Available online at www.sciencedirect.com
SCIENCE DIRECT®

Automatica 40 (2004) 847 – 853

automatica

www.elsevier.com/locate/automatica

Brief Paper

A practical method for analyzing the stability of neutral type LTI-time delayed systems☆

Nejat Olgac*, Rifat Sipahi

Mechanical Engineering Department, University of Connecticut, Storrs, CT 06269-3139, USA

Received 16 June 2002; received in revised form 29 September 2003; accepted 17 December 2003

Abstract

A new paradigm is presented for assessing the stability posture of a general class of linear time invariant–neutral time delayed systems (*LTI-NTDS*). The ensuing method, which we name the direct method (*DM*), offers several unique features: It returns the number of unstable characteristic roots of the system in an explicit and *non-sequentially* evaluated function of time delay, τ. Consequently, the direct method creates exclusively all possible stability intervals of τ. Furthermore, it is shown that this method inherently verifies a widely accepted necessary condition for the τ-stabilizability of a LTI-NTDS. In the core of the DM lie a *root clustering* paradigm and the strength of Rekasius transformation in mapping a transcendental characteristic equation into an equivalent rational polynomial. In addition, we also demonstrate by an example that DM can tackle systems with unstable starting posture for $\tau = 0$, only to stabilize for higher values of delay, which is rather unique in the literature.

Keywords: Linear systems with delays; Linear neutral systems; Delay-dependent stability; Stability margins; Cluster treatment of characteristic roots (CTCR)

1. Introduction and problem statement

Neutral time delayed systems (NTDS) have been extensively studied for over four decades (Barnett, 1983; Bellman & Cooke, 1963; Chen, 1995; Cooke & van den Driessche, 1986; Halanay, 1966; Hale, 1977; Hale, 2001, Hale & Verduyn Lunel, 1993, 2001a, b; Hale, Infante, & Tsen, 1985; Hu & Hu, 1996; Niculescu, 2001). The most common problem in these investigations is the stability assessment of such dynamics along the delay axis. The present work offers a novel paradigm and a general methodology to address this issue for a broad class of systems: Linear time invariant (LTI)-NTDS. A widely accepted form of LTI-NTDS (Bellman & Cooke, 1963; Hale & Verduyn Lunel, 1993; Niculescu, 2001) is considered here:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{x}(t-\tau) + \mathbf{C}\dot{\mathbf{x}}(t-\tau), \qquad (1)$$

where $\mathbf{x}(n \times 1)$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C} \in \Re\ (n \times n)$, $\tau \in \Re^{+}$. Taking $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ as constant matrices, the only remaining factor that influences the stability in Eq. (1) is the time delay, τ. In all the earlier investigations the LTI-NTDS starts with a stable posture for $\tau = 0$. The objective, then, is to identify the stability margin in τ domain (Chen, 1995; Niculescu, 2001), i.e., $\tau_{\max}$ that can still assure the stability. In this work, however, we do not require the stability for non-delayed system, nor do we aim the evaluation of $\tau_{\max}$. The objective here is to determine all possible intervals (we call them "pockets") of stability in τ domain exclusively, regardless the initial posture of the dynamics.

The delay in Eq. (1) introduces exponential transcendentality in the characteristic equation of the system. This feature invites infinitely many (but not dense) characteristic roots all of which have to be studied to determine the stability. This is an impossible task, unless a practical procedure is developed. The core concept of such a procedure is presented in Olgac and Sipahi (2002) for retarded LTI-TDS.

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publishing in revised form by Associate Editor Hitay Ozbay under the direction of Editor Tamer Başar.

* Corresponding author. Tel.: +1-860-486-2382; fax: +1-860-486-5088.

E-mail addresses: olgac@engr.uconn.edu (N. Olgac), rifat@engr.uconn.edu (R. Sipahi).

0005-1098/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2003.12.010