SIAM J. CONTROL OPTIM.
Vol. 46, No. 1, pp. 143–155

EXTENDED KRONECKER SUMMATION FOR CLUSTER TREATMENT OF LTI SYSTEMS WITH MULTIPLE DELAYS*

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Abstract. A new procedure is presented for determining the kernel and the offspring hypersurfaces for general linear time invariant (LTI) dynamics with multiple delays. These hypersurfaces, as they have very recently been introduced in a concept paper [R. Sipahi and N. Olgac, *Automatica*, 41 (2005), pp. 1413–1422], form the basis of the overriding paradigm which is called the cluster treatment of characteristic roots (CTCR). In fact, these two sets of hypersurfaces exhaustively represent the locations in the domain of the delays where the system possesses at least one pair of imaginary characteristic roots. To determine the kernel and offspring we use the extraordinary features of the "extended Kronecker summation" operation in this paper. The end result is that the infinite-dimensional problem reduces to a finite-dimensional one (and preferably into an eigenvalue problem). Following the procedure described in this paper, we are able to shorten the computational time considerably in determining these hypersurfaces. We demonstrate these concepts via some example case studies. One of the examples treats a 3-delay system. For this case another interesting perspective, called the "building block," is also utilized to display the kernel in three-dimensional space in the domain of "spectral delays."

Key words. linear time-delayed systems, Kronecker sum, multiple delays, stability, robust stability

AMS subject classifications. 15A15, 15A09, 15A23

DOI. 10.1137/06065180X

1. Introduction and the problem statement.

We consider linear time invariant, retarded multiple time-delayed systems (LTI-MTDS), the general form of which is given as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \sum_{j=1}^{p} \mathbf{B}_j \mathbf{x}(t-\tau_j), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{A}$, $\mathbf{B}_j$, $j = 1 \dots p$, are all constant matrices in $\mathbb{R}^{n\times n}$ and the vector of time delays $\boldsymbol{\tau} = (\tau_1, \tau_2, \dots, \tau_p) \in \mathbb{R}^{p+}$ of which the elements are rationally independent from each other. As a note of formalism we use boldface capital notation for vector and matrix quantities in the text. We refer to the right (and left) half open complex plane as $\mathbb{C}^+$ ($\mathbb{C}^-$), while $\mathbb{C}^0$ is used to indicate the imaginary axis. Therefore $\mathbb{C}^+ \bigcup \mathbb{C}^- \bigcup \mathbb{C}^0 = \mathbb{C}$ represents the entire complex plane.

The characteristic equation of the system in (1) is

$$\begin{aligned} CE(s,\tau_1,\dots,\tau_p) &= \det\left[s\mathbf{I} - \mathbf{A} - \sum_{j=1}^{p} \mathbf{B}_j e^{-\tau_j s} \right] \\ &= \mathbf{A_0}(s) + \mathbf{A}_{p+1}(s,\tau_1,\dots,\tau_p) + \sum_{j=1}^{p} e^{-n_j\tau_j s}\mathbf{A}_j\,(s\,,\tau_1\,,\dots,\tau_{j-1},\tau_{j+1}\,,\dots,\tau_p), \end{aligned} \tag{2}$$

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*Received by the editors February 9, 2006; accepted for publication (in revised form) September 25, 2006; published electronically March 30, 2007. This research was supported in part by awards from DoE (DE-FG02-04ER25656), NSF (CMS-0439980), and NSF (DMI-0522910).
http://www.siam.org/journals/sicon/46-1/65180.html

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