SIAM J. CONTROL OPTIM.
Vol. 45, No. 3, pp. 957–971

COMPLETE STABILITY ANALYSIS OF NEUTRAL-TYPE FIRST ORDER TWO-TIME-DELAY SYSTEMS WITH CROSS-TALKING DELAYS*

RIFAT SIPAHI† AND NEJAT OLGAC†

Abstract. The stability robustness of first order linear time invariant dynamics of *neutral type* with multiple time delays against delay uncertainties is taken into consideration. We depart from a simpler investigation of Hale and Huang [*J. Math. Anal. Appl.*, 178 (1993), pp. 344–362], which studies the same problem for retarded-type systems. On this basis we further introduce two challenging features by including (a) terms that add neutral dynamics and (b) an additional term that introduces cross-talk between the multiple delays. To the best of the authors' knowledge, the stability posture of this class of systems can be treated only by a unique procedure. It is known as cluster treatment of characteristic roots (CTCR), which was recently developed for *retarded*-type dynamics. We first show the applicability of CTCR to the stability analysis of *neutral*-type multiple-delay dynamics. Next, we prove the well-known "small-delay" phenomenon for the dynamics at hand, interestingly, as a natural by-product of the CTCR paradigm. Finally, we present several case studies to display the steps and the strengths of CTCR. This deployment is scalable to treat similar problems with higher order dynamics, which have direct ramifications to some practical control applications.

Key words. neutral time-delayed dynamics, stability, multiple time delays, cluster treatment of characteristic roots (CTCR)

AMS subject classifications. 15A15, 15A09, 15A23

DOI. 10.1137/050633810

1. Introduction.

We consider a general class of delayed differential equations (DDEs) from the stability robustness perspective, which has not been successfully investigated in the literature. The class is of first order *neutral type*, linear time invariant (LTI) time-delay systems with two cross-talking delays. The dynamics is written in conventional form [6], [7], [8] as

$$\begin{aligned} &\frac{d}{dt}[x(t) - ax(t-\tau_1) - bx(t-\tau_2)] \\ &= cx(t-\tau_1) + dx(t-\tau_2) + fx(t-\tau_1-\tau_2) + gx(t), \end{aligned} \tag{1.1}$$

where a, b, c, d, f, g are all real scalars as well as the dependent variable $x(t)$, $(\tau_1, \tau_2) \in \mathbf{R}^{2+}$. We highlight the $fx(t-\tau_1-\tau_2)$ term as the "delay cross-talk" feature in the problem. The problem is to analyze the stability robustness of this system against time-delay uncertainties in the semi-infinite first quadrant of $(\tau_1, \tau_2) \in \mathbf{R}^{2+}$. The characteristic equation of these dynamics is transcendental,

$$CE = s(1 - ae^{-\tau_1 s} - be^{-\tau_2 s}) - ce^{-\tau_1 s} - de^{-\tau_2 s} - fe^{-(\tau_1+\tau_2)s} - g = 0, \tag{1.2}$$

where arguments of CE, (s, τ_1, τ_2) are suppressed. The stability robustness question of this system reduces to finding (τ_1, τ_2) regions where all the characteristic roots

*Received by the editors June 16, 2005; accepted for publication (in revised form) January 28, 2006; published electronically July 31, 2006. This work was supported partially by research funds from the DoE (DE-FG02-04ER25656) and the NSF (CMS-0439980, CMS-0539980, DMI 0522910).
http://www.siam.org/journals/sicon/45-3/63381.html

†Mechanical Engineering Department, University of Connecticut, Storrs, CT 06269-3139 (rsipahi @hds.utc.fr, olgac@engr.uconn.edu). The first author is currently at Université de Technologie de Compiègne as a postdoc under Chateaubriand Scholarship 2005–2006.