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Brief paper

Complete stability robustness of third-order LTI multiple time-delay systems☆

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Abstract

A unique procedure is presented in this paper, for a complete stability robustness of the third-order LTI multiple time-delay systems (LTI-MTDS). The uniqueness of the treatment is simply due to the fact that there is no comparable methodology, presently, in the literature. The end result of this procedure is an exhaustive and precise determination of the stable regions in the domain of time delays. The backbone of the method is a novel framework called "the cluster treatment of characteristic roots, (CTCR)". CTCR is constructed over two fundamental propositions. The first proposition claims the existence of a *bounded* number of so-called "*kernel curves*", where the *only* imaginary characteristic roots occur. The second proposition is on an interesting directional *invariance* property of the crossing tendencies of these imaginary roots. For simplicity of conveyance and without loss of generality, the number of time delays is taken as two in this document. The new methodology is expandable to higher-order dynamics with more time delays than two, as the authors intend to demonstrate in future publications.

Keywords: Multiple time delay; Robustness; Stability; Linear time invariant systems

1. Introduction and the problem statement

This work is on the stability assessment of LTI multiple time-delay system (LTI-MTDS). We present a novel procedure, which ultimately creates the complete and exact "map of stability" on the space of the time delays. This is an unprecedented undertaking compared with the existing literature. The text is intended to prepare the groundwork for future investigations offering a global procedure applicable to the general class of LTI-MTDS with higher-order dynamics and larger number of delays. Strictly for purposes of easier conveyance of the methodology, however, we adhere, for the time being, to a restricted class of systems with two delays, for all the proofs and descriptions.

First, we present some notational definitions. In the text $\mathbf{C}^+$, $\mathbf{C}^-$, $\mathbf{C}^0$ are used for right-half, left-half and the imaginary axis of the complex plane, respectively. Consequently, $\mathbf{C}=\mathbf{C}^+\cup\mathbf{C}^-\cup\mathbf{C}^0$ represents the entire complex plane. Matrices, vectors and sets are all denoted by bold face, while the scalar entities are with normal font, e.g. $\{\mathbf{a}\}=(a_1, a_2, \ldots)$. The class of LTI-MTDS under consideration is represented by a three-dimensional state space dynamics with two time delays

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}_1\mathbf{x}(t-\tau_1)+\mathbf{B}_2\mathbf{x}(t-\tau_2), \tag{1}$$

where $\mathbf{x}\in\Re^3$, $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$ are all $\Re^{3\times 3}$ and the time delays $(\tau_1, \tau_2)\in\Re^{2+}$. The characteristic equation of this dynamics is

$$CE(s, \tau_1, \tau_2)=\det(s\mathbf{I}-\mathbf{A}-\mathbf{B}_1\mathrm{e}^{-\tau_1 s}-\mathbf{B}_2\mathrm{e}^{-\tau_2 s})=0 \tag{2}$$

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