DELAY-DECOUPLING CONTROL, A NOVEL METHOD FOR MIMO SYSTEMS WITH MULTIPLE INPUT DELAYS

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Abstract: A novel control logic is introduced for linear time-invariant minimum phase MIMO plants with multiple control delays. It is shown that the application of this control law generates a delay decoupling effect within the characteristic equation. Ultimately the delays individually appear in factorized form without a delay cross-talk effect or commensurate delay formation in each one of the factors. Both of these features make the stability analysis extremely easy, which is the main novelty in this method. Additionally, the structure of the controller empowers the designer to achieve desired tracking performance as well as improved disturbance rejection. As an example, the method is applied to a fourth order MIMO plant which has a rigid body mode, and two rationally independent delays. The delay decoupling control logic effectively achieves the desired trajectory tracking. We also extend the study to address the issues involving the robustness of the procedure.

Keywords: Time Delay Systems, MIMO, Delay-Decoupling Control, Coprime Factorization.

1. INTRODUCTION

Stability analysis of time delay systems both for retarded and neutral classes is extensively investigated, especially in recent years [Olgac and Sipahi, 2005a; Gu et al., 2003; Niculescu, 2001 and references therein]. The problem becomes very complex for systems with multiple rationally independent delays particularly when the delays have commensurate features (i.e., existence of integer multiples of any delay in the characteristic equation) or when cross-talk between delays exists (i.e., the linear combination of the delays occurs), [Olgac and Sipahi, 2005b; Sipahi and Olgac, 2006; Fazelinia et al., 2007].

The topic of this paper is considerably different from the earlier effort. It is on the *synthesis of a stabilizing controller* for the systems, which have multiple feedback delays. Obviously, this control logic must assure stability, and provide desirable tracking as well as disturbance rejection capabilities. In the literature, we encounter a number of control strategies which are developed for time-delayed systems. A PI type feedback stabilization of first-order systems with single time delay is studied in [Silva et al., 2001]. For MIMO systems with multiple delays this task is more intricate [Gundes et al., 2007], where the authors utilize conservative H_∞ tools. Other researchers have focused on the design of the controllers to eliminate the cross-interaction between the components of the controller and the states [Liu et al., 2007; Wang et al., 2002]. In both investigations, the small gain theorem is applied for robust stability analysis. However, the exercise is performed in point-wise fashion, i.e., stability is assured for selected values of delays. Therefore, the effort is computationally very costly.

Although all of these control strategies have some interesting and novel contributions, none of them can prevent the formation of delay cross-talk (which are represented by the terms containing multiplicity of delays together) or commensurate delays (which are formed by integer multipliers of individual delays) in the system characteristic equation. Just as a qualitative remark, even when there is no commensurate and cross-talking delays in the characteristic equation, the stability study of the closed-loop system with multiple delays is quite complex [Hale 1993; Gu et al. 2005].

Herein, we propose a novel control law which will avoid these complications for LTI minimum phase MIMO systems with multiple input delays. Application of this controller results in complete delay decoupling within the characteristic equation. That is, a multiplicative part of the characteristic equation which is influenced by delays is represented as a product of several single-time-delay dynamics. This feature provides a great convenience from the stability analysis point-of-view. Because of this property, we name the technique "**Delay-Decoupling Control**" (DDC). The key operation behind this control logic is the coprime factorization which is implemented over the plant dynamics. The DDC logic applies without serious restrictions. Even for cases with unstable plants, the control law is successfully generated as we explain in the text. The designer can further select the controller dynamics to achieve the desired tracking and disturbance rejection performance.

The paper is composed of the following sections. Section 2 describes the synthesis problem. In Section 3, the delay-decoupling control law (DDC) is introduced. The application of the new controller to an unstable second order SISO system is given in section 4, as well as a case study on a fourth order cart-pendulum plant. Some improvements on the tracking performance of the second case study are also presented. Finally, the concluding remarks are made.

2. PROBLEM STATEMENT

Consider the system shown in Fig. 1. $G(s)$ and $C(s)$ are the transfer matrices of the plant and the controller, respectively, with rational polynomial entries as the terms of the matrices. They are square matrices of appropriate dimensions. The diagonal matrix Λ represents the input delay dynamics. It implies that different segments of the control, U, are carrying different time delays, which is very common in practice.

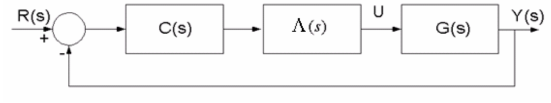

Fig. 1. Unity feedback MIMO system with control input delays

The elements of the matrices $G(s)$ and $C(s)$ are represented by $g_{ij}(s)$, $c_{ij}(s)$, i= 1, …, n, j= 1, …, n and $\Lambda(s) = diag(e^{-\tau_i s})$ where $\tau_i \in \Re^+$, i= 1, …, n are the n independent delays.

The equivalent transfer matrix between the reference input and output of the closed-loop system is

$$H(s) = (I + G\Lambda C)^{-1} G\Lambda C \quad (1)$$

where I is the n-dimensional identity matrix. The corresponding characteristic equation of this system can be written as

$$CE = |I + G\Lambda C| \times (Non - delayed\ polynomial) = P_1(s,\tau) \times P_2(s) \quad (2)$$

In this equation, the term $|I + G\Lambda C|$ is self-evident and it is the only term which contains delays while the denominators of g_{ij} and c_{ij} create the non-delayed polynomial $P_2(s)$. Typically the $P_1(s,\tau)$ term is complex, especially due to the delay cross-coupling elements (which are also called the delay cross-talks). The stability assessment of such a dynamics is one of the open problems in the field of time-delayed systems [Sipahi and Olgac 2006]. The stability robustness analysis of this class of systems against delay uncertainties, when there are two delays (and even with three delays), is still manageable. But we have encountered no case studies with higher number of delays to date. Obviously, absent a broader stability analysis methodology, the synthesis of a controller is impossible. In this paper, we propose a new approach through which one can form a control logic that has the capability of decoupling the effects of individual delay terms when it comes to the stability question. Therefore, the name "**Delay-Decoupling Control**, **(DDC)**" is adopted. The following section describes the procedure for the design of this controller.

3. CONTROLLER DESIGN STRATEGY

We start the process by rewriting the plant transfer matrix G as a product of two matrices

$$G(s) = A(s)B(s) \quad (3)$$

where $A_{n\times n}(s)$ must be made of elements which are proper or improper (but not strictly proper) transfer functions, and $B_{n\times n}(s)$ must be diagonal. This is a right coprime factorization operation, and it is essential for the main lemma which is presented next.

<u>Main lemma</u>:
For the minimum phase MIMO plant "$G(s)$" in equation (3), with control delays, "$\Lambda = diag(e^{-\tau_i s})$" in Fig. 1, the selection of control logic

$$C(s) = K(s)A^{-1}(s) \quad (4)$$

where $K_{n\times n}(s)$ is a diagonal transfer matrix and $A^{-1}(s)$ is proper (or strictly proper), results in an advantageous property. This control logic completely decouples the delays within the characteristic equation of the system. The delays in $P_1(s,\tau)$ term in (2) consist of n factors, each of which contains one of the delays only.

Proof:

The substitution of the proposed controller in the characteristic equation of (1), forms the delay influenced segment, $P_1(s,\tau)$, as

$$\begin{aligned} P_1(s,\tau) &= \left|I + AB\Lambda KA^{-1}\right| = \left|AA^{-1} + AB\Lambda KA^{-1}\right| \\ &= \left|A(I + B\Lambda K)A^{-1}\right| = \left|I + B\Lambda K\right| \end{aligned} \tag{5}$$

Based on the fact that the matrix $B\Lambda K$ is diagonal, $P_1(s,\tau)$ expression can be represented in a factorized form as

$$P_1(s,\tau) = \prod_{i=1}^{n}(1 + B_i K_i e^{-\tau_i s}) \tag{6}$$

where B_i and K_i are the i^{th} diagonal term of the respective matrices. Finally, the entire characteristic equation of the closed-loop system becomes

$$CE(s,\tau) = P_2(s)\prod_{i=1}^{n}(1 + B_i K_i e^{-\tau_i s}) = 0 \tag{7}$$

The influence of the delays τ_i, $i = 1,2,...,n$ on the system stability is, therefore, decoupled within the characteristic equation. That is, in order for the quasi-polynomial of (7) to be stable, every one of the n multipliers must be stable. Since each one of the n multipliers is dependent on one single delay only, the effects of the delays are decoupled. Furthermore, each one of these delays appears with no commensurate formation, which is a further advantage for the stability analysis.

QED ◊

In summary, the proposed controller, as given in (4), has the following properties:

1) It removes the cross-talk among the delays completely. It also reshapes the delayed part of the characteristic equation into a product of "*n*" single-delay expressions, with no commensurate terms. The stability analysis of the closed-loop system becomes substantially simplified. There are many procedures in the literature to achieve this. For instance, the authors' group has recently introduced a very efficient methodology, which is called the **Cluster Treatment of Characteristic Roots (CTCR)**, to address this class of stability problems [Olgac and Sipahi 2002, 2005b, 2006; Fazelinia et. al., 2007]. For each of the delays, the **CTCR** procedure creates an exact and complete set of delay intervals where the stability is assured. The method is numerically very efficient, can be executed in real time, and it also offers a closed form expression for the number of unstable roots for any setting of the delays. The combination of **DDC (Delay-Decoupling Control)** philosophy and the **CTCR** procedure creates a very effective methodology to determine the stable operating points of the system within the domain of the *n* delays.

2) The designer has still the option of selecting the matrix $K_{n\times n}(s)$, to achieve the desired performance as we describe in the examples section next. Obviously, the stability and trajectory tracking capabilities of the controlled system (Fig. 1.) are directly influenced by the selection of $K_{n\times n}(s)$. This selection, however, does not change the delay decoupling property mentioned above. For instance, one can opt for a higher order filtering dynamics in $K_{n\times n}(s)$ to smoothen the control action. One may also select these elements to shape the frequency response properties of the closed-loop system.

4. CASE STUDIES ON DDC

Example 1:

This example is given just to show the influence of *K(s)* in the closed-loop behavior of the system. We select, purposely a single delay dynamics, thus putting aside the delay decoupling aspect from the stabilizing controller design and controller performance. Consider the following unstable SISO plant.

$$G(s) = \frac{1}{s^2 - 0.1s + 2} \tag{8}$$

This plant can not be stabilized by implementation of a conventional P (proportional) controller when there is no delay. We will pursue here, with the main idea of the delay-decoupling control strictly for the stabilization of the delayed structure and its performance aspects. This resembles the "delay scheduling method" of [Olgac et al. 2005], where the additional delay is injected into the controller as a stabilizing tool. Such a proposition is counterintuitive, but it can function nicely as shown in the cited literature.

For this example, coprime factorization of $G(s)$ is simply written as

$$A(s) = 1 \tag{9}$$

$$B(s) = \frac{1}{s^2 - 0.1s + 2} \tag{10}$$

The controller according to DDC proposition in (4), including a control delay, becomes:

$$C(s) = K(s)A^{-1}e^{-\tau s} = K(s)e^{-\tau s} \tag{11}$$

which results in the following closed-loop transfer function

$$H(s) = \frac{K(s)e^{-\tau s}}{s^2 - 0.1s + 2 + K(s)e^{-\tau s}} \tag{12}$$

With an appropriate $K(s)$ such as P, PD or PID (proportional, integral and derivative) controller, this system can be stabilized and it performs well. For instance, one can select $K(s) = k_p$ (simple P-controller). The stability of the closed-loop system can easily be analyzed using the CTCR method

[Olgac and Sipahi, 2002, Sipahi and Olgac 2006]. For any given controller structure in (11) and the associated delay, CTCR can declare the stability of the system with a straightforward formula [equation (20) of Olgac and Sipahi 2002]. As an end product we can present a graphical display of the stability variation in the space of k_p v. τ in Fig. 2, which shows the shaded stable operating regions.

The conclusion from this example is two fold: a) One can stabilize this dynamics using time-delayed P-control, even though the non-delayed control can not stabilize the system. It is visible from Fig. 2, that there is no stability region on the axis with zero time delay, τ, and there are several of them for larger delays, b) The CTCR procedure generates the stability outlook of Fig. 2, exactly and efficiently.

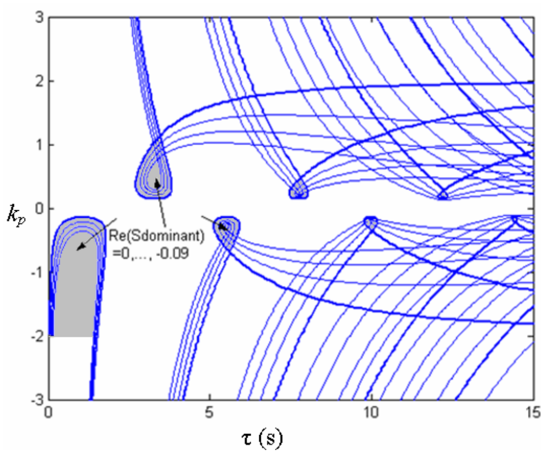

Fig. 2. Stable regions (shaded) for the system in (12)

Figure 2 also displays the trajectories in (τ, k_p) domain for which the Re($s_{dominant}$) are fixed values. The variations are marked from 0 to -0.09. The stability margin of the controller is improved as the operating point moves in the marked directions.

Stability robustness against uncertain time delays in the system is resolved using the CTCR paradigm as depicted in Fig. 2. We address next, the question of parametric uncertainty in the dynamics. Numerical efficiency of CTCR offers an ideal platform for this study. Namely, we can numerically investigate the stability robustness features of the system. In the parameter space let us demonstrate this capability on the example case study at hand.

If the stiffness parameter 2 is uncertain and its nominal value has ±10% fluctuations (i.e., 2 varies between 1.8 and 2.2), the stability picture of Fig. 2 can be recreated as in Fig. 3. Noting the fact that the root continuity with respect to parameters is guaranteed for retarded TDS [Hale 1993], the stability bounds in (τ, k_p) domain also exhibit a continuum. Therefore, when the stability regions are superimposed as in Fig. 3, one can claim the operating conditions of (τ, k_p) which offer robustly stable system. Those regions are shaded on Fig. 3.

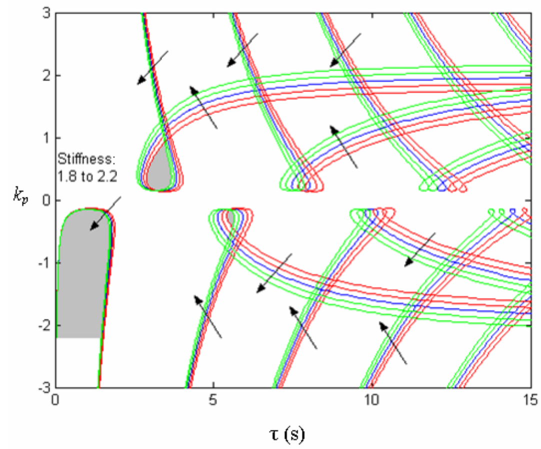

Fig. 3. Robust stability regions (shaded) for variation of stiffness 2 ± 10%

We state two expected observations from this study:

(i) It is trivially demonstrated (via simulation) but not given in the text, that the shaded regions of Fig. 3 offer robust stability.

(ii) These robustly stable regions are increasingly more restricted in comparison to certain (i.e., known) parameter case (of Fig. 2).

We should also make a remark that for increasing number of uncertain parameters, such an effort becomes numerically very costly. Nevertheless, it is a non-conservative robustness assessment tool. In that regard we consider CTCR as a novel contribution to this class of studies.

<u>Example 2:</u>

This example is given to demonstrate the full capability of DDC. Consider the cart-pendulum system shown in Fig. 4. The goal of this problem is to control the position of the cart "$x(t)$" and the angular displacement of the pendulum "$\theta(t)$" by deploying an appropriate feedback control to create force "F" and torque "T", both of which are executed with time delays.

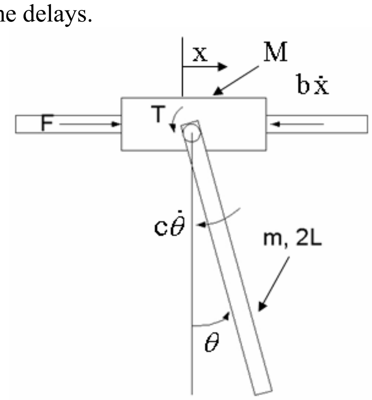

Fig. 4. Cart-pendulum system

The linearized equations of motion (for small θ excursions) of the system are borrowed from (Franklin et al., 2005) as

$$(M+m)\ddot{x}+b\dot{x}+mL\ddot{\theta}=F(t) \quad (13)$$

$$(I+mL^2)\ddot{\theta}+c\dot{\theta}+mgL\theta+mL\ddot{x}=T(t) \quad (14)$$

Laplace transformed version of them is given in (15, 16)

$$\begin{bmatrix} X(s) \\ \Theta(s) \end{bmatrix} = G(s)\begin{bmatrix} F(s) \\ T(s) \end{bmatrix} \quad (16)$$

where $G(s)$ is the self-evident (2×2) transfer matrix.

It is assumed that the force and torque are applied with transmission delays τ_1 and τ_2, respectively. The delay dynamics on individual feedback lines is represented by the matrix

$$\Lambda(s)=\begin{bmatrix} e^{-\tau_1 s} & 0 \\ 0 & e^{-\tau_2 s} \end{bmatrix} \quad (17)$$

which leads to the input-output relation of

$$\begin{bmatrix} X(s) \\ \Theta(s) \end{bmatrix} = G(s)\Lambda(s)\begin{bmatrix} F(s) \\ T(s) \end{bmatrix} \quad (18)$$

To find a stabilizing controller using the DDC method, the coprime factorization of the transfer matrix $G(s)$ is written as follows (19, 20)

$$A=\begin{bmatrix} (I+mL^2)s^2+cs+mgL & -mLs \\ -mLs^2 & (M+m)s+b \end{bmatrix} \quad (19)$$

It is obvious that the proposed coprime factorization is not unique, and the designer can certainly select other configurations. The proposed control law according to (4) is formed as (21).

As mentioned before, this method gives the designer the capability of selecting the matrix $K(s)$ to obtain a desired performance (such as better tracking and disturbance rejection). For instance the $K(s)$ factor in the control logic of (4) can be chosen as:

$$K(s)=\begin{bmatrix} k_1 & 0 \\ 0 & 1.1+\frac{k_2}{s}+0.13s \end{bmatrix} \quad (22)$$

which implies the P and PID class controllers on the first and second channels, respectively. k_1 and k_2 are the two free parameters to be decided based on the system stability analysis.

With this construct, we demonstrate next that, the closed-loop system can track the unit step reference inputs on either x or θ, successfully. The corresponding characteristic equation becomes:

$$\begin{aligned} CE &= [(MI+mI+MmL^2)s^4+(bI+bmL^2+cm+cM)s^3 \\ &+(mMgL+m^2gL+bc)s^2+bmgLs+k_1e^{-\tau_1 s}] \\ &\times[(MI+mI+MmL^2)s^3+(bI+bmL^2+cm+cM)s^2 \\ &+(mMgL+m^2gL+bc)s+bmgL+(1.1+\frac{k_2}{s}+0.13s)e^{-\tau_2 s}] \end{aligned} \quad (23)$$

Obviously, this equation contains a product of two factors which contain only one delay element in each. This formation is achieved as the most critical contribution of the DDC controller. It imparts the separation (decoupling) of the stability analysis of the two-delay system into two single-delay systems. The stability of the closed-loop dynamics requires the stability of both parts of the *CE*.

These parts have two independent single delay dynamics. The stability of each factor in (23) for fixed gains (k_1 or k_2) can easily be analyzed in $\boldsymbol{\tau}\in\Re^{2+}$ domain using the CTCR methodology [Olgac and Sipahi, 2002, Sipahi and Olgac 2006]. One can then sweep the gains, to create the complete stability picture in (τ_i, k_i) space (i = 1, 2).

For a numerical example, let us take the physical parameters of the system as: $m=0.231\ kg$, $M=0.911\ kg$, $L=0.32\ m$, $c=6\times10^{-4}\ Nm.s/rad$ and $b=8.4\ N.s/m$. We display the result of the CTCR procedure in the form of stability charts given in the planes of (τ_1,k_1) and (τ_2,k_2) as the generic parameter set. CTCR provides stability regions (shaded) in Figs. 5a and 5b.

In these figures, the curves represent exhaustively, the locations where the system possesses imaginary spectra. Again the shaded regions contain the stable operating points. Furthermore the application of the Routh's array gives the conditions, $0<k_1<3.0322$ and $0<k_2<3.6741$ to guarantee the stability of the delay-free case ($\tau_1,\tau_2=0$), which is in agreement with the figures.

$$\begin{bmatrix} X(s) \\ \Theta(s) \end{bmatrix} = \frac{\begin{bmatrix} (I+mL^2)s^2+cs+mgL & -mLs^2 \\ -mLs^2 & (M+m)s^2+bs \end{bmatrix}}{(MI+mI+MmL^2)s^4+(bI+bmL^2+cm+cM)s^3+(mMgL+m^2gL+bc)s^2+bmgLs}\begin{bmatrix} F(s) \\ T(s) \end{bmatrix} \quad (15)$$

$$B=\frac{\begin{bmatrix} \frac{1}{s} & 0 \\ 0 & 1 \end{bmatrix}}{(MI+mI+MmL^2)s^3+(bI+bmL^2+cm+cM)s^2+(mMgL+m^2gL+bc)s+bmgL} \quad (20)$$

$$C(s)=K(s)\frac{\begin{bmatrix} (M+m)s+b & mLs \\ mLs^2 & (I+mL^2)s^2+cs+mgL \end{bmatrix}}{(MI+mI+MmL^2)s^3+(bI+bmL^2+cm+cM)s^2+(mMgL+m^2gL+bc)s+bmgL} \quad (21)$$

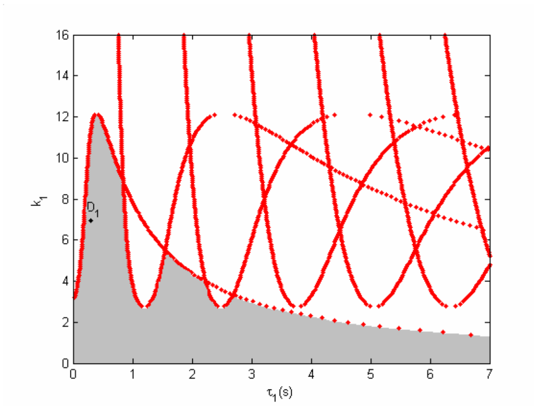

Fig. 5a. Stability region (shaded) in the plane of k_1 v. τ_1

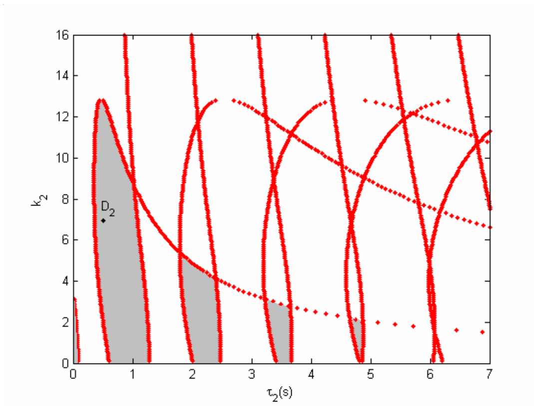

Fig. 5b. Stability regions (shaded) in the plane of k_2 v. τ_2

Another property of the proposed controller DDC is to make the imaginary spectra in delay domain $(\tau_1, \tau_2, ..., \tau_n)$ straight surfaces parallel to the major axes. For example, if we select $k_1 = 3$ and $k_2 = 4$, the stability regions, which are shaded in Fig. 6, become rectangles in the domain of the delays. It is observed that there can be two stable pockets for nonzero values of τ_2 for a given τ_1, while $\tau_2 = 0$ always indtrouces instability in the closed-loop system for this particular selection of k_1 and k_2.

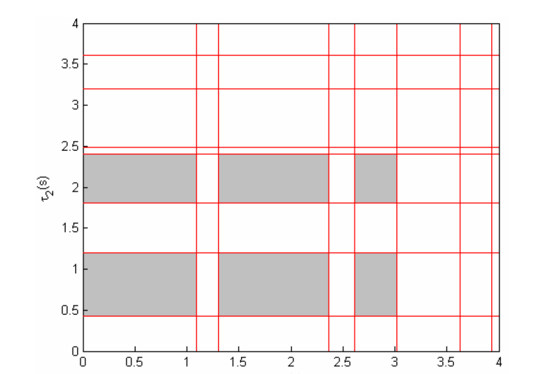

Fig. 6. Stable regions (shaded) in τ_1, τ_2 plane for $k_1 = 3$ and $k_2 = 4$

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Figures 7a and 7b show the response of the system for two different controlled regimes (which are differentiated by the gains, k and delays, τ, as marked on the figure). These regimes are marked by points **D₁** (0.3,6.95) and **D₂** (0.5,6.95) in Figs. 5a and 5b. Simulations are for the desired trajectory of unit step for *x(t)* (starting at t = 0 sec) and the desired trajectory for $\theta(t)$ as negative unit step (in degrees) which starts at t = 50 sec. These figures show the interaction between the two controlled states under the two regimes. Note that the excursions of the pendulum are high during the first 15 sec. They reach to about 8 degrees peak, in *θ(t)* which is somewhat excessive for the linearity assumptions of the dynamics. We also observe an overshoot for both *x(t)* and *θ(t)*, over 20% each.

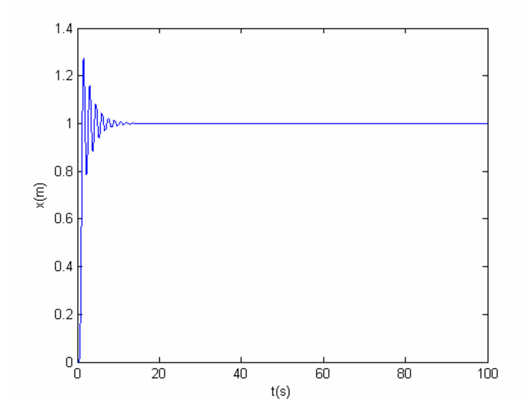

Fig. 7a. Tracking performance, *x*(m), for $\tau_1 = 0.3\,s$, $\tau_2 = 0.5\,s$ and $k_1 = k_2 = 6.95$

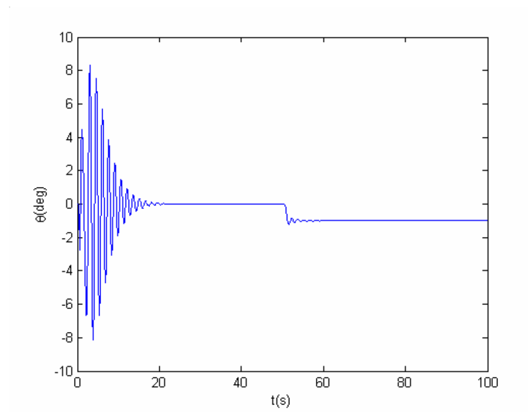

Fig. 7b. Tracking performance, θ(deg), for $\tau_1 = 0.3\,s$, $\tau_2 = 0.5\,s$ and $k_1 = k_2 = 6.95$

To suppress these undesirable excursions, we utilize a filtering logic within the controller (for instance $\frac{0.4}{0.4s+1}$ on both P and PID control channels). This operation creates a *K(s)* as

$$K(s) = \begin{bmatrix} \frac{0.4k_1}{0.4s+1} & 0 \\ 0 & \frac{0.4}{(0.4s+1)}(1.1 + \frac{k_2}{s} + 0.13s) \end{bmatrix} \quad (24)$$

The results are shown in Figs. 8a and 8b. The tracking is achieved under this control law without a discernible overshoot (both less than 0.8%).

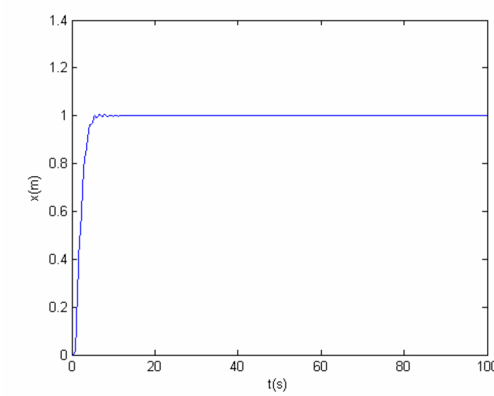

Fig. 8a. Tracking Performance, *x*(m), for $\tau_1 = 0.3\,s$, $\tau_2 = 0.5\,s$, $k_1 = k_2 = 6.95$, using filter

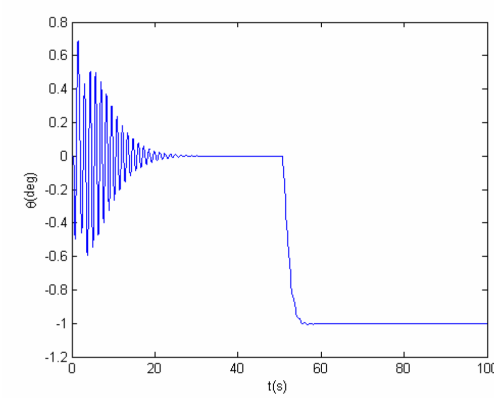

Fig. 8b. Tracking Performance, θ(deg), for $\tau_1 = 0.3\,s$, $\tau_2 = 0.5\,s$, $k_1 = k_2 = 6.95$, using filter

This application confirms that in DDC construction, selection of $K_{n\times n}(s)$ plays an important role in the desired control performance and it is another utility in the hands of the designer.

A brief discussion on the robustness of DDC

It is clear that the formation of DDC on a perfectly known system results in the intended delay-decoupling effect as shown in fig. 9.

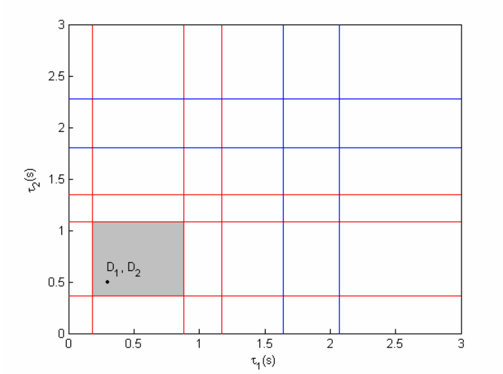

Fig. 9. Stable region for the nominal plant (i.e., *b* = 8.4 *N.m/s*)

If, however, there is some uncertainty in the system dynamics, two things happen: (a) The delay-decoupling feature will no longer hold, therefore, the characteristic equation can not be analysed as simply as we described. (b) Consequently, the stability robustness is to be further investigated absent the comfort of decoupling. Both of these aspects are beyond the scope of this concept paper. We simply wish to present a few example treatments here to display the complexities involved as well as the potential stability robustness outlooks.

Example 2.1: Take the cart-pendulum problem in example 2 and introduce a large variation to parameter *b*, from 7 to 10, but keep everything else in the formation of the DDC logic the same (e.g., $k_1 = k_2 = 6.95$, as well as *b* = 8.4 *N.m/s*) at its nominal value. Under the parametric uncertainty of $b \in [7,\ 10]$, the system characteristic equation would become:

$$CE_u = s^8 + (8.715820068 + 1.032258065b)s^7 + (8.993463588b + 54.51281871)s^6 + (52.00055771b + 435.1584700)s^5 + (412.2383697b + 742.0670790)s^4 + (5392.581042 + 649.7960412b)s^3 + (4722.049949b)s^2 + [(227.4691035 + 37.24722330b)s^4 + 1977.478518s^3 + 6196.578570s^2 + 45303.04016s]e^{-\tau_1 s} + [4.254817763s^6 + (36.09772339 + 4.392069949b)s^5 + 343.4818644s^4 + (336.3941728b + 979.9159054)s^3 + (6159.056174 + 858.0699697b)s^2 + 5393.219067bs]e^{-\tau_2 s} + (967.8395821s^2 + 8189.411849s + 51742.19304)e^{-(\tau_1+\tau_2)s} \quad (25)$$

which is not possible to factorize as in the case of the nominal system (23). Notice that in (25) the parameter *b* comes from the plant not the controller (which is formed considering the nominal value of *b*). Therefore the stability of the dynamics in (25) is truly in the domain of two independent delays, i.e., the two delays cannot be decoupled simply because of the parametric uncertainty on *b*.

The stability is still possible to analyze, but it becomes much more cumbersome as compared with the decoupled formation of delays. We use our recent paradigm CTCR to generate the **exact** (non-conservative) stability maps in the domain of the delays following [Fazelinia et. al. 2007]. The results are given in Figs. 10a and 10b. Theses figures show two features: (i) The stability regions (shaded) of the uncertain plant is largely in agreement with that of the nominal structure (of Fig. 9). Notice that for *b* = 8.4 *N.m/s*, Fig. 9 indicates that $0.18241 < \tau_1 < 0.87868$ and $0.36124 < \tau_2 < 1.08365$ rectangle is the only stable region in $(\tau_1, \tau_2) \in \Re^{2+}$. A large part of the stable region remains intact for *b* = 7 *N.m/s* as well as for 10 *N.m/s* (Figs. 10a and 10b). (ii) The controller can select a delay composition $(\tau_1,\ \tau_2)$ close to the center of the stable region in Fig. 9 in order to assure higher tolerance against the uncertain parameter.

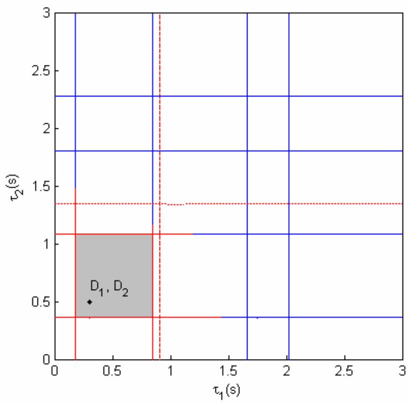

Fig. 10a. Robust stability region for $b = 7$ *N.m/s*

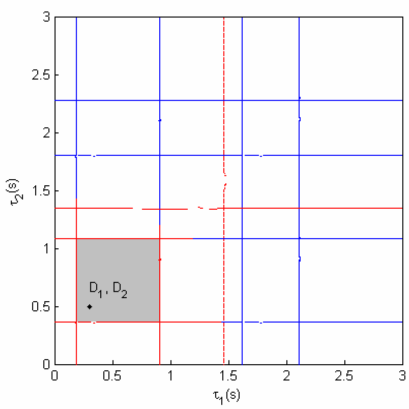

Fig. 10b. Robust stability region for $b = 10$ *N.m/s*

Example 2.2: We now take the parameter $m = 0.231$ *kg* in example 2, and vary it from 0.75×0.231 *kg* to $2 \times$ 0.231 *kg*. Figures 11a and 11b represent the respective **exact** stability maps obtained, again, using the CTCR paradigm. Similar arguments can be made as in example 2.1 for the common / robust stability regions in the domain of (τ_1, τ_2) for this selection of parametric variations. None of the separating lines are straight lines, but Fig. 11b shows the distortion much more visibly.

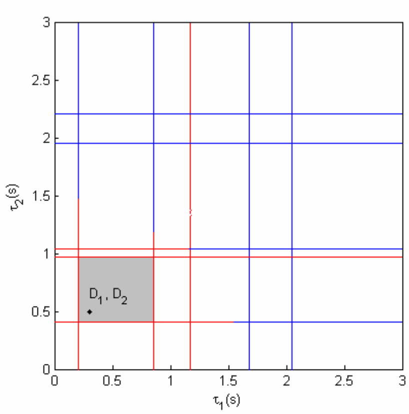

Fig. 11a. Robust stability region for $m = 0.154$*kg*

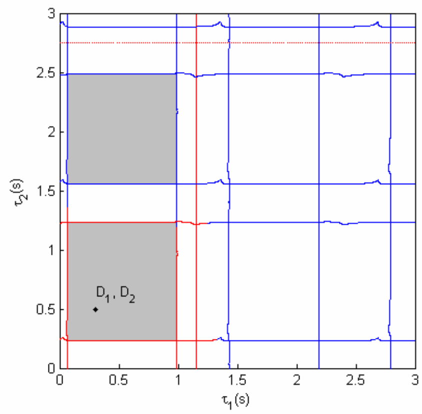

Fig. 11b. Robust stability regions for $m = 0.462$*kg*

These example cases display the simplicity that DDC offers, and the complexity of assessing the DDC logic from the perspective of stability robustness. This point is currently under investigation by the group of the authors.

5. CONCLUSION

A novel control law is presented for minimum phase MIMO plants with multiple control delays. A delay-decoupling controller (DDC) is designed considering these delays, using coprime factorization of the plant transfer matrix. Application of this controller separates the effect of the multiple delays from each other, within the characteristic equation. The delay influenced part of the characteristic equation can now be represented as a product of several single-time-delay dynamics which do not contain commensurate and cross-talk features of the delays. This formation of the characteristic equation makes the stability analysis very simple. DDC can be deployed for systems with larger number of delays (than two), which is a very strong feature. The strategy is implemented on two example systems which show the practicability of the new logic.

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