende, Jutan, & Corless, 1998; Xie, Zhou, Jin, & Xu, 2000), or used in combination with empirical models to form a model reference adaptive controller (e.g., Gundala, Hoo, & Piovoso, 2000).

Recursive formulations update the parameters of the process model as new plant measurements become available at each sampling instance (McIntosh, Shah, & Fisher, 1991; Maiti, Kapoor, & Saraf, 1994, 1995; Ozkan & Camurdan, 1998; Liu & Daley, 1999; Yoon, Yang, Lee, & Kwon, 1999; Zou & Gupta, 1999; Chikkula & Lee, 2000). Recursive estimation schemes have well-known problems including: convergence problems if the data does not contain sufficient and persistent excitation, inaccurate model parameters if unmeasured disturbances or noise influence the measurements, and sensitivity to process dead times and high noise levels.

A more practical adaptive strategy uses a gain and time constant schedule for updating the process model (McDonald & McAvoy, 1987; Chow, Kuznetsoc, & Clarke, 1998). An extension of this method is to use multiple models to update the process model. Linear models that described the system at various operating points are developed based on plant measurements. Past researchers (e.g., Banerjee et al., 1997) have illustrated that linear models can be combined in order to obtain an approximation of the process that approaches its true behavior. Two different multiple model controller design methods can be employed to maintain the performance of the controller over all operating levels.

In one case, a controller is designed for each level of operation. In the past, this methodology has been applied to generalized predictive control and proportional-integral-derivative controllers. The controller moves are then weighted based on the prediction error calculated for each controller. The resulting weights are obtained using recursive identification such that the prediction error is minimized (Yu, Roy, Kaufman, & Bequette, 1992; Schott & Bequette, 1994).

Although the concept used in this paper is similar to those listed above, there are some important differences. One of the differences of this approach is that the strategy is applied directly to the DMC algorithm. The method is to design and combine multiple linear DMC controllers, each with their own step response model. Another contribution is that the proposed methodology does not introduce additional computation complexity.

For the other case, a single controller is used. Although this concept is not used in the proposed strategy, the method is related. Gendron et al. (1993) developed a multiple model pole placement controller. The process models were weighted based on the current process variable measurement, and then the weighted model is used in a single controller. Rao, Aufderheide, and Bequette (1999) and Townsend and Irwin (2001) designed a multiple model adaptive model predictive controller. The processes models were combined based on the prediction error and then the weighted model was again sent to a single controller.

Townsend et al. (1998) developed a nonlinear DMC controller that replaces the linear process model with a local model network. This local model network contains local linear ARX models and is trained using a hybrid learning technique. From this local model network, the DMC controller is supplied with a weighted step response model.

Narendra and Xiang (2000) design multiple controllers using both fixed and adaptive process models. Based on the prediction error for each of these process models, a procedure is designed that switches between the controllers corresponding to the process model with the lowest prediction error. This allows the controller to incorporate both time-invariant dynamics along with time-varying dynamics.

3. Multivariable DMC

Multivariable DMC has been discussed extensively by past researchers (Cutler & Ramaker, 1980; Marchetti, Mellichamp, & Seborg, 1983) and is summarized here for the convenience of the reader. For a system with S controller outputs and R measured process variables, the multivariable DMC quadratic performance objective function has the form (García & Morshedi, 1986)

$$\operatorname*{Min}_{\Delta\bar{\boldsymbol{u}}} J = [\bar{\boldsymbol{e}} - \boldsymbol{A}\Delta\bar{\boldsymbol{u}}]^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}[\bar{\boldsymbol{e}} - \boldsymbol{A}\Delta\bar{\boldsymbol{u}}] + [\Delta\bar{\boldsymbol{u}}]^{\mathrm{T}}\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}[\Delta\bar{\boldsymbol{u}}], \quad (1)$$

subject to

$$\begin{aligned} &\hat{y}_{r,\min} \leqslant \hat{y}_r \leqslant \hat{y}_{r,\max}, \\ &\Delta\bar{\boldsymbol{u}}_{s,\min} \leqslant \Delta\bar{\boldsymbol{u}}_s \leqslant \Delta\bar{\boldsymbol{u}}_{s,\max}, \\ &\bar{\boldsymbol{u}}_{s,\min} \leqslant \bar{\boldsymbol{u}}_s \leqslant \bar{\boldsymbol{u}}_{s,\max}. \end{aligned} \quad (2)$$

A closed form solution to the multivariable DMC performance objective (Eq. (1)) results in the unconstrained multivariable DMC control law (García & Morshedi, 1986):

$$\Delta\bar{\boldsymbol{u}} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{A} + \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}\bar{\boldsymbol{e}}. \quad (3)$$

Here, $\boldsymbol{A}$ is the multivariable dynamic matrix formed from unit step response coefficients of each controller output to measured process variable pair; $\bar{\boldsymbol{e}}$ is the vector of predicted errors for the R measured process variables over the next P sampling instants (prediction horizon); $\Delta\bar{\boldsymbol{u}}$ is the vector of controller output changes for the S controller output computed for the next M sampling instants (control horizon); $\hat{y}_r$ is the predicted process variable profile for the rth measured process variable over the next P sampling instances; $\boldsymbol{\Gamma}^T\boldsymbol{\Gamma}$ is the matrix of controlled variable weights and $\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}$ is the matrix of move suppression coefficients.

$\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}$ is a square diagonal matrix of dimensions $(MS \times MS)$. The leading diagonal elements of the ith $(M \times M)$ matrix block along the diagonal of $\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}$ are λ_i^2. All off-diagonal elements are zero. Hence, in the multivariable DMC control law (Eq. (3)), the move suppression coefficients that are added to the leading diagonal of the system matrix, $(\boldsymbol{A}^T\boldsymbol{\Gamma}^T\boldsymbol{\Gamma}\boldsymbol{A})$, are λ_i^2 $(i = 1, 2, \ldots, S)$. Similarly, the $(PR \times PR)$ matrix of controlled variable weights, $\boldsymbol{\Gamma}^T\boldsymbol{\Gamma}$, has the leading diagonal elements as γ_i^2 $(i = 1, 2, \ldots, R)$. Again, all off-diagonal elements are zero.

The implementation of DMC involves using step response coefficients to predict the future process variable behavior, $\hat{y}_r(k+1)$, over the prediction horizon. This profile is corrected by adding to it estimates of the disturbance. The disturbance estimates are calculated as the difference between the current measurement of the process variable and the current value of the predicted process variable at the present sample time. The disturbance estimate is assumed constant over the prediction horizon. Then, Eq. (1) or (3) is solved on-line to determine the optimal values of the controller output moves. Only the first element of the vector is implemented and the entire procedure is repeated at the next sampling instance.

4. Formulation of an MMAC strategy for DMC

The method of approach in this work focuses on weighting a minimum of three linear DMC controllers based on the current measurement of the process variable. Three linear controllers are used here because as mentioned previously, collecting plant data is difficult and time consuming. The method can easily be expanded to more models if desired by the practitioner.

4.1. Non-adaptive DMC Implementation

For comparison, a non-adaptive DMC controller is designed and present along side the adaptive method using the tuning rules given by Shridhar and Cooper (1997, 1998). Table 1 displays the guidelines for determining the tuning parameters for non-adaptive DMC. The tuning parameters and the step response coefficients are calculated offline prior to the start-up of the non-adaptive DMC controller and remain constant during operation.

As presented in Table 1, step 1 of these rules is based on fitting the controller output to measured process variable dynamics for each sub-process relating the sth controller output to the rth process variable at the design level of operation with a first-order plus dead time (FOPDT) model approximation. Although an FOPDT model approximation does not capture all the features of higher order processes, it often reasonably

Table 1
Non-adaptive DMC tuning strategy

1. Approximate the process dynamics of all controller output to measured process variable pairs with FOPDT models:

$$\frac{y_r(s)}{u_s(s)} = \frac{K_{rs}\mathrm{e}^{-\theta_{rs}s}}{\tau_{rs}s+1} \quad (r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S)$$

2. Select the sample time as close as possible to:

$$\begin{aligned} T_{rs} &= \mathrm{Max}(0.1\tau_{rs}, 0.5\theta_{rs}), \\ & \qquad\qquad (r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S) \\ T &= \mathrm{Min}(T_{rs}), \end{aligned}$$

3. Compute the prediction horizon, P, and the model horizon, N:

$$P = N = \mathrm{Max}\left(\frac{5\tau_{rs}}{T} + k_{rs}\right) \quad \text{where } k_{rs} = \left(\frac{\theta_{rs}}{T} + 1\right) \quad (r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S)$$

4. Compute a control horizon, M:

$$M = \mathrm{Max}\left(\frac{\tau_{rs}}{T} + k_{rs}\right) \quad (r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S)$$

5. Select the controlled variable weights, γ_r^2, to scale process variable units to be the same.
6. Compute the move suppression coefficients, λ_s^2:

$$\lambda_s^2 = \frac{M}{10}\sum_{r=1}^{R}\left[\gamma_r^2 K_{rs}^2\left\{P - k_{rs} - \frac{3}{2}\frac{\tau_{rs}}{T} + 2 - \frac{(M-1)}{2}\right\}\right] \quad (s = 1, 2, \ldots, S)$$

7. Implement DMC using the traditional step response matrix of the actual process and the initial values of the parameters computed in steps 1–6.

describes the process gain, overall time constant and effective dead time of such processes (Cohen & Coon, 1953; Stauffer, 2001).

An FOPDT model fit provides four critical pieces of information useful for controller design. Specifically, K indicates the size and direction of the process variable response to a control move, τ describes the speed of the response, and θ tells the delay prior to when the response begins. Previous research for tuning DMC (Shridhar & Cooper, 1997, 1998) has demonstrated that this information is often sufficient to achieve desirable closed loop DMC performance at the specified design level of operation. As long as the FOPDT model parameters are identified such that they describe the system reasonably, the tuning strategy should be successful even for higher order processes (Shridhar & Cooper, 1997, 1998).

Step 2 involves the selection of a sample time, T. The FOPDT parameters, from step 1, provide a convenient method for obtaining T. The value of T given in Table 1 balances the desire for a low computation load (a large T) with the need to properly track the evolving dynamic behavior (a small T). Ljung (1987) confirms these findings, stating that too slow of a sampling rate will lead to information losses, and too fast of a sampling rate could lead to numerically sensitive procedures. Many control computers restrict the choice of T (e.g., Franklin & Powell, 1980; Åström & Wittenmark, 1984). Recognizing this, the remaining tuning rules permit values of T other than the recommended value given in Table 1.

Step 3 computes the prediction horizon, P, and the model horizon, N, in samples as the settling time of the slowest sub-process in the multivariable system. Note that both N and P cannot be selected independent of the sample time.

A larger P improves the nominal stability of the closed loop. For this reason P is calculated such that it includes the steady-state effect of all past controller output moves, i.e. it is calculated as the open loop settling time of the slowest FOPDT model approximation.

In addition, it is important that N be equal to the open loop settling time of the slowest sub-process to avoid truncation error in the predicted process variable profiles. Table 1 computes N as the settling time of the slowest FOPDT model approximation. This value is long enough to avoid the instabilities that can otherwise result since truncation of the model horizon misrepresents the effect of controller output moves in the predicted process variable profile (Lundström, Lee, Morari, & Skogestad, 1995).

Step 4 computes the control horizon, M, equal to 63.2% of the settling time of the slowest sub-process in the multivariable system. This ensures M to be long enough such that the results of the control actions are clearly evidenced in the response of the measured process variable.

Step 5 requires the selection of the controlled variable weights, γ_r^2. In most cases, the controlled variable weights are set equal to one. However, the practitioner is free to select these values to recast the measured process variables into the same units. Or, the practitioner can use these parameters to achieve tighter control of a particular measured process variable by selectively increasing its relative weight.

Step 6 computes the move suppression coefficients, λ_s^2. Its primary role in DMC is to suppress aggressive controller actions. When the control horizon is 1 ($M = 1$), no move suppression coefficient is needed ($\lambda = 0$). If the control horizon is greater than 1 ($M > 1$), then the analytical equation given in Table 1 is used.

Step 7 summarizes the tuning parameters for multivariable DMC. Once the tuning parameters have been determined, the unit step response coefficients, $\boldsymbol{a}_{rs,i}$ ($i = 1, 2, \ldots, N$; $r = 1, 2, \ldots, R$; $s = 1, 2, \ldots, S$), for controller output s on measured process variable r are calculated.

Step response coefficients for the internal DMC process model do not use an FOPDT approximation. Rather, actual process data is employed as is typical for DMC. The process data is generated by introducing a positive step in one controller output with the process at steady state and all the controllers in manual mode. In addition, all other controller output variables must remain constant. From the instant the step change is made, the response of each process variable is recorded as it evolves and settles at a new steady state. For a step in the controller output of arbitrary size, the response data is normalized by dividing through by the size of the controller output step to yield the unit step response. This is performed for each controller output to measured process variable pair, and it is necessary to make the controller output step large enough such that noise in the process variable measurement does not mask the true process behavior.

For simplicity, the remainder of the algorithm is formulated here for a system with two controller outputs and two process variables. The method can be directly extended to more complex processes.

For a two-by-two system, the multivariable dynamic matrix, $\boldsymbol{A}$, is formulated using the first P step response coefficients:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}_{2P \times 2M}, \tag{4}$$

where $\boldsymbol{A}_{11}$ is constructed from the step response of process variable 1 (PV1) obtained by a step change in controller output 1 (CO1). $\boldsymbol{A}_{ij}$ is given by

$$\boldsymbol{A}_{ij} = \begin{bmatrix} a_{ij,1} & 0 & 0 & \cdots & 0 \\ a_{ij,2} & a_{ij,1} & 0 & & 0 \\ a_{ij,3} & a_{ij,2} & a_{ij,1} & \ddots & 0 \\ \vdots & \vdots & \vdots & & 0 \\ a_{ij,M} & a_{ij,M-1} & a_{ij,M-2} & & a_{ij,1} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{ij,P} & a_{ij,P-1} & a_{ij,P-2} & \cdots & a_{ij,P-M+1} \end{bmatrix}_{P\times M}. \tag{5}$$

Using Eqs. (1) and (2) for constrained DMC or Eq. (3) for unconstrained systems, a vector of (SM) controller output moves is computed over the control horizon:

$$\boldsymbol{\Delta\bar{u}} = \begin{bmatrix} \Delta u_1(n) \\ \Delta u_1(n+1) \\ \Delta u_1(n+2) \\ \vdots \\ \Delta u_1(n+M-1) \\ \Delta u_2(n) \\ \Delta u_2(n+1) \\ \Delta u_2(n+2) \\ \vdots \\ \Delta u_2(n+M-1) \end{bmatrix}_{2M\times 1}, \tag{6}$$

where $\Delta u_1(n)$ is the move implemented for CO1 and $\Delta u_2(n)$ is the move implemented for controller output 2 (CO2).

4.2. The adaptive strategy

The adaptive DMC strategy builds on the non-adaptive formal tuning rule and the DMC control move calculation. As displayed in Fig. 1, the approach for the adaptive strategy involves designing three non-adaptive DMC controllers. Each controller has a model-based optimizer and a model-based predictor. The approach described here involves designing and combining three non-adaptive DMC controllers. However, this technique can involve designing and combining any number of non-adaptive controllers.

As explained below, all use the same values for T, P, N, M, and γ_r^2, while λ_s^2 varies for each controller. The three controllers each compute their own control action. These are then weighted and combined based on the value of the current measurements of each process variable to yield a single set of control moves forwarded to the final control elements.

Although three controllers are employed here, the method can be expanded to include as many local linear controllers as the practitioner would like. The use of three linear DMC controllers is the minimum needed to reasonably control a nonlinear process. The more linear controllers that are used, the better the adaptive controller will perform. There are no theoretical guidelines to illustrate how many linear controllers should be used in the adaptive control strategy to give optimal performance (Yu et al., 1992). While this method will often not capture the severe nonlinear behaviors associated with many processes, it will provide significant benefit over the non-adaptive DMC controller.

Implementation begins by collecting sets of step test data at three levels of operation, one at a lower, middle and upper level of the expected operating range for each sub-process relating the sth controller output to the rth measured process variable. For example, if there are two controller outputs and two measured process variables, then 12 sets of step test data will be needed for the adaptive DMC strategy. Each of the process models should describe the process dynamics around the point in which the data was collected. Two of the operating points should be at the upper and lower extremes of the expected operating region to ensure that the nonlinear approximation reasonably describes the actual process over the entire operating range (Di Marco et al., 1997). The third operating point should be around the middle of the expected operating region. The operating levels are defined as a specific value for each of r measured process variables, y_{rl}, where $l = 1, 2, 3$ are for the lower, middle and upper level of operation, respectively.

Each data set is fit with a linear FOPDT model for use in the tuning correlations. The data itself is used to formulate the step response coefficients. The tuning parameters for the adaptive DMC strategy are computed by employing the formal tuning rules given in Table 1.

Here, T is selected as close as possible to the smallest T_{rsl} from the data sets, or:

$$T_{rsl} = \mathrm{Max}(0.1\tau_{rsl}, 0.5\theta_{rsl}),$$
$$T = \mathrm{Min}(T_{rsl}),$$
$$(r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S;\ l = 1, 2, 3). \tag{7}$$

This ensures that when the process is operating in the level with the fastest dynamics, the sample time is fast enough to capture the process behavior. Since many control computers restrict the choice of T (e.g., Franklin & Powell, 1980; Åström & Wittenmark, 1984), the remaining tuning rules permit values of T other than that computed by Eq. (7) to be used.

The tuning parameters P, N, and M are selected as the maximum values:

$$P = N = \mathrm{Max}\left(\frac{5\tau_{rsl}}{T} + k_{rsl}\right)$$
$$(r = 1, 2, \ldots, R;\ s = 1, 2, \ldots, S;\ l = 1, 2, 3), \tag{8a}$$

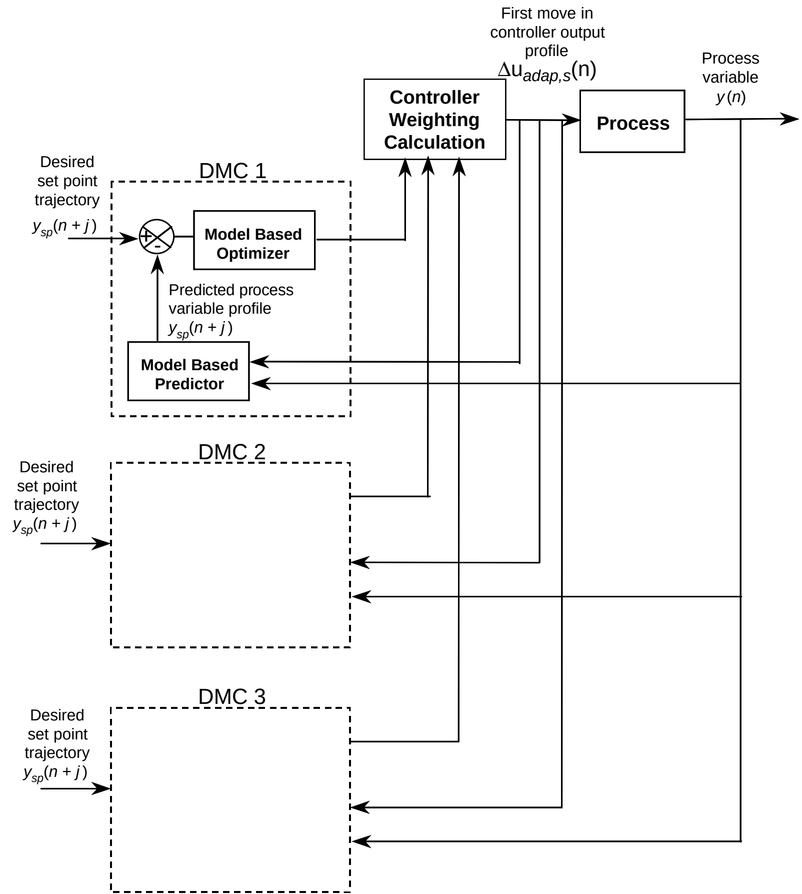

Fig. 1. Schematic of multiple model adaptive DMC strategy.

where

$$k_{rsl} = \left(\frac{\theta_{rsl}}{T} + 1\right), \tag{8b}$$

$$M = \mathrm{Max}\left(\frac{\tau_{rsl}}{T} + k_{rsl}\right)$$
$$(r = 1, 2, 3, \ldots, R;\ s = 1, 2, \ldots, S;\ l = 1, 2, 3). \tag{8c}$$

Thus, the horizons will always be long enough to capture the slowest dynamic behaviors in the range of operation. Truncation of any of the horizons (prediction, model or control) can result in instabilities in the closed system.

The controlled variable weights are usually set equal to one. However, the practitioner is free to select these values to recast the measured process variables into the same units. For example, the span of each process variable can be used to convert the units for the process variable to percentage. Or, the practitioner can use these parameters to achieve tighter control of a particular measured process variable by selectively increasing its relative weight.

Even though the above tuning parameters remain fixed upon implementation, success in this adaptive strategy requires that λ_s^2 vary based upon each data set. Since each data set will have different values for K_{rs}, τ_{rs}, and θ_{rs}, the value of λ_{sl}^2 calculated for each data set must reflect this difference, or

$$\lambda_{sl}^2 = \frac{M}{10}\sum_{r=1}^{R}\left[\gamma_{rl}^2 K_{rsl}^2\left\{P - k_{rsl} - \frac{3}{2}\frac{\tau_{rsl}}{T} + 2 - \frac{(M-1)}{2}\right\}\right]$$
$$(s = 1, 2, \ldots, S;\ l = 1, 2, 3). \tag{9}$$

Note that the calculation of λ_{sl}^2 is based upon the overall M and not on the control horizon calculated for each test set of data for each sub-process in the multivariable system. This allows λ_{sl}^2 to suppress aggressive control actions over the entire control horizon. Similar to non-adaptive DMC, Eq. (8) is valid for a control horizon greater than 1 ($M > 1$), and if the control horizon is 1 ($M = 1$), then no move suppression coefficient is used ($\lambda_{sl}^2 = 0$).

Upon implementation, the MMAC strategy for DMC calculates three non-adaptive DMC controller output moves, one for each level of operation. The adaptive controller output moves, $\Delta u_{adap,s}$, are a weighted average of each linear controller output move:

$$\Delta u_{adap,s} = \sum_{l=1}^{3} x_{l,s}\Delta u(1)_{l,s}$$
$$(s = 1, 2, \ldots, S;\ l = 1, 2, 3), \tag{10}$$

where $x_{l,s}$ is a weighting factor. If $y_{meas,r}$ is the actual value of the measured process variable r at the current sample time, then

If $y_{meas,r} \geqslant y_{3,r}$ then

$$x_{1,s} = 0, \quad x_{2,s} = 0, \quad x_{3,s} = 1. \tag{11}$$

If $y_{2,r} < y_{meas,r} < y_{3,r}$ then

$$x_{1,s} = 0, \quad x_{2,s} = 1 - x_{3,s}, \quad x_{3,s} = \frac{y_{meas,r} - y_{2,r}}{y_{3,r} - y_{2,r}}. \tag{12}$$

If $y_{1,r} < y_{meas,r} < y_{2,r}$ then

$$x_{1,s} = 1 - x_{2,s}, \quad x_{2,s} = \frac{y_{meas,r} - y_{1,r}}{y_{2,r} - y_{1,r}}, \quad x_{3,s} = 0. \tag{13}$$

If $y_{meas,r} \leqslant y_{1,r}$ then

$$x_{1,s} = 1, \quad x_{2,s} = 0, \quad x_{3,s} = 0. \tag{14}$$

In the event that $y_{meas,r} = y_{2,r}$, then the adaptive controller output move equals the value associated with the middle data set. Hence, the weighting factors are in the range of [0,1]. The values of the adaptive controller outputs finally implemented are calculated as

$$u(n)_s = u(n-1)_s + \Delta u_{adap,s} \quad (s = 1, 2, \ldots, S). \tag{15}$$

Theoretical studies are needed to address the issue of closed loop stability over the entire range of nonlinear operation. It has been shown by past researchers (Greene & Willsky, 1980) that the overall MMAC system may not be stable even if each individual controller is stable over the entire range of operation. In addition, Narendra and Xiang (2000) address the issue of stability for linear time-invariant discrete systems using MMAC. The results included a proof of global stability for the overall system.

5. Demonstration of multivariable adaptive DMC

The adaptive DMC algorithm is demonstrated here for three process simulations, a synthetic transfer function model, a multi-tank process, and rigorous distillation column.

5.1. Transfer function model

The transfer function model has two measured process variables and two controller outputs. A change in either controller output affects both process variables. Each controller output to measured process variable pair is a sub-process. In this example there are four sub-processes, which include controller output 1 (CO1) to process variable 1 (PV1), controller output 1 (CO1) to process variable 2 (PV2), controller output 2 (CO2) to process variable 1 (PV1), and controller output 2 (CO2) to process variable 2 (PV2).

To form nonlinear models, three different transfer functions are combined using a linear weighting function. The general form of each transfer function is

$$G_p(s) = \frac{K e^{-\theta S}}{(\tau_{P_1} s + 1)(\tau_{P_2} s + 1)}. \tag{16}$$

Each of the three transfer functions has different parameter values, and each exactly describes the behavior of the process at a specific value of the measured process variable. At intermediate values of the measured process variable, the transfer function contributions are combined to yield a continually changing dynamic behavior. This methodology is used to construct nonlinear models for the four sub-processes.

Table 2 lists the parameters used for each of the three transfer functions. As listed, a model is defined at measured process variables value of 20%, 50%, and 80%. To make the application nonlinear, the process gains change by 600% over the range of operation. Also, dead time and time constants change by as much as 200%.

Dynamic tests are performed by pulsing each controller output at each level of operation, yielding six sets of test data. Three data sets were collected at PV1 levels of 20%, 50%, and 80%, and three data sets were collected at PV2 levels of 20%, 50% and 80%. Following the adaptive DMC design procedure described previously, an FOPDT model is fit to each data set to yield the parameters listed in Table 3. The FOPDT parameters are then used in the equations in Table 1 to obtain the non-adaptive DMC tuning parameters listed in Table 4.

Table 4 also lists the tuning parameters for the adaptive DMC strategy obtained by using Eqs. (7)–(9). Note that as described in the adaptive strategy, all three controllers use the same value for T, P, N, M, and γ_r^2, while λ_s^2 varies for each controller. This ensures that the sample time is short enough to capture the fastest dynamic behaviors while the horizons are long enough to capture the slowest dynamic behaviors in the range of operation.

The control objective in this study is set point tracking of PV1 across the range of nonlinear operation. The design goal is a fast rise time with no peak overshoot ratio (POR). Non-adaptive DMC uses the tuning parameters associated with the middle level of operation (i.e. the measured process variable equals 50%). It is reasonable to design the non-adaptive controller based on the middle level of operation because this will yield a compromise in performance over the range of dynamic behaviors.

Fig. 2 shows the response of PV1 for both the non-adaptive and adaptive DMC implementations. As illustrated by the figure, the performance of the non-adaptive DMC varies as the dynamic behavior of the process changes. As the set point is stepped from 80% down to 20%, the performance of the non-adaptive controller varies from an under-damped response to one

Table 2
General model parameters for the transfer function model

	CO1 to PV1	CO1 to PV2	CO2 to PV1	CO2 to PV2
Process variable value (%)	20	20	20	20
SOPDT model parameters				
K_P	1	0.5	0.5	1
τ_{P1} (time units)	10	15	15	10
τ_{P2} (time units)	5	10	10	5
θ_P (time units	3	4	4	3
Process variable value (%)	50	50	50	50
SOPDT model parameters				
K_P	3	1.5	1.5	3
τ_{P1} (time units)	20	25	25	20
τ_{P2} (time units)	10	15	15	10
θ_P (time units	6	5	5	6
Process variable value (%)	80	80	80	80
SOPDT model parameters				
K_P	6	3.0	3.0	6
τ_{P1} (time units)	30	35	35	30
τ_{P2} (time units)	15	20	20	15
θ_P (time units)	9	6	6	9

Table 3
FOPDT model parameters for the transfer function model

	CO1 to PV1	CO1 to PV2	CO2 to PV1	CO2 to PV2
Process variable value (m)	20	20	20	20
FOPDT model fit parameters (for use in tuning equations only)				
K_P	1.0	0.5	0.5	1.0
τ_P (time units)	12.3	20.3	20.3	12.3
θ_P (time units)	6.0	9.6	9.6	6.0
Process variable value (m)	50	50	50	50
FOPDT model fit parameters (for use in tuning equations only)				
K_P	3.0	1.5	1.5	3.0
τ_P (time units)	26.0	34.3	34.3	26.0
θ_P (time units)	11.4	13.0	13.0	11.4
Process variable value (m)	80	80	80	80
FOPDT model fit parameters (for use in tuning equations only)				
K_P	6.0	3.0	3.0	6.0
τ_P (time units)	36.6	44.6	44.6	36.6
θ_P (time units)	18.3	17.8	17.8	18.3

that is over-damped and sluggish in nature. However, the adaptive strategy is able to maintain the performance of the controller over the operating range for PV1.

Specifically, as the set point is stepped from 80% to 60%, the response of the process variable for the non-adaptive controller displays a fast rise time with a 20% POR. For the set point change from 40% to 20%, the non-adaptive controller exhibits a sluggish rise time with no POR. The adaptive controller is able to maintain a consistent rise time with no POR over the entire range of operation.

Table 4
Non-adaptive and adaptive DMC tuning parameters for the transfer function model

	Lower level	Middle level	Upper level
Non-adaptive DMC tuning parameters			
PV1 (%)		50	
PV2 (%)		50	
T (time units)		6	
P (samples)		31	
N (samples)		31	
M (samples)		8	
γ_1^2		1	
γ_2^2		1	
λ_1^2		187.9	
λ_2^2		187.9	
Adaptive DMC tuning parameters			
PV1 (%)	20	50	80
PV2 (%)	20	50	80
T (time units)	3	3	3
P (samples)	80	80	80
N (samples)	80	80	80
M (samples)	20	20	20
γ_1^2	1	1	1
γ_2^2	1	1	1
λ_1^2	155.9	1257	4195
λ_2^2	155.9	1257	4195

The performance of the adaptive DMC remains constant as the dynamic behavior of the process changes by weighting the DMC controller output moves from each model to account for the changing dynamic behavior of the process. In effect, the adaptive strategy is changing the amount of control effort needed as the process dynamics are changed.

5.2. Multi-tank process

The multi-tank process, shown in Fig. 3, is a four tanks process where there are two sets of freely draining tanks positioned side by side. This simulation is one of the case studies available in Control Station®. Control Station is a controller design and tuning tool and a process control training simulator used by industry and academic institutions worldwide for control loop analysis and tuning, dynamic process modeling and simulation, performance and capability studies, hands-on process control training. More information and a free demo are available at www.controlstation.com.

The general model for each tank is described using a material balance. DMC level controller manipulates the flow rate entering their top tanks, respectively. The two measured process variables are the liquid levels in the lower tanks. The process represents a challenging control problem due to its modest nonlinear dynamic behavior and multivariable control loop interactions. This process displays a nonlinear behavior in that the

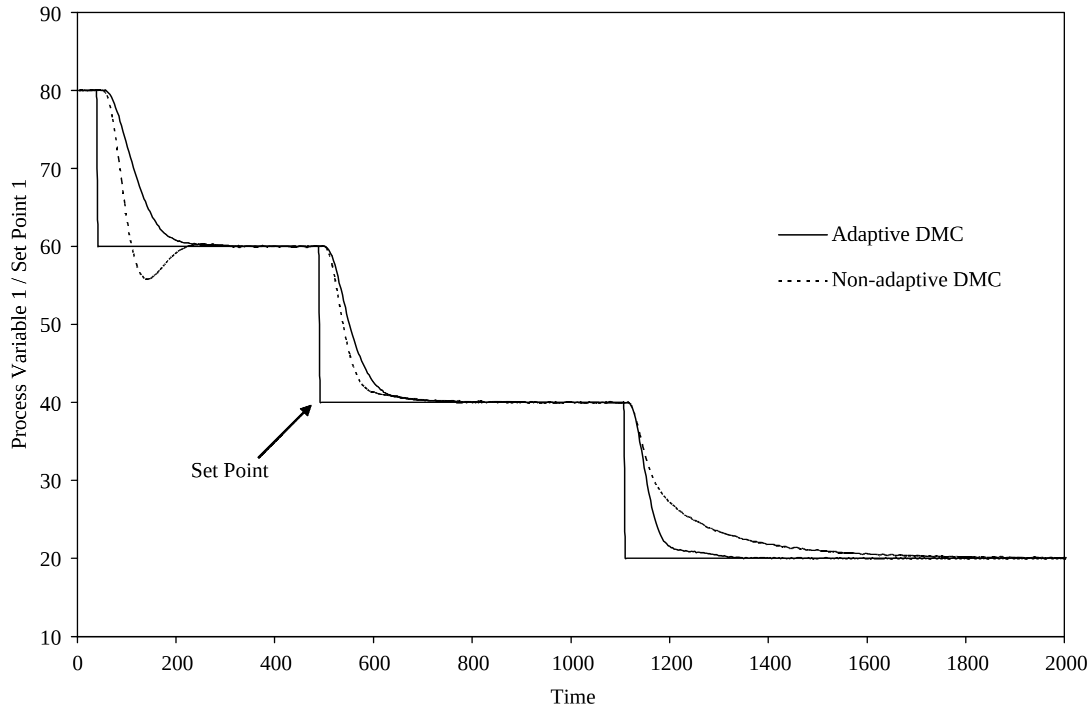

Fig. 2. Response of PV1 for the transfer function model using non-adaptive and adaptive DMC.

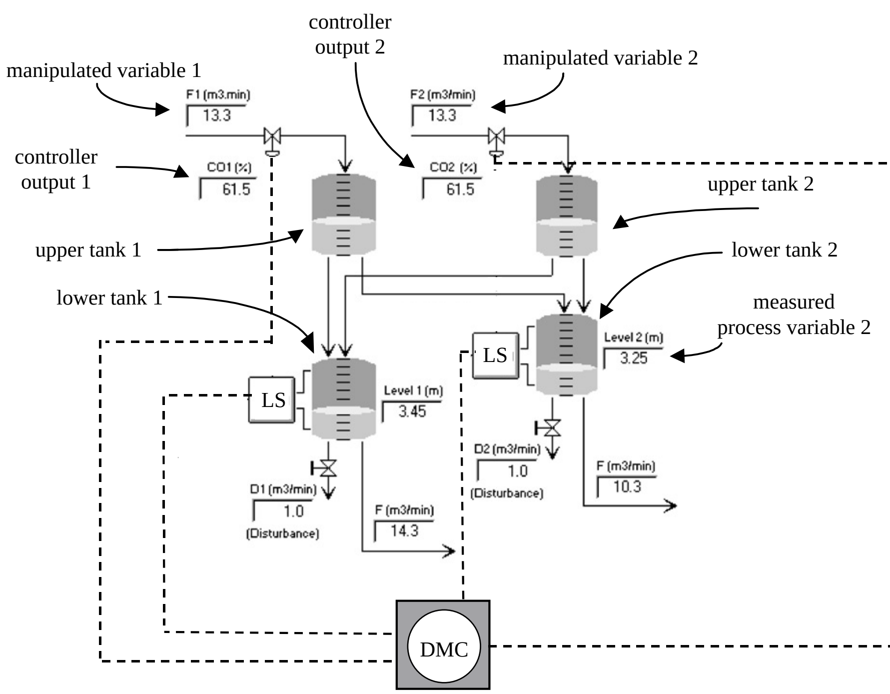

Fig. 3. Multi-tanks graphic.

process gains for each sub-process change by at least a factor of 2 over the range of operation studied in this example.

Dynamic tests are performed by pulsing the controller output at each level of operation, generating six sets of test data. Three data sets were obtained at levels in tank 1 of (measured PV1) of 1.5, 3.5 and 6.6 m. The other three data sets were obtained at levels in tank 2 of (measured PV2) of 1.4, 3.3 and 6.3 m. Following the procedure just described in the previous example, each

Table 5
FOPDT model parameters for the multi-tanks simulation

	CO1 to level 1	CO1 to level 2	CO2 to level 1	CO2 to level 2
Process variable value (m)	1.5	1.4	1.5	1.4
FOPDT model fit parameters				
K_P	0.05	0.02	0.03	0.05
τ_P (min)	12.2	12.7	12.5	12.9
θ_P (min)	5.4	5.9	5.2	5.8
Process variable value (m)	3.5	3.3	3.5	3.3
FOPDT model fit parameters				
K_P	0.07	0.04	0.04	0.07
τ_P (min)	14.9	15.9	13.9	15.4
θ_P (min)	6.9	7.6	6.8	7
Process variable value (m)	6.6	6.3	6.6	6.3
FOPDT model fit parameters				
K_P	0.10	0.05	0.06	0.10
τ_P (min)	17.0	18.4	17.0	18.3
θ_P (min)	8.9	9.8	8.6	9.5

Table 6
Non-adaptive and adaptive DMC tuning parameters for the multi-tanks simulation

	Lower level	Middle level	Upper level
Non-adaptive DMC tuning parameters			
Level 1 (m)		3.5	
Level 2 (m)		3.3	
T (s)		180	
P (samples)		29	
N (samples)		29	
M (samples)		8	
γ_1^2		1	
γ_2^2		1	
λ_1^2		0.088	
λ_2^2		0.088	
Adaptive DMC tuning parameters			
Level 1 (m)	1.5	3.5	6.6
Level 2 (m)	1.4	3.3	6.3
T (s)	180	180	180
P (samples)	34	34	34
N (samples)	34	34	34
M (samples)	10	10	10
γ_1^2	1	1	1
γ_2^2	1	1	1
λ_1^2	0.068	0.14	0.25
λ_2^2	0.079	0.14	0.26

data set is fit with an FOPDT model (results listed in Table 5) and these parameters are used to compute the non-adaptive and adaptive DMC tuning values (results listed in Table 6).

The control objective in this study is set point tracking across the range of nonlinear operation. The design goal in this study is a fast rise time with no POR. Non-adaptive DMC uses the tuning parameters associated with the middle level of operation (i.e. PV1 = 3.5 and PV2 = 3.3).

Fig. 4 shows the response of PV1 for both the non-adaptive and adaptive DMC implementations. As the set point is stepped from 5.5 to 1.5 m the behavior of PV1 for non-adaptive DMC ranges from a response that is under-damped to a response that is over-damped. As the process reaches higher tank levels, the process variables response becomes more oscillatory.

As the set point is stepped from 5.5 to 4.5 m, the response of the non-adaptive controller displays a 5% POR. For the set point step from 2.5 to 1.5 m, the non-adaptive controller exhibits a sluggish rise time with no POR. The adaptive controller exhibits no problems in maintaining the design goal of a fast rise time with no POR over the range of operation.

The disturbance rejection capabilities of the adaptive and non-adaptive DMC controller were also studied. The disturbance is a secondary flow out of the lower tanks from a positive displacement pump, and is independent of the liquid level except when the tanks are empty. The disturbance flow rate was stepped from 1 to 2 m^3/min and then back to 1 m^3/min.

Fig. 5 shows the response of PV1 for both the non-adaptive and adaptive DMC implementations at a set point level of approximately 3.5 m. At this level of operation both the adaptive and non-adaptive QDMC controllers give similar performance. This is because the non-adaptive controller was designed for a level of 3.5 m. This is verified in Fig. 5.

Fig. 6 displays the response of PV1 for both the non-adaptive and adaptive DMC implementations at a set point level of approximately 1.5 m. At this level of operation the adaptive DMC controller should exhibit better disturbance rejection capabilities. This is because the tuning and model parameters for the non-adaptive controller are no longer valid. As displayed in Fig. 6, the adaptive controller outperforms the non-adaptive controller. The adaptive controller is able to reject the disturbance quicker and return the height of the tank back to its set point faster.

As shown by these figures, the adaptive DMC controller is able to maintain better performance over all operating ranges. The adaptive strategy weights the multiple controller output moves in order to achieve the desired performance at each level of operation.

5.3. Distillation column process

The distillation column simulation, shown in Fig. 7, is a tray-by-tray simulation model that is highly nonlinear and has strong interactive characteristics. This is one of the many multivariable simulations available in Control Station. It is based on the system given by McCune and Gallier (1973). For each tray, the reboiler and the

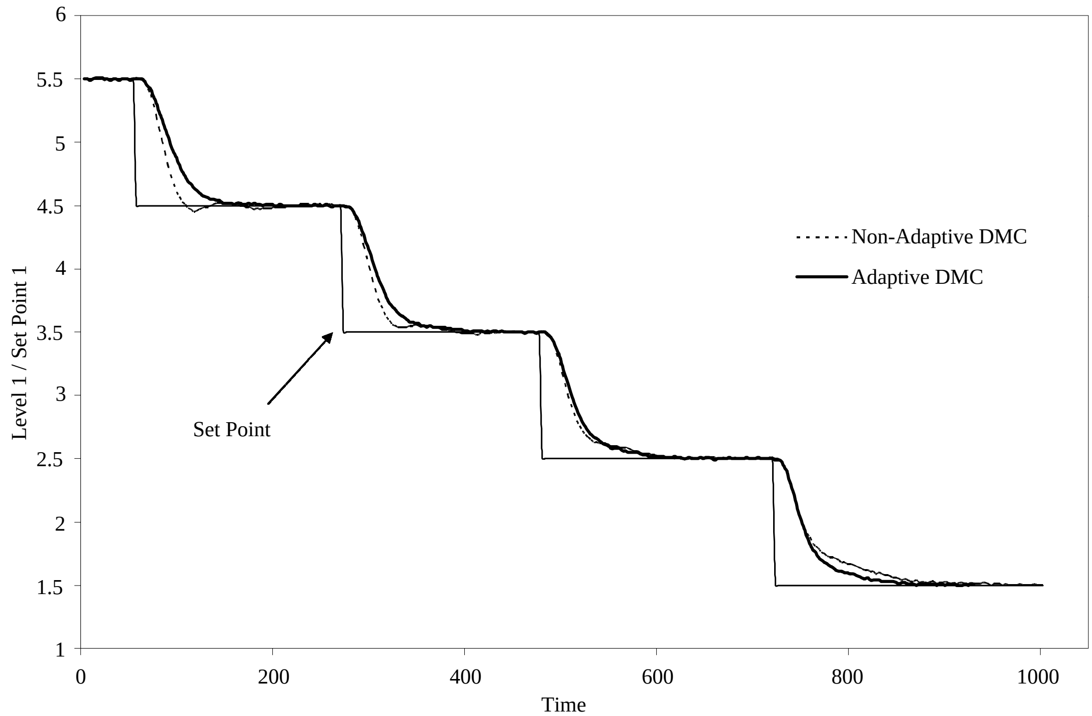

Fig. 4. Response of PV1 (level 1) for the multi-tanks process using non-adaptive and adaptive DMC.

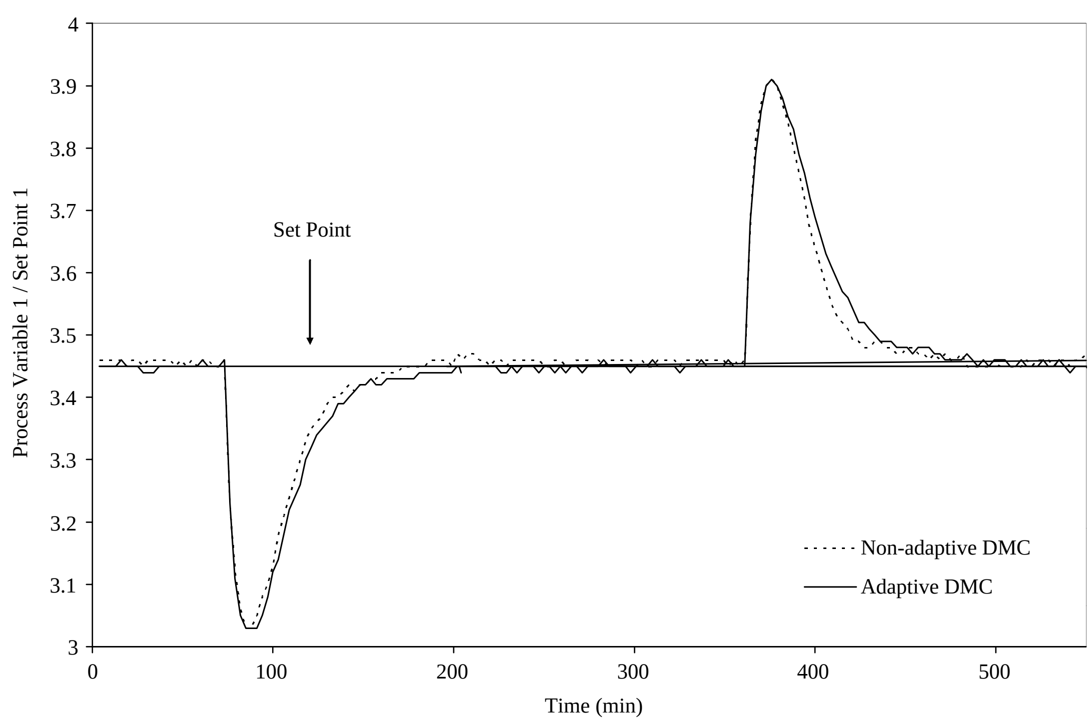

Fig. 5. Response of PV1 for the multi-tanks process using non-adaptive and adaptive DMC for disturbance rejection capabilities at a set point of approximately 3.5 m.

condenser, differential equations are used to describe the overall and component mass balances and algebraic equations are used for the energy balances. The stage efficiencies are represented by the Murphy tray efficiency, and a linear hydraulic relationship is used to describe the liquid flow from each tray as a function of

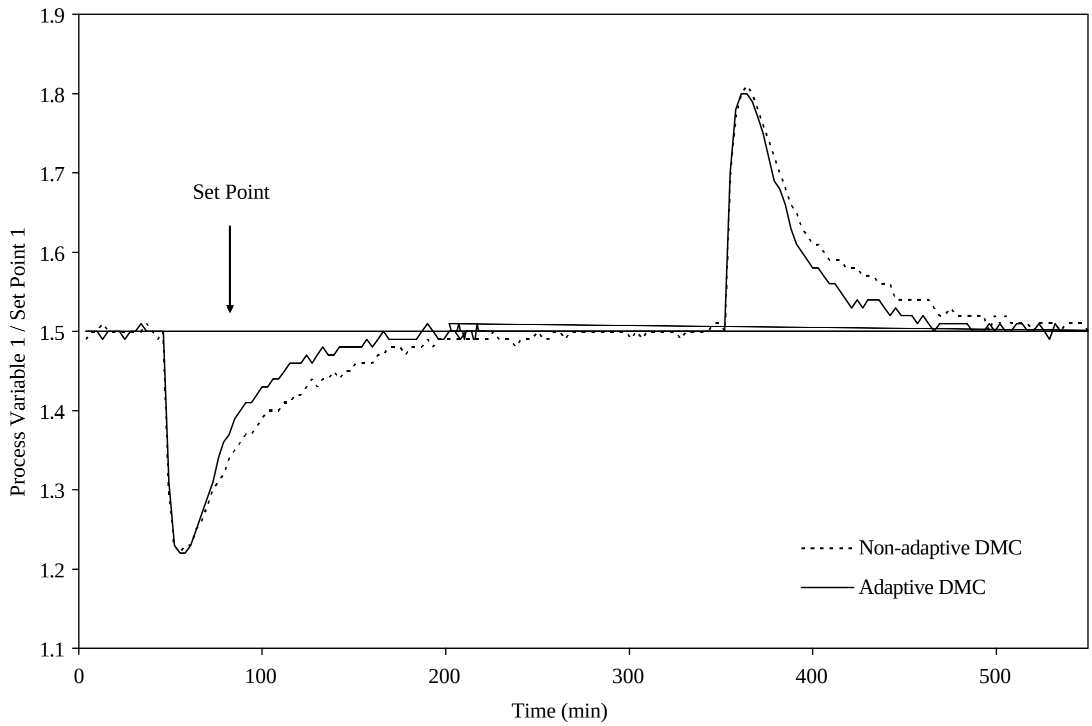

Fig. 6. Response of PV1 for the multi-tanks process using non-adaptive and adaptive DMC for disturbance rejection capabilities at a set point of approximately 1.5 m.

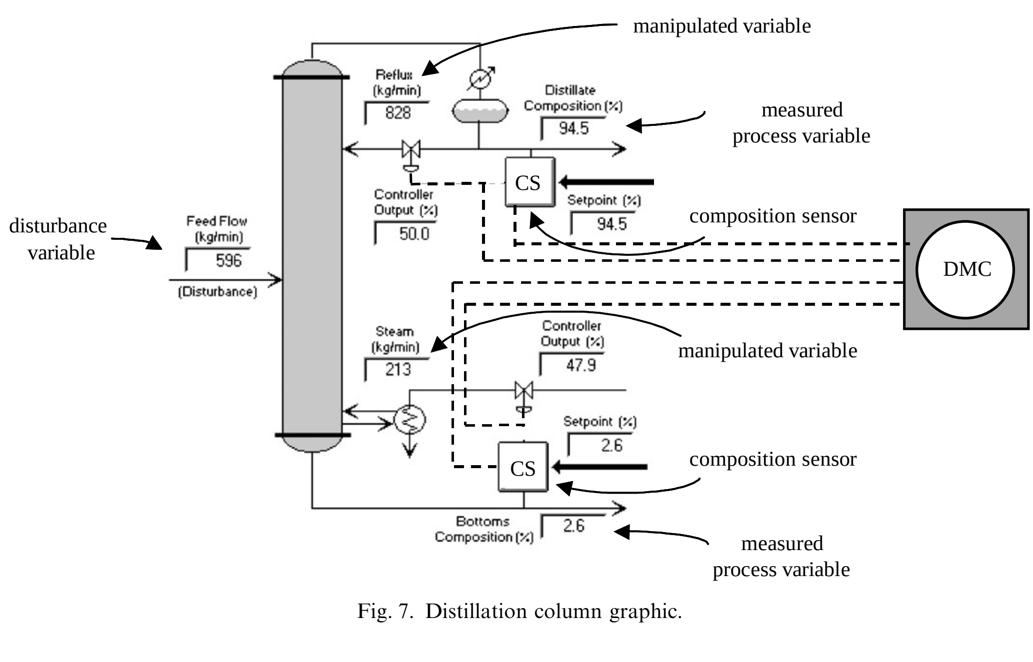

Fig. 7. Distillation column graphic.

the mass holdup. The simulation model separates benzene and toluene at constant pressure.

The column has two measured process variables and two manipulated variables. The reflux rate is used to control the composition of benzene in the distillate stream and steam rate is used to control the composition of benzene in the bottoms stream. The feed flow rate acts as a disturbance to the column.

Dynamic tests are performed by pulsing the controller output at each level of operation, generating six sets of test data. Three data sets were obtained at distillate compositions (measured PV1) of 89.3%, 94.4% and

Table 7
FOPDT model parameters for the distillation column simulation

	CO1 to PV1	CO1 to PV2	CO2 to PV1	CO2 to PV2
Process variable value (%)	89.3	1.5	89.3	1.5
FOPDT model fit				
K_P	1.1	0.11	−0.94	−0.12
τ_P (min)	43.6	35.4	44.5	37.3
θ_P (min)	21.9	21.0	14.7	5.9
Process variable value (%)	94.4	2.6	94.4	2.6
FOPDT model fit				
K_P	0.94	0.41	−0.80	−0.40
τ_P (min)	64.8	68.0	63.7	70.2
θ_P (min)	27.9	22.0	21.0	6.8
Process variable value (%)	98.7	12.9	98.7	12.9
FOPDT model fit				
K_P	0.11	1.28	−0.09	−1.1
τ_P (min)	50.6	47.5	44.1	45.4
θ_P (min)	16.7	23.6	16.7	15.0

Table 8
Non-adaptive and adaptive DMC tuning parameters for the distillation column simulation

	Lower level	Middle level	Upper level
Non-adaptive DMC tuning parameters			
Distillate composition (%)		94.4	
Bottoms composition (%)		2.6	
T (s)		420	
P (samples)		52	
N (samples)		52	
M (samples)		13	
γ_1^2		1	
γ_2^2		1	
λ_{RG}		7.0	
c		578.8	
λ_1^2		0.71	
λ_2^2		0.55	
Adaptive DMC tuning parameters			
Distillate composition (%)	89.3	94.4	98.7
Bottoms composition (%)	1.5	2.6	12.9
T (s)	240	240	240
P (samples)	90	90	90
N (samples)	90	90	90
M (samples)	23	23	23
γ_1^2	1	1	1
γ_2^2	1	1	1
λ_{RG}	5.4	7.0	7.0
c	305.4	323.7	450.7
λ_1^2	5.2	3.7	4.8
λ_2^2	4.1	2.9	4.0

98.7%. The other three data sets were obtained at bottom compositions (measured PV2) of 1.5%, 2.6% and 12.9%. Following the procedure just described in the previous example, each data set is fit with an FOPDT model (results listed in Table 7) and these parameters are used to compute the non-adaptive and adaptive DMC tuning values (results listed in Table 8).

The control objective in this study is set point tracking across the range of nonlinear operation. The design goal in this study is a fast rise time and a quick settling time with no more than a 15% POR. Non-adaptive DMC uses the tuning parameters associated with the middle level of operation (i.e. PV1 = 94.4% and PV2 = 2.6%).

Fig. 8 displays the response of the top composition for both the non-adaptive and adaptive DMC implementations. As the set point is stepped from 96.5% to 94.5%, the response of the non-adaptive controller displays no POR and a very long settling time compared to the performance of the adaptive DMC controller. For the set point step from 94.5% to 92.5%, the non-adaptive controller exhibits no POR and a long settling time compared to the performance of the adaptive DMC controller. For the set point step from 92.5% to 90.5%, the non-adaptive controller displays a slightly longer settling time than the adaptive controller. The adaptive controller exhibits no problems in maintaining the design goal of a fast rise time and settling time with a 15% POR over the range of operation.

The performance of the adaptive DMC implementation is able to maintain the performance for PV1 over all operating ranges. The adaptive strategy continues to weight the multiple controller output moves in order to achieve the desired performance.

6. Conclusions

A multiple model adaptive strategy for DMC has been developed and the application and benefits of this adaptive strategy is demonstrated through simulation. For the non-adaptive DMC algorithm, the process variable responses varied from over-damped to under-damped depending on the operating level. The adaptive DMC controller is able to maintain consistent set point tracking performance over the range of nonlinear operation. This work develops an adaptive strategy that builds upon linear controller design methods for creating a robust MMAC for DMC.

The contributions of the method presented here include an adaptive DMC strategy that:

- is straightforward to implement and use,
- requires minimal computation for updating model parameters,
- relies on the linear control knowledge of plant personnel, and
- is reliable for a broad class of process applications.

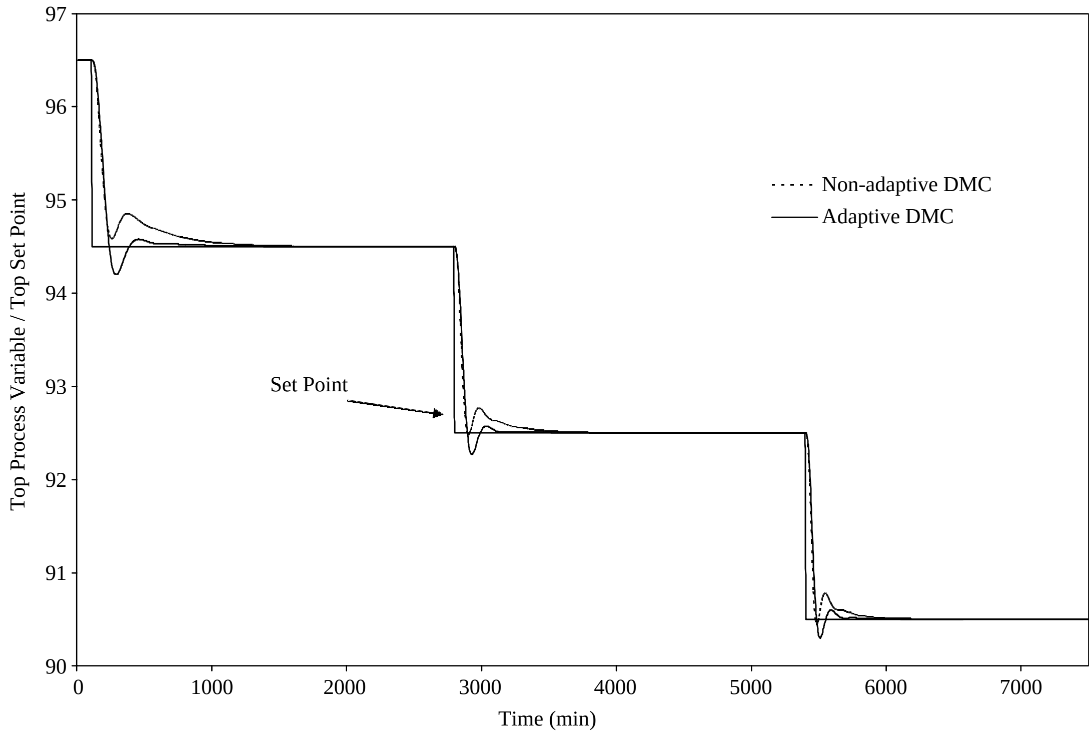

Fig. 8. Response of the top process variable for the rigorous distillation column simulation using non-adaptive and adaptive DMC.

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