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A unique methodology for the stability robustness of multiple time delay systems

Rifat Sipahi[1], Nejat Olgac*

University of Connecticut, Mechanical Engineering Department, Storrs, CT 06269-3139, USA

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Abstract

The stability robustness is considered for linear time invariant (LTI) systems with rationally independent multiple time delays against delay uncertainties. The problem is known to be notoriously complex, primarily because the systems are infinite dimensional due to delays. Multiplicity of the delays in this study complicates the analysis even further. And "rationally independent" feature of the delays makes the problem prohibitively challenging as opposed to the TDS with commensurate time delays (where time delays are rationally related). A unique framework is described for this broadly studied problem and the enabling propositions are proven. We show that this procedure analytically reveals all possible stability regions exclusively in the space of the delays. As an added strength, it does not require the delay-free system under consideration to be stable. Our methodology offers a resolution to this question, which has been studied from variety of directions in the past four decades. None of these respectable investigations can, however, deliver an exact and exhaustive robustness declaration. From this stand point the new method has a unique contribution.

Keywords: Cluster treatment of characteristic roots (CTCR); Stability analysis; Linear time invariant; Multiple time delay systems

1. Introduction and the problem statement

This study is on the stability robustness of linear time invariant multiple time delay system (LTI-MTDS) against uncertainties in the delays. We present a new procedure for the most generic form of two time delay and second order retarded LTI systems, with the intension of extending it to higher order dynamics with more delays. Such systems are of increasing interest in many practical applications such as data network congestion [12], neural network dynamics [2], machine tool chatter [17].

First, we present some notational definitions. In the text $\mathbf{C}^+$, $\mathbf{C}^-$, $\mathbf{C}^0$ are used for right half, left half and the imaginary axis of the complex plane, respectively. Consequently, $\mathbf{C} = \mathbf{C}^+ \cup \mathbf{C}^- \cup \mathbf{C}^0$ represents the entire complex plane. One-dimensional sets are denoted with curly brackets, e.g. $\{\mathbf{a}\} = (a_1, a_2, \ldots)$, and $a_j,\ j = 1, 2, \ldots$ are scalar. We use double subscript on these scalar members to identify a particular assigned value, such as a_{30}, a_{31} or a_{32}, etc. imply some values of a_3.

The most generic form of the characteristic equation for a second order dynamics with two time delays is

$$\mathrm{CE}_\tau(s, \tau_1, \tau_2) = a_0(s) + a_1(s)\mathrm{e}^{-\tau_1 s} + a_2(s)\mathrm{e}^{-\tau_2 s} + a_3(s)\mathrm{e}^{-(\tau_1+\tau_2)s} = 0, \quad \tau_1, \tau_2 > 0, \tag{1}$$

where $a_j(s)$, $j = 0, 1, 2, 3$ are polynomials of s with degrees of 2, 1, 1, 0, respectively. The highest degree of s in (1) is 2 and it appears in $a_0(s)$ only, where there is no time delay influence, making (1) a "retarded system" [12,5,7,8,19,11,1]. The delays are rationally unrelated, i.e., $\tau_1/\tau_2 =$ irrational number. The problem is to determine the stability mapping of this system in $\{\boldsymbol{\tau}\} = (\tau_1, \tau_2)$ space. Please make a mental mark of the last term in the equation, which represents the "cross-talk" between the two delays.

The stability of TDS when there is only one single delay has been studied extensively. Some of these studies search for the stability switches in the space of the *only* time delay, $\tau \in \Re^+$ [8,4,3,13,18,16,7]. The same pursuit in MTDS is understandably much more complex simply because the stability influence

* Corresponding author. Tel.: +1 860 486 2382; fax: +1 860 486 5088.
E-mail address: olgac@engr.uconn.edu (N. Olgac).
[1] Currently at l'Université de Technologie de Compiegne, France.

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