Design of Microresonators Under Uncertainty

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***Abstract*—A methodology for robust design analysis of microelectromechanical systems is presented by considering the example of comb microresonators and uncertainty in parameters governing the resonant frequency and the transconductance values. Analytical models for the variability in the resonant frequency and transconductance are developed as function of the parameter uncertainty, and used as objective functions sought to be minimized in a robust design endeavor. An enumeration search over the design space is utilized to determine the optimal design of microresonators that minimize the variability subject to constraints on performance requirements. The results are presented over a wide interval of operating resonant frequency, and can be used in the robust design of microresonators. [0960]**

***Index Terms*—Comb microresonator, microelectromechanical systems (MEMS), parameter uncertainty, reliability, robust design.**

NOMENCLATURE

C	Capacitance [F].
d	Finger gap [$\mu\mathrm{m}$].
E	Youngs modulus [GPa].
G_d	Reference transconductance [S].
G_x	Transconductance [S].
h	Beam thickness [$\mu\mathrm{m}$].
k	Augmented constant [$\mathrm{V}^2\mathrm{F}^2/\mathrm{m}^2\mathrm{kg}^{0.5}$].
L	Beam length [$\mu\mathrm{m}$].
M_E	Effective mass [kg].
M_E'	Effective mass per unit thickness [kg/m].
M_P	Mass of the plate [kg].
M_u'	Unit mass per unit thickness [kg/m].
N	Number of interdigitated fingers.
Q	Quality factor.
V_P	dc bias voltage [V].
V_S	Bias voltage between the structures and the electrode [V].
W	Beam width [$\mu\mathrm{m}$].

Greek Symbols

$\Delta(.)$	Maximum variance.
ϵ	Misalignment eccentricity.
ε_0	Permittivity of free space.
ω_d	Reference resonant natural frequency [kHz].
ω_0	Resonant natural frequency [kHz].
ω_0^*	Target resonant natural frequency [kHz].
Γ_x	Normalized transconductance.
Ω_0	Normalized resonant frequency.
Ω_0^*	Target normalized resonant frequency.

I. INTRODUCTION

RESONATORS are critical components in wide variety of microelectromechanical systems (MEMS) applications, such as in radio frequency systems [1] and microengines [2]. Microresonators are usually driven electrostatically to oscillate on one side, and sensed capacitively on the other side of a microelectronic circuit configuration. Thus, one of the most important mechanical design properties is the resonant natural frequency of the resonators, which is crucial for the performance of the device in which the resonators serve as components. An important electrical functional property is the transconductance, which relates to the magnitude of the amplification gain that the resonators, as electronic devices, are capable of delivering.

With growing number of applications of microresonators, and microstructures in general, a robust design methodology is needed to ensure the highest product performance, which is greatly influenced by manufacturing process variations due to the small dimensions and high feature complexity. With the present microfabrication techniques, process variation is inevitable and the manifestation is the variability in the product features which also leads to variability in the performance [3]. While manufacturing techniques continue to be improved so as to fabricate microstructures to tighter tolerances, the issue of variability can be alleviated using designs that, given the inevitable features variation in any manufacturing process, are engineered to perform with reduced variability.

Work on sensitivity analysis on MEMS resonators has been reported in the literature. Dewey *et al.* [4] utilized a statistical method to obtain a specific design for a resonator that operates at certain frequency. Liu *et al.* [5] presented sensitivity analysis by considering manufacturing-induced variability in the width of the resonator. The sensitivity information was derived using an analytical model and the authors provided an example of a robust design. The principal focus of the present paper is on reducing the performance variability in microresonators through the application of principles of design under uncertainty. To this end, this paper presents an analysis and design of microresonators under uncertainty, by considering uncertainty in several geometric and material property parameters simultaneously, and investigating the interactive effects of the parameter uncertainty on the variability in the resonant frequency and the transconductance of microresonators. Parameters closely influenced by the manufacturing variability, namely, the dimensions of the resonator beams and the Youngs modulus of the material of the beams, are considered uncertain, and a worst-case analysis is used to determine the magnitude of the performance variation.

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