The Cluster Treatment of Characteristic Roots and the Neutral Type Time-Delayed Systems

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A new methodology is presented for assessing the stability posture of a general class of linear time-invariant—neutral time-delayed systems (LTI-NTDS). It is based on a "Cluster Treatment of Characteristic Roots CTCR" paradigm, which yields a procedure called the Direct Method (DM). The technique offers a number of unique features: It returns exact bounds of time delay for stability, as well as the number of unstable characteristic roots of the system in an explicit and nonsequentially evaluated function of time delay. As a direct consequence of the latter feature, the new methodology creates entirely, all existing stability intervals of delay, τ. It is shown that the Direct Method inherently enforces an intriguing necessary condition for τ-stabilizability, which is the main contribution of this paper. This, so-called, "small delay" effect, was recognized earlier for NTDS, only through some cumbersome mathematics. Furthermore, the Direct Method is also unique in handling systems with unstable starting posture for $\tau=0$, which may be τ-stabilized for higher values of delay. Example cases are provided.
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1 Introduction and Problem Statement

Neutral time-delayed systems (NTDS) have attracted attention in the dynamic systems community for over four decades [1–14]. The most common problem in these investigations is the stability analysis of such dynamics. The present study concentrates on a subclass: Linear Time Invariant-NTDS (LTI-NTDS). It presents a unique and general methodology for the stability analysis, which also reveals some key conditions for the so called "τ-stabilizability."

A widely accepted form of LTI-NTDS [2,8,14] is considered here:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{x}(t-\tau) + \mathbf{C}\dot{\mathbf{x}}(t-\tau) \tag{1}$$

where $\mathbf{x}(n\times 1)$, $\mathbf{A},\mathbf{B},\mathbf{C}\in\Re(n\times n)$, $\tau\in\Re^+$. Taking $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ as constant matrices, the single parameter that influences the stability of Eq. (1) is the time delay, τ. Notice that the highest order dynamics, $\dot{\mathbf{x}}$, is time delayed here. This is the attribute of NTDS. For the retarded time delay systems (RTDS) this property does not hold i.e., $\mathbf{C}=0$. As we will discuss later in detail, the stability properties of these two classes of systems are identical except a significant difference for *small delay* cases. This point alone has occupied prominent investigators in recent years [6–11,14]. It is very exciting that we can show the otherwise very tedious-to-prove feature of small delay NTDS dynamics falls out as a by-product from our new methodology.

This text is strictly on the NTDS dynamics. The earlier investigations typically start with a stable nondelayed system. They aim the determination of stability margins in τ domain [3,14], i.e. the $\tau_{\max}$ that can still assure the stability. In this work, however, we do not assume the stability for nondelayed system, and we do not aim the determination of stability margins. Our objective is to discover all possible pockets of stability in τ domain, disregarding the initial posture of the state (i.e., stable or unstable for $\tau=0$).

The delay in Eq. (1) makes it infinite dimensional. The transcendental nature of the respective characteristic equation brings about infinitely many characteristic roots, which have to be examined for declaring the system stability. The complexity of the problem is obvious even for a fixed τ. The focus of this paper is to resolve this question within the semi-infinite domain of $\tau\in\Re^+$. The methodology we present here follows the framework called the *Direct Method*, which was recently introduced primarily for the *retarded* LTI-TDS [15]. A strong distinction, however, between the retarded and neutral systems is reported in the literature particularly for small delays. The treatment explained here also addresses this "small delay" phenomenon. We will show that this phenomenon is linked directly to a necessary condition for τ-stabilizability of NTDS, and this link appears as a natural by-product of our new methodology, DM. This point is the primary contribution of the study along with others mentioned later.

The characteristic equation of the system in (1) is

$$\mathrm{CE}(s,\tau) = \det(s\mathbf{I} - \mathbf{A} - \mathbf{B}e^{-\tau s} - \mathbf{C}se^{-\tau s}) = 0 \quad \text{with } \tau > 0 \tag{2}$$

which yields a scalar equation in the general form of

$$\mathrm{CE}(s,\tau) = a_n(s)e^{-n\tau s} + a_{n-1}(s)e^{-(n-1)\tau s} + \cdots + a_0(s) = \sum_{k=0}^{n} a_k(s)e^{-k\tau s} = 0 \tag{3}$$

where $a_k(s)$ are polynomials of degree n (or less) in s with real coefficients. This system is called "neutral" if any one of the polynomials $a_j(s)$, $j=1,\ldots,n$ contains an s^n term, i.e., the highest order dynamics being influenced by the delay, τ, becomes n. Equation (3) represents, in general terms, an n-toppled commensurate time delay case, implying that the delays are integer multiples of τ.

By definition (e.g., [16]), the linear system in (1) is asymptotically stable if and only if all the characteristic roots of the transcendental equation (3) are on the left half of the complex s plane. Since there are infinitely many such roots, to examine their loca-

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