Correlation of Thermal Conductivities of Unidirectional Fibrous Composites Using Local Fractal Techniques

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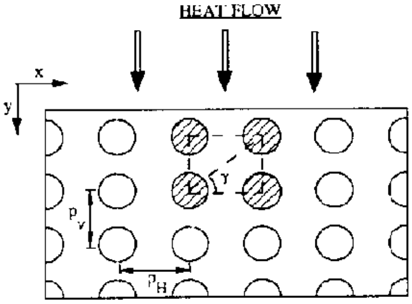
The arrangement of fibers strongly influences heat conduction in a composite. Traditional approaches using unit cells to describe the fiber arrangements work well in the case of ordered arrays, but are not useful in the context of disordered arrays, which have been analyzed in the literature by statistical means. This work presents a unified treatment using the tool of local fractal dimensions (although, strictly speaking, a composite cross section may not be an exact fractal) to reduce the geometric complexity of the relative fiber arrangement in the composite. The local fractal dimensions of a fibrous composite cross section are the fractal dimensions that it exhibits over a certain small range of length scales. A generalized unit cell is constructed based on the fiber volume fraction and local fractal dimensions along directions parallel and transverse to the heat flow direction. The thermal model resulting from a simplified analysis of this unit cell is shown to be very effective in predicting the conductivities of composites with both ordered as well as disordered arrangement of fibers. For the case of square packing arrays, the theoretical result of the present analysis is identical to that of Springer and Tsai (1967).

1 Introduction

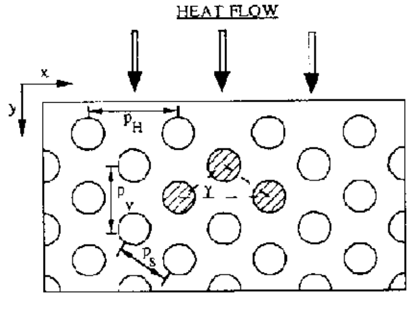
With the increasing use of composites in various thermal environments, the thermal properties of unidirectional fibrous composites have been the focus of many investigations in the last few years. These studies have essentially been along two parallel, but related paths: One is a model-based approach, where the fiber arrangements are modeled by simplified geometric equivalents; the second is a statistical approach, which employs statistical techniques to determine upper and lower bounds on the effective properties. This paper deals with the problem of determining the effective transverse thermal conductivity of fibrous composites. The effective thermal conductivity is defined as the conductivity of an equivalent homogeneous medium, which exhibits the same steady-state thermal characteristics as the composite material.

The model-based studies assume an ordered arrangement of fibers in the matrix with the fiber centers located at the vertices of either a rectangle (called rectangular arrays) or a triangle (called staggered arrays), as shown in Fig. 1. The effective thermal conductivities of these arrays can be obtained by analyzing representative rectangular or staggered unit cells, of which the composite cross section is assumed to be composed. Literature abounds with both analytical (Springer and Tsai, 1967; Behrens, 1968) and numerical (Han and Cosner, 1981; James and Keen, 1985; Muralidhar, 1990) results for the effective thermal conductivity of ordered arrays. However, with the exception of the work by Han and Cosner (1981), the other model-based studies are restricted mainly to the case of square packing arrays, for which γ equals 45 deg in Fig. 1.

Investigations found in the literature on disordered media (Hashin, 1970; Milton, 1982; Smith and Torquato, 1989) are aimed at determining upper and lower bounds on the effective thermal conductivities. The bounds are generally in terms of the fiber volume fraction (v), the fiber-matrix conductivity ratio (β), and a microstructural parameter ζ_2, which is a multifold integral that depends upon the choice of the probability distribution function for the composite (Smith and Torquato, 1989). Given only β and v, Hashin (1970) has obtained the best estimate of the bounds on the effective conductivity of transversely isotropic fiber-reinforced materials. Other re-



(a) Rectangular Array



(b) Staggered Array

Fig. 1 Schematic of ordered fiber arrangements: (a) rectangular array; (b) staggered array

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