

A generalized analysis of capillary flows in channels

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Abstract

Investigations on the motion of a fluid in capillary geometries have been extensively reported in the literature using both experimental and theoretical approaches. In this paper, the theories for capillary flow are generalized to a unified nonlinear second-order differential equation which takes the effects of the entrance, the inertial forces, and the dynamic contact angle into account. An analytical solution of the differential equation is obtained in the form of a double Dirichlet series. The readily evaluated analytical solution is compared with experimental and numerical results in the literature, which shows a good agreement. It is demonstrated that this analytical approach can be used to predict capillary flows for a wide range of fluids and parallel-plate and tube geometries in a unified manner.

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1. Introduction

Research on capillary-driven flows, dating its origin back to more than one century ago, is still a subject of interest due to the widespread applications in materials manufacturing, oil production, ink printing, and transportation of propellant, among other modern engineering fields [1–3]. For a fully developed capillary driven flow, the analysis on the balance of viscous force, surface tension force, and gravitational force yields the classical Washburn equation on the penetration depth, h , evolution with time, t [4]. While this analysis holds for describing the long-time capillary flow behavior, it does not account for several forces that prevail in the initial stages of a capillary flow process.

Several modifications to the Washburn equation have been reported to include the inertial force and entrance pressure loss effects [3,5–10]. A simple method to incorporate the entrance pressure loss was adopted in Ref. [5] by evaluating the energy loss using an empirical energy loss coefficient. Duarte et al. [6] employed a hydrodynamical approach by averaging the Navier–Stokes equation along the liquid column in the capillary geometry, and the velocity variation at the entrance was

taken into consideration. In this approach, the details of the liquid motion in the vicinity of the flow front are neglected, and the obtained equation is not applicable to the initial time $t = 0$. A more elaborate approach considered the motion of the fluid in a wider region of the flow field, including the fluids both in the capillary tube and in the fluid reservoir [1,2,7–10].

Most studies on capillary flows assumed constant contact angle between the advancing liquid front and the solid wall; however, experimental observations have shown a strong dependence of the contact angle on the velocity of the meniscus (flow front) movement [11–13]. Several empirical relations are available to describe the dynamic contact angle as a function of time [14–17]. The Hoffman function [14] can be applied to the case of capillary flow in horizontal channels where the effects of the inertial force and gravity are negligible. Bracke and Sciffer relations [15,16] are not applicable to the case when the dynamic contact angle starts with an initial value of 90° and gradually approaches the equilibrium value. The Newman model [17] may be applied to all types of capillary flows; however, a fitting parameter in the model must be determined from experimental data.

Based on the different considerations of inertial force, entrance effect, and dynamic contact angle mentioned above, a set of nonlinear first- or second-order differential equations were derived in the literature [1–10]. The majority of the studies

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