Stability Robustness Analysis of Multiple Time-Delayed Systems Using "Building Block" Concept

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***Abstract*—An intriguing perspective is presented in studying the stability robustness of systems with multiple independent and uncertain delays. It is based on a holographic mapping, which is implemented over the domain of the delays. This mapping considerably alleviates the problem, which is otherwise known to be notoriously complex. It creates a dramatic reduction in the dimension of the problem from infinity to manageably small number. Ultimately the process is reduced to studying the problem within a finite dimensional cube with edges of length 2π in the new domain, what we call the *building block*. In essence, the mapping collapses the entire set of potential stability switching points onto a small (upperbounded) number of *building hypersurfaces*. We further demonstrate that these *building hypersurfaces* can be implicitly defined and they are completely isolated within the above mentioned cube. It is also shown that the exhaustive detection of these building hypersurfaces is necessary and sufficient in order to arrive at the complete stability robustness picture we seek. As a consequence, this concept yields a very practical and efficient procedure for the stability assessment of such systems. This novel perspective serves very well for the preparatory steps of the authors' earlier contribution in the area, cluster treatment of characteristic roots (CTCR). We elaborate on this combination, which forms the main contribution of the paper. Several example case studies are also provided.**

***Index Terms*—Building block, cluster treatment of characteristic roots (CTCR), robustness, stability, time delay.**

I. Introduction and the Problem Statement

In this study, we consider linear time-invariant, retarded multiple time-delayed systems (LTI-MTDS). The general state-space form of this class of systems is given as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \sum_{j=1}^{p} \mathbf{B}_j \mathbf{x}(t-\tau_j) \qquad (1)$$

where $\mathbf{x} \in \Re^n, \mathbf{A}, \mathbf{B}_j, j = 1 \ldots p$ are all constant matrices in $\Re^{n\times n}$ and the vector of time delays $\boldsymbol{\tau} = (\tau_1, \tau_2, \ldots, \tau_p) \in \Re^{p+}$ of which the elements are positive and rationally independent from each other. As a note of formalism we use boldface characters for vector and matrix quantities and sets in the text. We refer to right (and left) half open complex plane as $C^+(C^-)$, while C^0 is used to indicate the imaginary axis. Therefore $C \triangleq C^+ \cup C^- \cup C^0$ represents the entire complex plane.

The determination of the stability robustness of this system against delay uncertainties is the main objective in this paper. This problem has been studied for over four decades resulting in some respectable volume of literature [1]–[6]. Stability posture of these systems for a given delay vector, $\boldsymbol{\tau}$, has been one of the research foci. The determination of the stability robustness against uncertainties in delays $(\boldsymbol{\tau})$ and other parameters ($\mathbf{A}$ and $\mathbf{B}_j$) is also investigated widely [7]–[12]. We focus, here, on the delay uncertainty only. This problem is also known to be of complexity class $\mathcal{N}$-$\mathcal{P}$ hard [13]. Several investigations appeared on significantly simplified forms of the problem [14]–[16]. A recent paradigm, cluster treatment of characteristic roots (CTCR), of the authors', relaxed these simplifications via a practical and numerically efficient procedure when there are two delays $(p = 2)$ [17], [18]. Indeed, CTCR produces a complete stability robustness tableau within the domain of the delays, $\boldsymbol{\tau} \in \Re^{p+}$. The numerical efficiency of CTCR is also tested for cases $n = 3, p = 2$ [18], and $n = 6, p = 3$ [19], however with $\mathcal{N}$-$\mathcal{P}$ hard complexity feature still remaining.

As we review later in the text, the key procedure that enables the CTCR paradigm is a holographic mapping of the delays (known as Rekasius substitution [17], [18], [20]). This mapping successfully converts spectrally infinite-dimensional problem into a finite dimensional one. We can metaphorically describe this operation as looking at the problem under special optics. To improve the process further we suggest an additional perspective here, which further confines the domain of analysis into a finite dimensional cube. This cube is called the "building block" and it is the main contribution of this paper. First, we elaborate on this new mapping, and present its salient features. Next, we explain its impact on the steps of the CTCR procedure. As the "building block" concept is valid for the most general class of multiple time delay systems (1), this paper contains the first treatment of n -dimensional p -time delay LTI-MTDS in full rigor.

It is important to note that, along the steps of the "building block" procedure described here, we completely adhere to the underlying guidelines of the CTCR paradigm [17], [18]. We highlight these characteristics in the next section for the completeness of the present work.

The text is structured as follows: Section II reviews the steps of the CTCR paradigm with its two novel propositions that give rise to the concepts of "*kernel*" and "*offspring*" hypersurfaces.

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