FULL-STATE FEEDBACK CONTROL DESIGN WITH "DELAY SCHEDULING" FOR CART-and-PENDULUM DYNAMICS

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Abstract

A new approach is proposed to design fixed full-state feedback controllers for linear time invariant (LTI) systems with multiple time delays. This approach takes advantage of the recently introduced "delay scheduling" concept, which opens a new direction in synthesizing the control. "Delay scheduling" strategy suggests further prolonging the existing (and unavoidable) delays in order to recover stability or to improve the control performance features. To be able to do this, however, system should have multiple stable operating zones in the domain of the delays. The main contribution of this paper is to develop a procedure for designing a control law which facilitates such pockets. It starts with a selection of the feedback gains for non-delayed systems. We utilize a recent paradigm, Cluster Treatment of Characteristic Roots (CTCR), to examine the stability outlook when the delays are introduced in the dynamics. A scheme is also introduced to modify this gain structure so that the system exhibits a desirable set of stable pockets to facilitate the "delay scheduling". The paper describes the methodology, without loss of generality, on a fully-actuated cart-pendulum system. Relevant experiments are carried out to show the viability of the proposed idea.

Nomenclature

$\boldsymbol{x}$	Cart position (m)	$\boldsymbol{c}$	Viscous friction constant on pendulum hub (Nms/rad)
$\boldsymbol{\theta}$	Pendulum angle (rad)	$\boldsymbol{c_0}$	Coulomb friction torque on pendulum hub (Nm)
$\boldsymbol{x_d}$	Desired trajectory for x (m)	$\boldsymbol{K_{T1}}$	Torque constant of cart motor (Nm/A)
$\boldsymbol{\theta_d}$	Desired trajectory for θ (rad)	$\boldsymbol{K_{T2}}$	Torque constant of pendulum motor (Nm/A)
$\boldsymbol{r}$	Radius of cart driver pinion (m)	$\boldsymbol{F_{fric}}$	Nonlinear friction force on cart (N)
$\boldsymbol{I_1}$	Control current into the cart motor (A)	$\boldsymbol{T_{fric}}$	Nonlinear friction torque on pendulum hub (Nm)
$\boldsymbol{I_2}$	Control current into the pendulum motor (A)	$\boldsymbol{M}$	Mass of cart (kg)
$\boldsymbol{F}$	Force applied to cart (N)	$\boldsymbol{I}$	Pendulum moment of inertia about its CG (kgm^2)
$\boldsymbol{T}$	Torque on the pendulum hub (Nm)	$\boldsymbol{m}$	Mass of pendulum (kg)
$\boldsymbol{b}$	Viscous friction constant on cart (Ns/m)	$\boldsymbol{l}$	Half length of pendulum (m)
$\boldsymbol{b_0}$	Coulomb friction force on cart (N)	$\boldsymbol{g}$	Gravitational acceleration (m/s^2)

I. Introduction and Problem Statement

"Delay scheduling" is a relatively new concept as it is utilized for time-delay system stabilization [4, 5]. Assuming that a system has some unavoidable inherent time delays in the feedback control line, which beget unstable dynamics, the answer to the following question is very critical: Is it possible to find some larger delay compositions that recover the stability? The controller can artificially prolong the present delays simply by introducing "hold buffers" in the feedback line. These extended delays may re-establish the stability; they may also introduce more attractive performance features. This concept of selectively increasing the delays is called the "*delay scheduling*" from which some counterintuitive results arise, as explained later.

In order to be able to use "delay scheduling" in a systematic manner, one needs a tool that explicitly determines the stability robustness outlook of the system against uncertainties in the delays. Without such a tool, it is not possible to judge the corrective prolongations on the delays while still maintaining the stability. To respond to this need, a recent methodology, which is called *Cluster Treatment of Characteristic Roots (CTCR)* is deployed.

Since the main idea behind the "delay scheduling" lies in the existence of multiplicity of stability pockets in the domain of the delays, the following questions arise: Will every closed loop time-delayed system show multiple zones of stability in the domain of the delays? What if there is only one limited stable region and the inherent delays are not located in this pocket, thus the anticipated operation is unstable? Evidently, a practical strategy is needed to introduce multiplicity of stable operating zones in the space of delays. This paper introduces such a strategy for the time-delayed systems with fixed full-state feedback control laws. To better display the capabilities of the proposed concept, an example dynamics: fully-actuated cart-pendulum system; is taken without loss of generality of the treatment.

The control objective is to make the cart and the pendulum track two different desired trajectories. The actuators are two DC servo motors which are controlled in current modes. Distinctive delays are assumed to be present in the feedback lines and the idea is to schedule (i.e. prolong) these delays to some larger values than the current delays, while maintaining successful trajectory tracking in both degrees of freedom (x and θ).

The outline of the paper is as follows: In Section II, the cart-and-pendulum control system is described. Section III covers the discussions on the full-state feedback control design to facilitate the "delay scheduling" capability. In Section IV, example cases are studied and in section V, experimental verification of the highlights of this paper is presented.

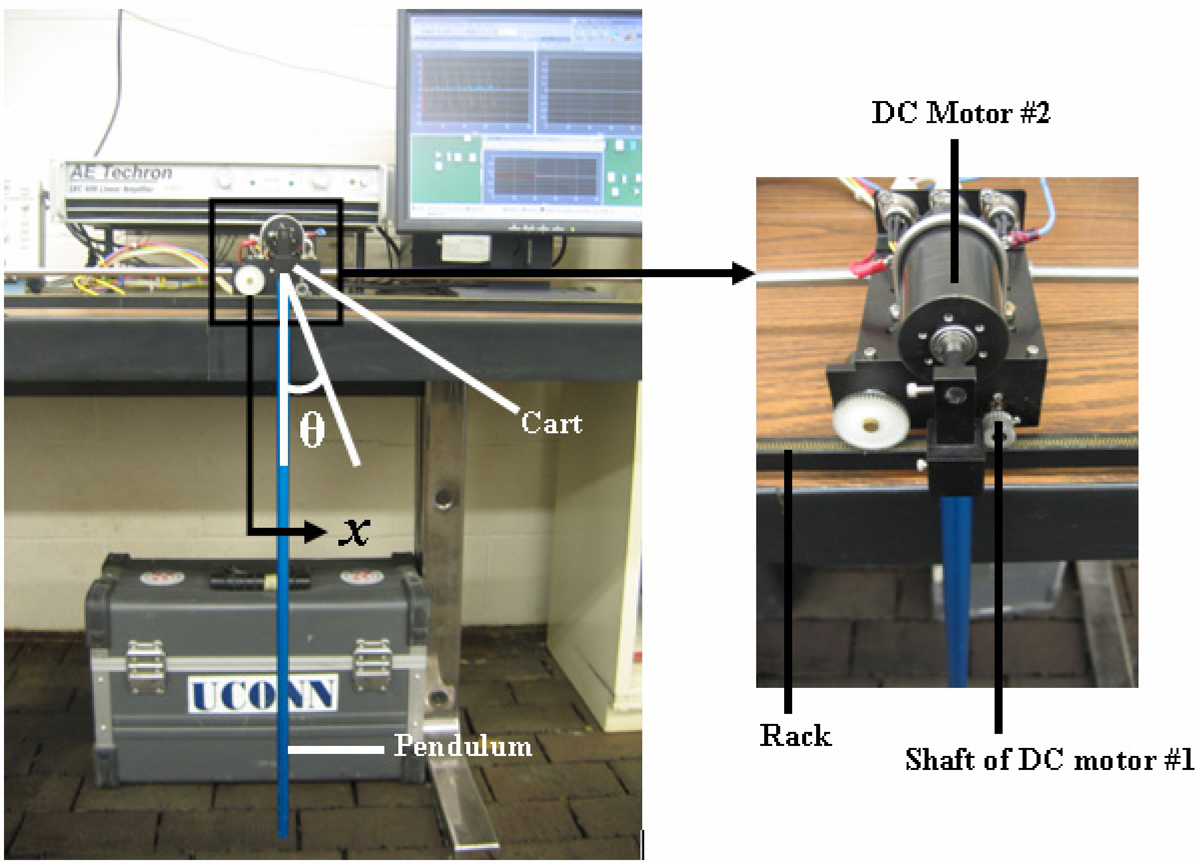

Fig. 1 - Fully-actuated cart-pendulum dynamics.

II. Dynamic Model of Cart-Pendulum System and Control Objective

The mathematical model of the cart-and-pendulum system is adapted from [6] as:

$$(M+m)\ddot{x}+m\ell\ddot{\theta}\cos\theta-m\ell\dot{\theta}^2\sin\theta+F_{fric}=F \quad (1)$$

$$(I+m\ell^2)\ddot{\theta}+T_{fric}+m\ell\ddot{x}\cos\theta+mg\ell\sin\theta=T \quad (2)$$

where $I = m\ell^2/3$ is the moment of inertia and all the other terms are as described in the nomenclature. The feedback control force (F) and control torque (T) are generated by two DC-servo motors #1 (of the cart) and #2 (of the pendulum), respectively. Considering small angular excursions for the pendulum, equations (1) and (2) are linearized as:

$$(M + m)\ddot{x} + m\ell\ddot{\theta} + F_{fric} = F \tag{3}$$

$$(I + m\ell^2)\ddot{\theta} + T_{fric} + m\ell\ddot{x} + mg\ell\theta = T \tag{4}$$

F and T are related to the corresponding motor currents via:

$$F = \frac{K_{T1} I_1}{r} \quad \text{and} \quad T = K_{T2} I_2 \tag{5}$$

where I_1 and I_2 are the motor control currents that are fed to the motors #1 and #2, respectively. The parameters of the test system are identified using conventional methods as listed below:

M=0.911 kg	b_0=0.25 N	K_t=0.045 Nm/A	r=0.68 cm
m=0.231 kg	c=0.0011 Nms/rad	K_e=0.022 Vs	I=0.00788 kgm^2
ℓ=0.32 m	c_0=0.0023 Nm	b=4.55 Ns/m	g=9.8 m/s^2

The objective is to design a feedback control law for I_1 and I_2. A critical requirement in this problem is the capability of this control law to be tolerant to some feedback delays, to make possible “delay scheduling” concept while maintaining a desirable tracking ability of cart and pendulum trajectories.

III. Controller Design For Time-Delayed Systems Using “Delay Scheduling”

The equations (3) and (4) are rewritten in state-space as:

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BI}_{\mathbf{in}} + \mathbf{E} \tag{6}$$

where $\mathbf{X} = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$ is the state, $\mathbf{I}_{in} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ is the controlled current input vectors.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{3mg}{(4M+m)} & \dfrac{-4b}{(4M+m)} & \dfrac{3c}{\ell(4M+m)} \\ 0 & \dfrac{-3g(m+M)}{\ell(4M+m)} & \dfrac{3b}{\ell(4M+m)} & \dfrac{-3c(M+m)}{m\ell^2(4M+m)} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{4K_{T1}}{r(4M+m)} & \dfrac{-3K_{T2}}{\ell(4M+m)} \\ \dfrac{-3K_{T1}}{\ell r(4M+m)} & \dfrac{3(M+m)K_{T2}}{m\ell^2(4M+m)} \end{bmatrix}$$ are

the state and control gain matrices, respectively. $\mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{3c_0 \operatorname{sgn}(\dot{\theta})}{\ell(4M+m)} - \dfrac{4b_0 \operatorname{sgn}(\dot{x})}{(4M+m)} \\ \dfrac{-3c_0(M+m)\operatorname{sgn}(\dot{\theta})}{m\ell^2(4M+m)} + \dfrac{3b_0 \operatorname{sgn}(\dot{x})}{\ell(4M+m)} \end{bmatrix}$ is the uncertain disturbance term which entails only the Coulomb friction from the cart and the pendulum hub as well as the gear-meshes. Notice that linear viscous friction components are kept in the matrix **A**.

The desired trajectories of the pendulum and the cart are taken as $\theta_d(t)$ and $x_d(t)$. The nominal (desired) control $\mathbf{I_d}$ is derived from the inverse dynamics of (6) considering zero Coulomb friction [10, 11].

$$\dot{\mathbf{X}}_\mathbf{d} = \mathbf{A}\mathbf{X}_\mathbf{d} + \mathbf{B}\mathbf{I}_\mathbf{d} \tag{7}$$

$$\text{where } \mathbf{I_d} = \begin{bmatrix} I_{1d} \\ I_{2d} \end{bmatrix} = \begin{bmatrix} \dfrac{r(M+m)}{K_{T1}}\ddot{x}_d + \dfrac{rb\dot{x}_d}{K_{T1}} + \dfrac{rm\ell\ddot{\theta}_d}{K_{T1}} \\ \dfrac{4m\ell^2\ddot{\theta}_d}{3K_{T2}} + \dfrac{m\ell\ddot{x}_d}{K_{T2}} + \dfrac{mg\ell\theta_d}{K_{T2}} + \dfrac{c\dot{\theta}_d}{K_{T2}} \end{bmatrix}$$

$\mathbf{I_d}$ is the feedforward control current vector whose elements (I_{1d} for motor #1, I_{2d} for motor #2) correspond to the computed torque values for each motor for the desired trajectories, $x_d(t)$ and $\theta_d(t)$, while ignoring the Coulomb friction terms.

Subtracting (6) from (7) yields the error dynamics:

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{B}(\mathbf{I_d} - \mathbf{I_{in}}) - \mathbf{E} = \mathbf{Ae} + \mathbf{Bu} - \mathbf{E} \tag{8}$$

where $\mathbf{e} = \begin{bmatrix} x_d - x \\ \theta_d - \theta \\ \dot{x}_d - \dot{x} \\ \dot{\theta}_d - \dot{\theta} \end{bmatrix}$ is the error vector. $\mathbf{u} = \mathbf{I_d} - \mathbf{I_{in}}$ is the intended full-state linear feedback control input which, will be constructed as $\mathbf{u} = -\mathbf{Ke}$. Fig. 2 depicts the formation of this control structure. The control composition is:

$$\mathbf{I_{in}} = \mathbf{I_d} + \mathbf{Ke} \tag{9}$$

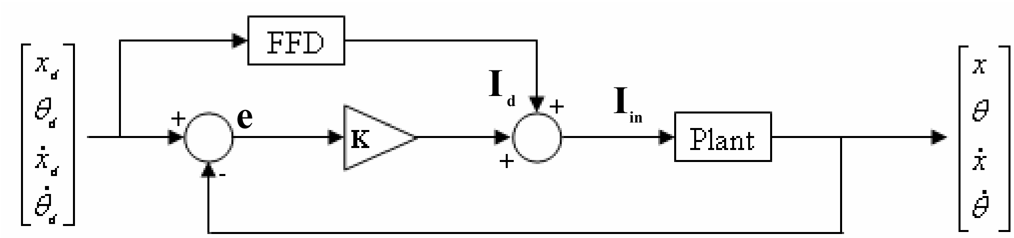

Fig. 2 - Feedforward and feedback control structure.

Full-state feedback control law (**K**) ensures the system to reject the disturbances in a satisfactory fashion, which are primarily due to Coulomb type friction in the cart or in the pendulum substructures. Obviously, this selection of **K** has a direct effect on the stability analysis of the system. When $\mathbf{K} = 0$ (i.e. uncontrolled system), the (cart-pendulum) dynamics is stable. When **K** is nonzero but small, the system is still expected to be stable. Even, when the delays appear in the feedback line, such systems with small feedback gains can be, what is called, "delay independent stable" [12]. An intuitive interpretation for this is, the control gain **K** is so feeble that regardless of the delay in the feedback, system always remains stable. As K increases, however, the stability becomes "delay dependent" and the stability is achieved only in some pockets of delays. The key question is how large can K be so that it will execute acceptable control performance while creating some tolerance to delays (i.e. in some pockets of delays)? This forms the main theme of the present paper.

We should stress that for a general time delayed system, there is no commonly accepted methodology in the literature that can be used to design a stabilizing control law. Well-known procedures such as LQR do not apply because of the infinite dimensionality of the system. In this paper, we start the controller design for the nondelayed dynamics for which the system is finite dimensional [4, 5, 9]. With this fixed control structure, we assess the stability robustness of the system against nonzero delays. This assessment is carried out utilizing the cluster treatment of characteristic roots (CTCR) paradigm. This procedure is based on two critical propositions regarding the general dynamics of LTI time delayed systems (LTI-TDS). For LTI-TDS with fixed control laws, the CTCR determines the stability robustness of the system in the semi-infinite delay space exhaustively, exactly and efficiently. Although we utilize the CTCR procedure heavily in this paper, we leave the details of it to concept papers [1-3].

In our cart-pendulum dynamics, we consider a delay, τ_1, in the feedback line of x, $\dot{x}$ and another delay, τ_2, for $\theta, \dot{\theta}$. We rewrite the error dynamics including these delays as:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\mathbf{K}_1\mathbf{e}(t-\tau_1) + \mathbf{B}\mathbf{K}_2\mathbf{e}(t-\tau_2) \tag{10}$$

where the control law is denoted by $\mathbf{u} = -\mathbf{K}_1\mathbf{e}(t-\tau_1) - \mathbf{K}_2\mathbf{e}(t-\tau_2)$ and $\mathbf{K}_1 + \mathbf{K}_2 = \mathbf{K}$ where **K** is earlier mentioned control gain matrix. Equation (10) represents a general class of linear-time invariant multiple time delayed system (LTI-MTDS). We consider all parameters known and constant, and query the stability robustness against uncertain delays, deploying the CTCR paradigm. It gives the complete stability picture of the system in the space of the delays. As discussed earlier the selected controller (which is designed for zero delay) may not result in multiple stability pockets that are desired for a "delay scheduling" procedure. Even worse, the inherent delays in the system may be located in the unstable zone.

We argued earlier that it is possible to obtain the preferred multiplicity of stable zones in the space of delays by making the feedback gain matrix, **K**, smaller. Reiterating the earlier discussions: If we select $\mathbf{K} = \mathbf{0}$ (i.e., there is no control action), then the closed loop system would be delay independent stable. If we select **K** based on zero delay, then the system may be stable in a very limited region in the delay space. The former case is unacceptable, and the latter may be too restrictive for the purposes of "delay scheduling". Our aim is to find a composition for K which is between these two extreme cases, such that the multiplicity of pockets with larger delays is created while the system performance is still satisfactory. Since this process requires repeated creation of the stability robustness picture, the tool used for this objective must be very efficient. The CTCR provides this feature.

With this point in mind, we pursue in the following steps:

(i) First we consider non-delayed case for (10) and determine an acceptable **K** matrix for the system. Although many different methods can be used for this selection of **K**, we simply follow an ad-hoc procedure for this paper.

(ii) We then deploy CTCR for this gain matrix, $\mathbf{K}$, and assess the stability chart in the delay space.

(iii) If the stability chart obtained in (ii) is not desirable (i.e., stability pockets are too restrictive), we reduce **K** by introducing a scaling constant, $\alpha < 1$, which multiplies **K** and the resultant gain matrix is denoted by $\mathbf{K'}$ as follows:

$$\mathbf{K'} = \alpha\mathbf{K} \quad \text{where } \alpha < 1 \tag{11}$$

We then repeat (ii) for the new gain matrix $\mathbf{K'}$.

(iv) The procedure is stopped when the stability chart of (ii) is acceptable for the most likely composition of delays and for the deployment of "delay scheduling".

These steps are studied in the following two examples.

IV. Example Case Studies

i) Case I:

For the fully-actuated cart-pendulum system, we make an ad-hoc selection for the control gain matrix (**K**) as which provides acceptable experimental properties.

$$\mathbf{K} = \begin{bmatrix} 40 & 0 & 1 & 0 \\ 0 & 85 & 0 & 1 \end{bmatrix} \tag{12}$$

The split gains corresponding to different feedback lines in Eq. (10) are $\mathbf{K}_1 = \begin{bmatrix} 40 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{K}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 85 & 0 & 1 \end{bmatrix}$.

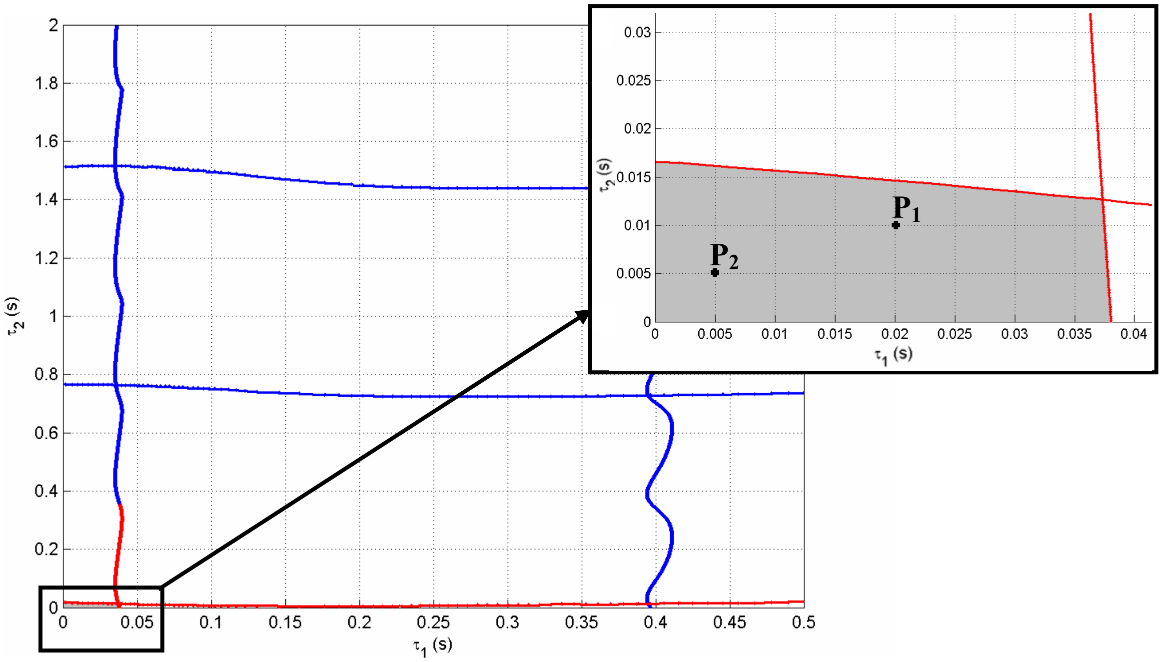

Fig. 3 - Stability map for case I.

The CTCR procedure creates the stability robustness picture as in Fig. 3. Expectedly, there is only one small stable operating region which is shaded (can be seen in the zoomed inset) If, for instance, the inherent delay composition is ($\tau_1 = 0.2\text{s}, \tau_2 = 1\,\text{s}$), the system is unstable. The objective is to design a feedback control structure so that these delays still yield desirable performance.

ii) Case II:

For the same system in example case I, control gain matrix (**K**) is altered according to the proposed procedure using $\alpha = 0.1$ (i.e., new control gain matrix is selected as ten percent of the one in case I).

$$\mathbf{K}' = \begin{bmatrix} 4 & 0 & 0.1 & 0 \\ 0 & 8.5 & 0 & 0.1 \end{bmatrix} \tag{13}$$

and using the splitting format of this gain as discussed earlier, one obtains $\mathbf{K}_1 = \begin{bmatrix} 4 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

and $\mathbf{K}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 8.5 & 0 & 0.1 \end{bmatrix}$. The CTCR procedure creates the corresponding stability outlook as displayed in Fig. 4 (shaded regions are stable).

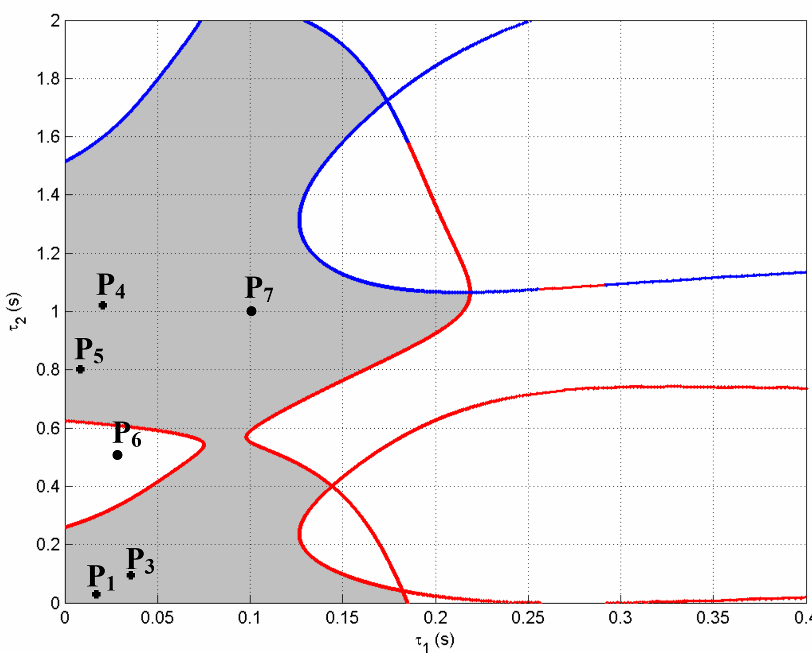

Fig. 4 - Stability map for case II.

As argued earlier, the reduced feedback gains result in considerably larger stable zone. For instance, contrary to case I, delay pair ($\tau_1 = 0.2\text{s}, \tau_2 = 1\,\text{s}$) renders stable behavior this time. With the precise description of this stability map, we can now deploy the "delay scheduling" procedure. If the existing delays are in an unstable region, one has an option to increase the delays artificially to bring them into the next stable zone. For example, if present delay pair is P_6 ($\tau_1 = 0.03\text{s}, \tau_2 = 0.5\,\text{s}$), it is possible to prolong the delays to P_7 ($\tau_1 = 0.1\text{s}, \tau_2 = 1\text{s}$) in order to have stable operation. Another interesting question is how to select the best delay composition in a stable region? The answer to that question is provided in the following section.

V. EXPERIMENTAL STUDY

Analytically determined stability features of the system as depicted in Fig. 4 are also experimentally validated in this section. The digital control structure utilized in our test set-up has the following features: sampling speed is 1 kHz, inherent loop delay within the system (from sensing to actuation) is 5 ms, that is, the control loop has a minimum 5 ms unavoidable delay in both channels. The resolution of the angular motion (θ) is 0.088° which corresponds to an encoder which has 4096 pulses/rev. Another similar encoder measures the cart position (x) with a resolution of 23 μm. A filtered differentiator whose bandwidth is much larger than the bandwidth of the system is deployed in order to obtain velocities $\dot{x}$ and $\dot{\theta}$ from the respective encoder measurements. Selections of the desired trajectories are made such that both actuators remain within their linear range and without an occurrence of saturation throughout the tests.

We report a number of tests which validate the proposed method of designing full-state feedback using "delay scheduling" concept.

i) Trajectory tracking performance:

Analytically determined stability outlook of the system as depicted in example cases i and ii are validated in this section via experiments. The objective of the control, again, is to track a desired trajectory $x_d(t)$ and $\theta_d(t)$ successfully. As mentioned before, cart's position and velocity feedback are provided with a time delay, τ_1, while the pendulum's angle and angular velocity feedback arrive with another delay, τ_2, throughout the exercise. The desired trajectories are taken as:

$$x_d(t) = 10\sin(0.8\pi t)\ \text{cm} \quad \text{and} \quad \theta_d(t) = 10\sin(1.2\pi t)\ ^\circ \tag{14}$$

For an experiment-based comparison, a common stable operating point P_1 ($\tau_1 = 20ms, \tau_2 = 10ms$) on Figs. 3 and 4 is selected. The results are shown in Fig. 5 for case i and in Fig. 6 for case ii.

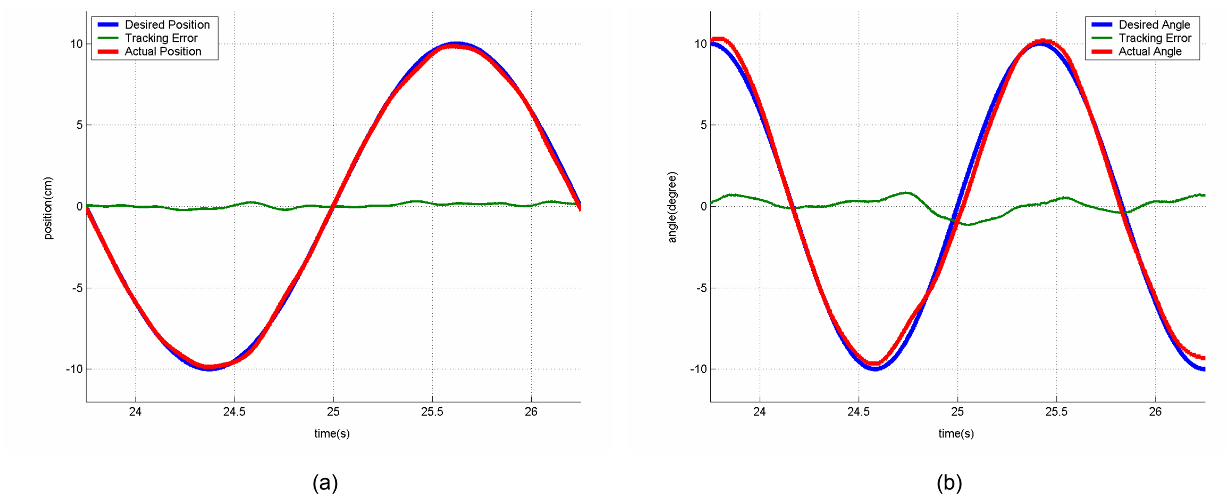

Fig. 5 - (a) position (x) and (b) angle (θ) variations during the trajectory tracking for point P_1 on Fig. 3.

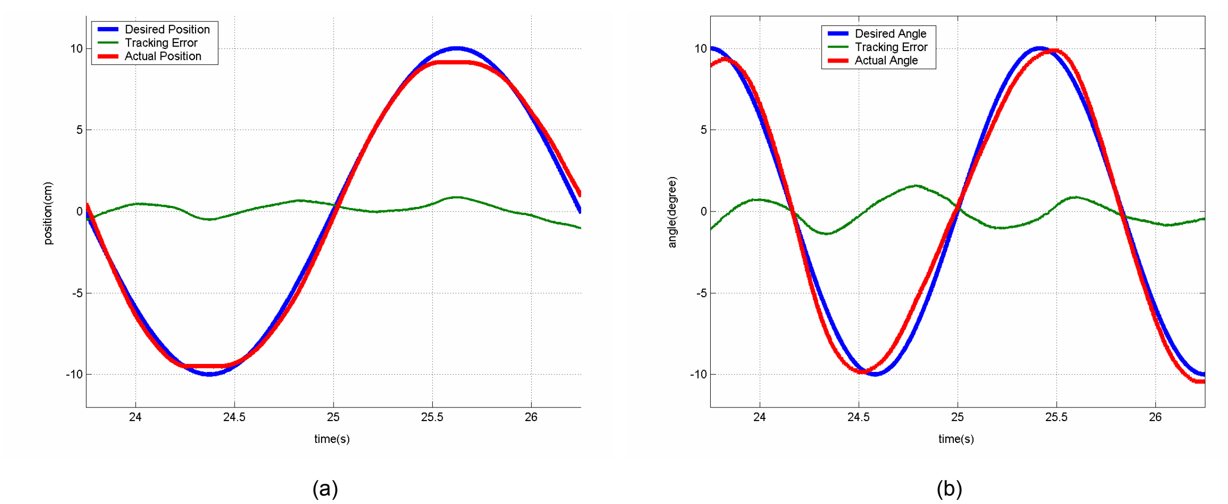

Fig. 6 - (a) cart position and (b) pendulum angle variations during the trajectory tracking for point P_1 on Fig. 4.

Both Fig. 5a and Fig. 6a display a very good tracking ability of the cart motion (as blue and red lines coincide for the most part). Clearly, lower gain controller in case ii caused worse trajectory tracking performance but the errors are still in acceptable levels. This occurrence is a typical compromise between robustness against large delays and trajectory tracking ability. Figs. 5b and 6b display the similar comparative picture for the pendulum angle.

ii) Disturbance Rejection Performance:

Control performance variations depending on the control gain matrix are also numerically compared for the stability maps of Figs. 3 and 4. Recently developed numerical approximation codes [7, 8] are used to evaluate the rightmost characteristic roots of the system for a given point in (τ_1, τ_2) space. The corresponding time constants, γ_{dominant}, are directly related to the disturbance rejection speed:

$$\gamma_{\text{dominant}} = \frac{1}{\left|\Re e(s_{\text{dominant}})\right|} \tag{15}$$

where s_{dominant} represents the rightmost characteristic root. For the two example cases, $\Re e(s_{\text{dominant}})$ distributions in the delay space are given in Figs. 7 and 8 with color coding.

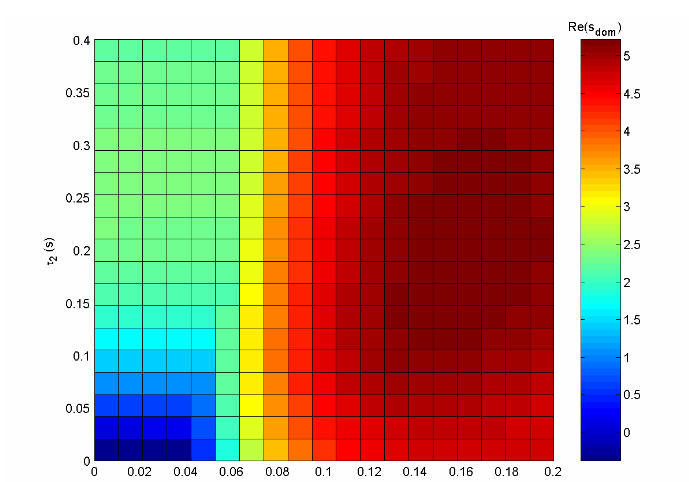

Fig. 7 - $\Re e(s_{\text{dominant}})$ distribution in the delay space for case i.

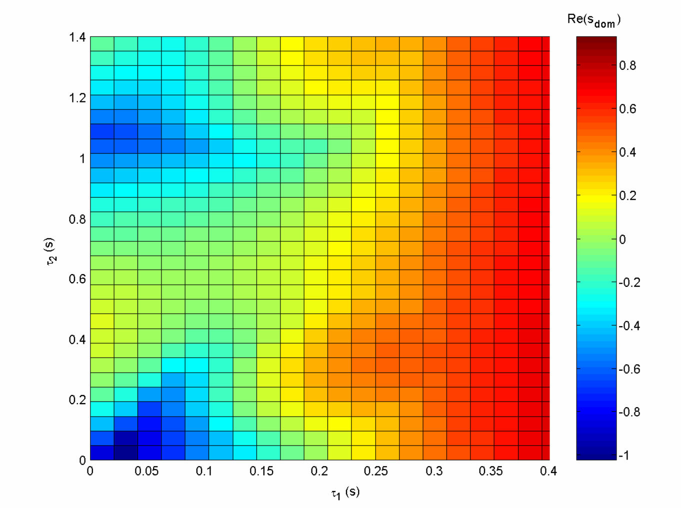

Fig. 8 - $\Re e(s_{\text{dominant}})$ distribution in the delay space for case ii.

We can extract two important details from these figures. (i) The stable zones (designated by $\Re e(s_{\text{dominant}}) < 0$) and unstable zones ($\Re e(s_{\text{dominant}}) > 0$) in Figs. 7 and 8 agree with the CTCR generated exact stability maps of Figs. 3 and 4, respectively. (ii) The fastest settling time appears at the point where $\Re e(s_{\text{dominant}})$ is the smallest negative number. For case i, this point is the zero delay case (obviously in reality this is the point with inherent delays) marked with P_2 ($\tau_1 = 0.005\text{s}, \tau_2 = 0.005\,\text{s}$) on Fig. 3 since the inherent time delay in the system is 5 ms and corresponding $\Re e(s_{\text{dominant}})$ is -0.38. In the meanwhile for case ii, it's at P_3 ($\tau_1 = 0.03s, \tau_2 = 0.07s$) on Fig. 4, where $\Re e(s_{\text{dominant}})$ is -1.02. That means with lower gains, not only extension of stable zone is achieved but also better settling time (i.e., better disturbance rejection speed) is achieved for increased time delays. Furthermore, for the point P_4($\tau_1 = 0.025s, \tau_2 = 1.06s$) on Fig. 4, $\Re e(s_{\text{dominant}})$ is -0.8 which shows very close settling time compared to P_3. In conclusion, the case with lower gains can accommodate much larger delays and even with improved performance. Therefore, the system is perfectly amenable to "delay scheduling" with the smaller gains.

For the stability map in Fig. 4, the corresponding rightmost pole distribution is given in Fig. 8. By considering this figure, it is possible to find the best delay composition in the same stable zone for re-scheduling. For example, if the present delays are P_5 ($\tau_1 = 0.01s, \tau_2 = 0.8s$), designer can improve the disturbance rejection ability by prolonging (i.e., rescheduling them) them to P_4 ($\tau_1 = 0.02s, \tau_2 = 1.06s$). This demonstrates how "delay scheduling" advances the control performance. We test these disturbance rejection features at operating points P_3 and P_4 by applying an impulsive torque via the DC motor of the cart while both x and θ are following their individual desired trajectories. In agreement with the above mentioned rightmost pole locations at each point, Fig. 9 displays similar disturbance rejections.

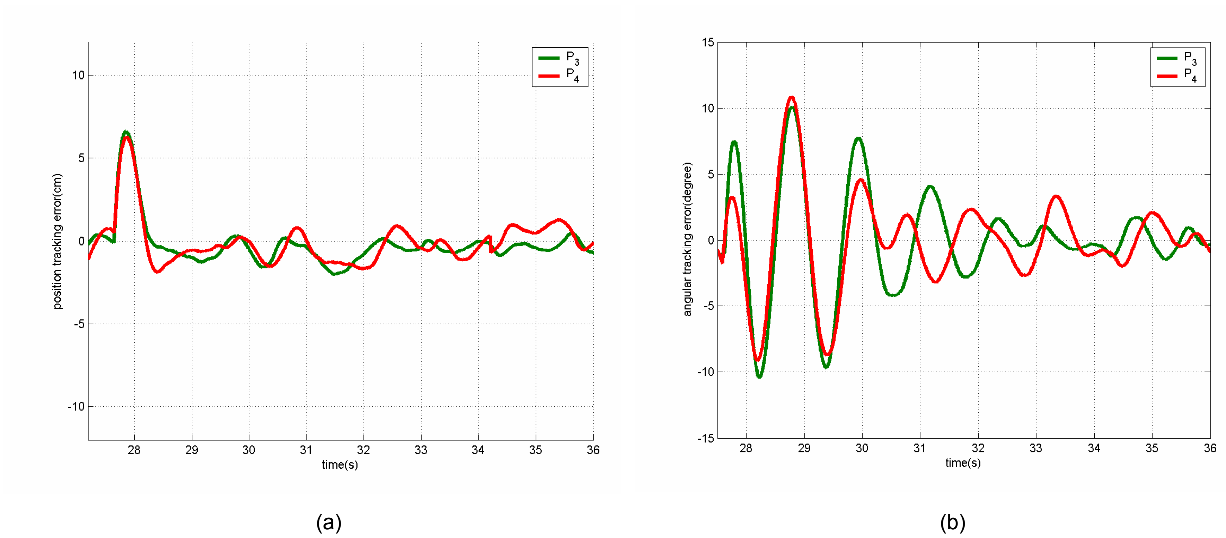

Fig. 9 - Tracking errors in (a) position and (b) angle for the operating points P_3 and P_4 on Fig. 4, under impulsive disturbance.

Conclusion

For the LTI systems with multiple delays, newly introduced “delay scheduling” procedure is shown to be intriguingly advantageous in terms of recovering stability or even having better control performance features. If there is an unavoidable delay composition that renders unstable behavior, the “delay scheduling” suggests increasing the delays further up to the next stable zone in order to recover stability. The contribution in this paper is a novel full-state feedback control design strategy in order to obtain multiple stability pockets in the domain of the delays. This strategy directly supports the “delay scheduling” concept. Two example cases are studied over a fully-actuated cart-pendulum dynamics with two feedback delays both analytically and experimentally.

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