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Viscous Fingering in a Hele-Shaw Cell With Finite Viscosity Ratio and Interfacial Tension

A volume tracking method was developed to simulate time-dependent unstable viscous fingering in a Hele-Shaw cell. The effect of finite viscosity ratio μ_r between displacing and displaced fluids and their interfacial tension σ on finger morphology is investigated. It is shown that there exist four distinct finger patterns, depending upon the viscosity ratio, μ_r, and Ca', the modified capillary number for constant flow rate, or $\Delta P \cdot W/\sigma$, for constant driving pressure difference. Morphology diagrams are developed to identify the ranges of the dimensionless parameters corresponding to the various finger patterns. The simulation results are validated with experiments. [DOI: 10.1115/1.1524589]

1 Introduction

Viscous fingering in a Hele-Shaw cell, [1], originating from industrial oil recovery, was first studied by Saffman and Taylor [2], and the complexity of fundamentals behind the phenomenon itself has since attracted much attention. A Hele-Shaw cell consists of two parallel plates with a narrow spacing; when a less viscous fluid is driven to displace a more viscous one in the cell, an initially flat interface between the two fluids evolves into a so-called *viscous fingering* pattern. The flow is governed by the Hele-Shaw equation, [3], which, if both fluids are Newtonian, is written as

$$\mathbf{u}_i = -\frac{b^2}{12\mu_i}\nabla P_i \tag{1}$$

where i refers to each of the two fluids, $\mathbf{u}_i$ is velocity averaged over the cell thickness b, μ_i dynamic viscosity, and P_i pressure. Usually, incompressibility is assumed $(\nabla \cdot \mathbf{u}_i = 0)$, which reduces Eq. (1) to a Laplacian equation for the pressure in both fluids

$$\nabla^2 P_i = 0. \tag{2}$$

Across the interface, the pressure is continuous if the two fluids are miscible; otherwise, a pressure jump condition should be met

$$P_1 - P_2 = \sigma \cdot \kappa \tag{3}$$

where σ is interfacial tension and κ the curvature of the interface projected onto the Hele-Shaw cell plane.

In most studies so far, the viscosity of the displacing fluid is neglected, which renders a uniform pressure distribution in its side. The interface then becomes part of boundaries enclosing the more viscous displaced fluid. As indicated by the experiments of Park and Homsy [4], Maxworthy [5], and Kopf-Sill and Homsy [6], in the absence of the displacing fluid viscosity the finger shape and morphology could be determined by a single parameter, the *modified capillary number*

$$Ca' = \frac{U\mu}{\sigma}\left(\frac{W}{b}\right)^2, \tag{4}$$

where U is the velocity of the finger, μ the viscosity of the displaced fluid, σ interfacial tension, W the half Hele-Shaw cell width, b the cell thickness. For low Ca', a single, long, steady finger called a *Saffman-Taylor finger* was obtained; for large Ca', unstable, branched fingers were reported, [4–6]. McLean and Saffman [7] solved for the steady finger width analytically as a function of Ca', but the mechanism, by which the finger asymptotically approaches half-cell width, was resolved later, [8,9].

Time-dependent direct simulations have also been developed to investigate both stable and unstable fingers. One approach is the *random-walker* method, of which good examples are given by Kadanoff [10], Liang [11], Tang [12], and by Arneodo and coworkers [13]. In this purely stochastic method, the solution of the Laplacian pressure equation, Eq. (2), is obtained by a probabilistic scheme. Particles released from one end of the cell, walk randomly until they encounter and stick to a cluster forming from the other end, which eventually grows into a finger-like pattern. Although the behavior of a random-walker is precisely formulated by the discretized Laplacian pressure equation, it is restricted to the case of vanishing displacing fluid viscosity. The approach can not be used to simulate viscous fingering under finite viscosity ratio between the two fluids.

Numerical solutions of the Eqs. (1)–(3) governing the viscous fingering phenomenon have been reported using boundary integral method by DeGregoria and Schwartz [14,15], Hou et al. [16,17], and Nie and Tian [18], the vortex-in-cell method by Tryggvason and Aref [19], and Meiburg and Homsy [20], and the volume tracking method by Whitaker [21]. In a particular approach of the boundary integral method, called the conformal mapping method (for example, [22]), the moving interface is transformed into a fixed boundary in a complex domain, and the interface velocities are derived from the complex boundary velocities. In the vortex-in-cell method, a vorticity form of the Hele-Shaw equation is used to obtain the interface as the only location that is not irrotational, and the velocities are reconstructed from the vorticities. Good agreement with McLean and Saffman's analytical solution, [7], for stable fingers was reported by using either of these two methods with zero displacing fluid viscosity, and the authors also extended the use of the method to selected unstable fingering cases.

Both of the above-mentioned methods feature a Lagrangian approach to tracking the interface. The volume tracking method, [21], on the other hand, employs an Eulerian formulation. Here "volume" refers to a value F assigned to each cell that forms a fixed mesh over the whole computational domain. Specifically, this value F represents the fraction of a cell that is occupied by the displacing fluid. A time-dependent advection equation, given below, is solved to obtain F throughout the computational domain, from which the interface is constructed through the cells having $0 < F < 1$.

$$\frac{\partial F}{\partial t} + \frac{\partial (uF)}{\partial x} + \frac{\partial (vF)}{\partial y} = 0 \tag{5}$$

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