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STABILITY ROBUSTNESS OF RETARDED LTI SYSTEMS WITH SINGLE DELAY AND EXHAUSTIVE DETERMINATION OF THEIR IMAGINARY SPECTRA*

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Abstract. In this paper we consider the stability robustness of the general class of vector LTI (linear time invariant) equations with a single delay, $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{x}(t-\tau)$, $\mathbf{x} \in \mathbf{R}^n$. The robustness is against the uncertain, but constant delay, $\tau \in \mathbf{R}^+$. We first present a set of novel propositions and state that the solution must start from the complete knowledge of imaginary spectra of the system, and the corresponding delays. The propositions claim that such spectra form a set of manageably small number of members, and this number is upper bounded by n^2 regardless of the composition of **A** and **B** matrices. They also claim that the infinite-dimensional system at hand has an outstanding discipline regarding these imaginary spectra. This discipline invites the recently developed concept called the cluster treatment of characteristic roots (CTCR). The CTCR procedure requires a complete and precise determination of the imaginary spectra of the system. There are many procedures in the literature to achieve this. They are, in fact, some variations of the five main methods of different levels of precision and complexity. There is, however, no study known to the authors for presenting a comparison among these methods. This paper addresses this need. We first offer an overview of each of the five methods and then compare their numerical performances over an example case study.

Key words. linear time-delayed dynamics, quasi polynomial, imaginary spectra detection, robust stability

AMS subject classifications. 15A15, 15A09, 15A23

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1. Problem statement and an explicit function for stability.

The stability of linear time invariant retarded time delayed systems (LTI–TDS) has been a very active research topic during the past several decades [1, 2, 3, 4, 5]. Numerous contributions by renowned investigators can be found in the literature on the subject. Although at present the focus of attention in the time delayed systems (TDS) community is directed toward much more complex dynamics (such as parametric uncertainties, robustness, time-varying time delays, nonlinear TDS), the LTI–TDS has an undisputed knowledge base which offers plenty of insight into some realistic problems. Furthermore it is the authors' belief that the LTI–TDS field still remains rich with challenging and unsolved problems. Some existing methods, for instance, present new knowledge, which have not been recognized until recently [4, 6, 7, 26]. Some others suggest variations on the earlier techniques—to overcome some subtle and hidden impracticalities—mainly from a numerical deployment point of view [8].

The general dynamics in question is

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{x}(t-\tau), \tag{1.1}$$

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