SAMS: Stochastic Analysis With Minimal Sampling—A Fast Algorithm for Analysis and Design Under Uncertainty

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Design of processes and devices under uncertainty calls for stochastic analysis of the effects of uncertain input parameters on the system performance and process outcomes. The stochastic analysis is often carried out based on sampling from the uncertain input parameters space, and using a physical model of the system to generate distributions of the outcomes. In many engineering applications, a large number of samples—on the order of thousands or more—is needed for an accurate convergence of the output distributions, which renders a stochastic analysis computationally intensive. Toward addressing the computational challenge, this article presents a methodology of Stochastic Analysis with Minimal Sampling (SAMS). The SAMS approach is based on approximating an output distribution by an analytical function, whose parameters are estimated using a few samples, constituting an orthogonal Taguchi array, from the input distributions. The analytical output distributions are, in turn, used to extract the reliability and robustness measures of the system. The methodology is applied to stochastic analysis of a composite materials manufacturing process under uncertainty, and the results are shown to compare closely to those from a Latin hypercube sampling method. The SAMS technique is also demonstrated to yield computational savings of up to 90% relative to the sampling-based method. [DOI: 10.1115/1.1866157]

1 Introduction

Many engineering systems and processes operate in an environment of uncertainty in the parameters governing the system or process behavior. The interactive effects of the uncertainty lead to variability in the system performance or the process outcomes, and uncertainty analysis is an important tool for investigating these effects. For applications where a closed-form linear model is available, uncertainty analysis can be performed by simply identifying the worst-case combinations of the input parameters, which are usually the extreme values of the respective parameter ranges, and using the linear model to obtain the limits of the output variation [1–4]. However, in most engineering applications, a closed-form model is not usually available; furthermore, the model is highly nonlinear, for which the worst cases of the output do not necessarily correspond to the extremes of the uncertain inputs. If the uncertainty in the input parameters can be identified as a probability distribution, the shape of the output distribution may not reflect the identical distribution as those of the inputs due to nonlinearity in the model. Thus, for most engineering applications, a sampling-based uncertainty analysis, or stochastic analysis, is necessary for obtaining reliable and robust designs.

As illustrated in Fig. 1, stochastic analysis involves quantifying the input parameter uncertainty in the form of appropriate distribution functions, sampling from the distributions, and using a deterministic physical model to simulate the process or the system behavior for the combinations of input parameter samples, and to construct the output variability distributions. The output distributions are, in turn, used to obtain reliability or robustness measures. For applications in which the phenomena are governed by a non closed-form, nonlinear model, the stochastic analysis using sampling may require massive computational resources [5–7]. The models based on the solution of a system of partial differential equations using finite element or finite difference methods are computationally tedious, and since sampling-based analysis calls for multiple deterministic simulations—often on the order of hundreds or thousands for a single stochastic simulation—the stochastic simulations are computationally intensive. Furthermore, for solving design under uncertainty problems which include optimization over a stochastic model, the computational burden is tremendously intensified [8,9]. For example, in the application of a stochastic analysis of a manufacturing process discussed in Ref. [9], the CPU times for one optimization, run using a stochastic model with just 50 samples, average about 24 h.

One approach to alleviating the computational requirements is to make the sampling more effective, thus reducing the required number of samples for the stochastic analysis. Two such examples are the Latin hypercube sampling (LHS) [10] and the Hammersley sampling method [11]. LHS is a stratified sampling method in which, if n number of samples from a one-dimensional distribution are required, the distribution is divided into n intervals (strata) of equal probability, and one sample is picked randomly from each interval to generate the samples, while the Hammersley sampling provides representative sample for multivariate probability distributions by way of a low-discrepancy design for placing n points uniformly in a k-dimensional cube. Being stratified sampling methods [10,11], LHS and HSS generate samples that better represent the entire distribution compared to the Monte Carlo technique, in which the samples are selected randomly and may not cover the entire distribution [12]. Other approaches to minimizing computational time include developing statistical models to approximate the complex nonlinear model [13], and approximating the sensitivity of uncertain parameters without resorting to sampling [14]. It is, however, desirable to derive the entire set of output distributions without sampling or with minimum sampling. The availability of output distributions provides

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