Detection and Localization of Multiple Unresolved Extended Targets via Monopulse Radar Signal Processing

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Abstract

When the size of targets is comparable to the range resolution of monopulse radars, these targets should be considered as extended rather than point targets. If several closely-spaced targets fall within the same radar beam and between adjacent matched filter samples in range, the full monopulse information from all of these samples can and should be used to resolve these targets, i.e., estimate the number of targets and their respective angles-of-arrival and ranges. To detect and localize multiple unresolved extended targets, we establish a model for monopulse radar returns from extended objects, and present a maximum likelihood estimator to localize the targets. Rissanen’s minimum description length (MDL) criterion will be used to decide the number of existing extended objects. We also derive the upper limit on the number of targets and their scattering centers that can be resolved, and we provide necessary conditions for these targets to be uniquely identified. We compare the new extended target monopulse processing scheme with previously developed point-target monopulse techniques in the simulations.

Index Terms

Monopulse Radar, Extended Target, Unresolved Targets, Measurement Extraction, Tracking, Super-resolution

1. INTRODUCTION

Monopulse radar is widely used to provide accurate angle measurements. In one angular dimension, e.g., elevation, the monopulse radar transmits energy in a beam and observes the
Fig. 1. A monopulse radar uses 4 squinted radar sub-beams to provide a sum of all sub-beams, azimuth and elevation difference channels. On the right side, the Cross sections complex returns of a pair of squinted sub-beams steered relatively above and below the expected target location. By comparing the difference and sum of returns from the two sub-beams, an angle estimate can be obtained; for radars that estimate the direction of arrival (DOA) in two angular dimensions, four squinted sub-beams are used, one for each Cartesian quadrant. See Figure 1 for a four beam monopulse radar pattern and the cross-section of four monopulse beams. Let A, B, C & D represent the corresponding complex returns shown as in Figure 1. The complex sum, horizontal difference and vertical difference are given by

\[ s = A + B + C + D \]  
\[ d_h = (C + D) - (A + B) \]  
\[ d_v = (A + C) - (B + D) \]  

Conventional monopulse ratio processing is appropriate to the case that only one target falls in a given range bin: the real part of the monopulse ratio encapsulates the observations in one statistic, and a single direction of arrival is estimated. However, if two or more unresolved targets fall in the same range bin and beam, the measurements from these targets are merged together. In this case, conventional monopulse processing fails [1][11][32][33].

There has been work with two unresolved targets and a single (complex) matched filter sample: [22], [27] and [28] modified the antenna configuration to aid in the resolution process. Blair and
Brandt-Pearce [8]-[10] used complex monopulse ratio processing\(^1\), and extended the concept of the monopulse ratio-based detection and estimation techniques to the case of two unresolved targets. It could be argued that the approaches in [10] are not optimal in the estimation sense, and hence in [34] Sinha et al. presented a numerical maximum likelihood (ML) angle estimator for both Swerling I and III targets. Although some performance improvement over [10] was observed in some cases (notably for sea-skimming targets), in [37] it was found that the simple and fast approach of Blair and Brandt-Pearce is difficult to improve on. In [36] the ML computation of [34] is made explicit, and hence competitive with [10] in terms of numerical needs. Interestingly, it can be shown (using the approach from [40] and [41]) that the above works with unresolved targets can identify no more than two targets. This limit is imposed by the lack of observation information diversity within their models.

Farina et al. [13] and Gini et al. [16][17] developed a method to jointly estimate complex amplitudes, Doppler frequencies and DOA of multiple unresolved targets for a rotating radar, again with only one receiving channel. Their method utilizes the spatial diversity from the nature of a rotating radar to detect and localize more than two targets; its processing strategy is similar to that of [40] and [41]. Brown et al. [12] explore spatial diversity for a monopulse radar to estimate the DOA of more than two unresolved targets.

With the exception of [12], the above consider radar returns at only one matched filter sampling point; that is, the targets are assumed to be located exactly where the matched filter is sampled, and hence that there is no “spillover” of target energy to adjacent matched filter returns. However, it was recently found [40][41] that target spillover ought to be considered. The monopulse returns from two — or more — consecutive matched filter sampling points are jointly processed by a maximum likelihood estimator to provide angles and sub-bin range estimates. Rissanen’s Minimum Description Length (MDL) [18][31] criterion is used to detect the number of targets present. It has been shown that with more information (i.e., from two adjacent sampling points used in a joint fashion), one can detect and get sub-bin range and angular estimates for up to **five targets** from just these two matched filter samples; and similarly more targets when more matched filter samples entered the processing.

The method in [40][41] can be categorized as a a super-resolution technique [6][14][26][35].

\(^1\)For a single target the usual approach is to use only the real part of the monopulse ratio.
However, it is different from the traditional concept of super-resolution, which utilizes spectral analysis in array signal processing. In the context of radar, the traditional super-resolution techniques can be used in pursuing higher resolutions for synthetic aperture radars (SAR) [6][26]. The techniques in [40], [41] and this paper, are designed for monopulse radars depicted in Figure 1. Note that with SAR, both the range and cross-range resolutions are good while the cross-range resolutions for conventional monopulse radars are typically much worse than the range resolution.

All previous work in monopulse radar signal processing of unresolved targets assumed point target models; that is, with narrow band radars, the range bin is large enough to allow targets to be considered as points. However, for radars with wider bandwidth, the range resolution can be comparable to the size of the targets. It is more appropriate to model the targets as extended, and their size needs to be taken into account.

As will be shown in Section II, we also observe that, for these radars with finer range resolutions, the cross-range resolution is typically much coarser than the range resolution. It is not uncommon that the range resolution can be less than 10 meters while the cross-range resolution remains a few hundred meters. Thus, targets can be viewed as “point targets” along the cross-range direction and “extended targets” along the range direction. We also notice that unresolved targets are likely to be the results of coarse cross-range resolution. In this paper, we propose a signal processing technique for this type of scenarios.

We will modify the joint-bin processing for point targets in [40] and [41], and establish a model for monopulse returns from extended targets. We propose a joint-bin maximum likelihood estimator (MLE) for the locations of the unresolved extended targets and we will use Rissanen’s Minimum Description Length (MDL) to determine the number of existing extended targets. We also derive the upper limit on the number of targets and their scattering centers and we provide necessary conditions for these targets to be uniquely identified.

In this paper, we assume a monopulse radar signal processing system depicted in Figure 2. The detection block includes usual radar signal processing elements, including demodulation, matched filtering, sampling, and detection of signal energy in each sample (thresholding). The output from the detection block is a sequence of samples labelled with “0” or “1”. “0” means there is no signal from targets at this sample, while “1” means there is signal energy from targets.

The sample sequence is then processed in the segmentation block, where the samples are seg-
Fig. 2. A monopulse radar signal processing system for unresolved targets.

mented into sets of consecutive samples. There are likely unresolved targets between the samples in a set. Various segmentation algorithms can be developed for different applications. We will not discuss in detail about segmentation in this paper, and we believe this topic deserves a separate treatment. We give a simple segmentation example here. When the target size is comparable to the range bin length, a simple segmentation algorithm can be, all consecutive samples with signal energy are aggregated into a set. For instance, a sample sequence “...001100011100...” will result in two sets with the “11” samples and the “111” samples respectively.

Each of these sets is then processed with the resolution and localization block to find out how many targets are associated with that set and what their DOA's are. This paper mainly concentrates on the resolution and localization block.

The paper continues in Section II with a statistical model and assumptions for the monopulse radar returns, and parameterizes the matched filter outputs in the multiple-bin cases — note that we define a “bin” to denote the range (or delay) between two matched filter samples. In Section III we present the maximum likelihood estimation and discuss the required constrained optimization. If the number of targets is not known beforehand, the optimization must be performed repeatedly for feasible combinations; ultimately to decide on the number of targets requires maximization of a penalized likelihood, and we discuss the use of the MDL approach. In Section IV we discuss the upper limits on the number of targets and their scattering centers so that the targets can be uniquely identified. In Section V we corroborate our analysis via simulations, with particular emphasis on the tracking results — an MHT with track management is used. We term our approach the monopulse radar signal processing with an extended target model (MRSP-E). It is unfair to compare MRSP-E to an unadorned point-target estimator, since assuming two scattering centers per target the latter will suggest that there are twice the number

\(^2\)These points can be seen as the centers of two adjacent resolution cells.
of targets as are truly present. As such, we have MRSP-E competing against MRSP-MP (*modified point target model*), which associate groups of nearby point targets to a single extended one prior to delivery to its MHT. We summarize in Section VI.

II. MODELS

In this section, a model for monopulse radar signal returned from extended targets will be presented. We first provide the radar application scenario and target model considered in this paper.

A. Target Model

With modern radars, range resolution of a few tens of meters is easily achievable [30]. Thus it is reasonable to assume the size of the targets of interest is comparable to the range resolution of these radars, and the targets can not be modeled as point targets.

We consider an extended target consisting of a few *scattering centers*, the distances between which are not negligible when compared to the range resolution. An example of such extended targets is an aircraft shown in Figure 3. The aircraft contains four scattering centers, and the length of the aircraft is comparable to the range resolution of the radar.

We further assume that each scattering center consists of a number of scatterers. The distances between these scatterers are much smaller compared to the range resolution, so that the scattering center can be viewed as a point.

We also notice that each scattering center should be modeled using Swerling fluctuation models [23][30], because of target motion and the multiple-scatter composition of a scattering center. More specifically, we assume the return signal from a scattering center follows a Swerling II model, which means pulse-to-pulse fluctuation, i.e., the returns of different pulses from the same scattering center are independent. The Swerling II model also assumes the amplitude of the return follows a Rayleigh distribution, and the phase of the return is assumed to be uniformly distributed in \([0, 2\pi)\).

The return signals different the scattering centers are also assumed to be independent, even if the scattering centers are from the same extended target.
Let us consider an X band radar (10 GHz) with bandwidth $B = 10$ MHz, and assume Nyquist sampling (rate=$B$) is used. The range resolution is given by [30]

$$\Delta \tau = \frac{c}{2B}.$$  \hspace{1cm} (4)

and in this case $\Delta \tau = 15$ meters.

The beamwidth of an antenna is approximately [30] $\lambda/D$ radian, where $\lambda$ is the wavelength and $D$ is the width of the antenna. The cross-range resolution is given by $\tau \lambda/D$, where $\tau$ is the range. So, if the antenna diameter is $D = 1$ meter and the wavelength $\lambda = 0.03$ meters, the cross-range resolution at 30 kilometers range is about 900 meters. In order to make the cross-range resolution similar to that in range (say, 15 meters), we would need a either 60-meter antenna, SAR, or to increase the carrier frequency to 600 GHz, unlikely for many existing monopulse systems.

As such, we assume a resolution cell of this radar at 30 kilometers range is depicted in Figure 4. The distance between two adjacent sampling points is approximately 15 meters and we have two 5-meter-long targets located in between. The key is that the range resolution is much better than the cross-range resolution: we assume that a practical extended target is extended only in range, while the azimuthal offset of scattering centers can be neglected.

We summarize the target model assumptions as follows.

A1 Each target of interest consists of multiple scattering centers, and each scattering center follows Swerling II fluctuation model (more details above and in Section II-B).
The size of each target (distances between scattering centers) is comparable to the range resolution of the radar, but it is much smaller compared to the cross-range resolution. All scattering centers of the same target have the same off-boresight angles (azimuth and elevation), because of A2. The return signals from different scattering centers are independent, even if the scattering centers are from the same target.

B. Monopulse Returns from Unresolved Extended Targets

Here we will model the monopulse returns from unresolved extended targets and in the next section estimate the locations and number of existing extended targets. Let us assume there are $K$ extended targets illuminated within the radar beam, and that the transmitted signal has pulse shape $\xi(t)$ (for example, a rectangular pulse$^3$). We also assume that

$^3$Note that $\xi(t)$ is a dimensionless quantity of unity power value.
the $k^{th}$ target consists of $N_k$ scattering centers. We model the returns from these targets as

\[ s(t) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} A_{kj} \xi(t - \tau_{kj}) \cos(\omega_c t - \phi_{kj}) + n_s(t) \]  

(5)

\[ d_h(t) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} A_{kj} \xi(t - \tau_{kj}) \cos(\omega_c t - \phi_{kj}) + n_{dh}(t) \]  

(6)

\[ d_v(t) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} A_{kj} \xi(t - \tau_{kj}) \cos(\omega_c t - \phi_{kj}) + n_{dv}(t) \]  

(7)

referring, respectively, to the above as the antenna sum, azimuth- and elevation-difference channels; $A_{kj}$ and $\tau_{kj}$ are respectively the amplitude and the round-trip delay of the radar return from the $j^{th}$ scattering center of the $k^{th}$ target. In (5)–(7), we have

\[ A_{kj} = \sqrt{\kappa} \beta_{kj} G_{\Sigma}(\theta_{hk,k}, \theta_{vk,k}) \]  

(8)

\[ \kappa = \text{constant proportional to the transmitted power} \]  

(9)

\[ \beta_{k} = \text{radar cross section (RCS) of the } j^{th} \text{ scattering center of the } k^{th} \text{ target} \]  

(10)

\[ \theta_{hk} = \text{horizontal off-boresight angle of the } k^{th} \text{ target} \]  

(11)

\[ \theta_{vk} = \text{vertical off-boresight angle of the } k^{th} \text{ target} \]  

(12)

\[ G_{\Sigma}(\theta_{hk,k}, \theta_{vk,k}) = \text{sum channel voltage gain at angle } (\theta_{hk,k}, \theta_{vk,k}) \]  

(13)

\[ G_{\Delta h}(\theta_{hk,k}, \theta_{vk,k}) = \text{horizontal difference channel voltage gain at angle } (\theta_{hk,k}, \theta_{vk,k}) \]  

(14)

\[ G_{\Delta v}(\theta_{hk,k}, \theta_{vk,k}) = \text{vertical difference channel voltage gain at angle } (\theta_{hk,k}, \theta_{vk,k}) \]  

(15)

\[ \eta_{hk} = \frac{G_{\Delta h}(\theta_{hk,k}, \theta_{vk,k})}{G_{\Sigma}(\theta_{hk,k}, \theta_{vk,k})} \text{ (“electronic” horizontal DOA of the } k^{th} \text{ target)} \]  

(16)

\[ \eta_{vk} = \frac{G_{\Delta v}(\theta_{hk,k}, \theta_{vk,k})}{G_{\Sigma}(\theta_{hk,k}, \theta_{vk,k})} \text{ (“electronic” vertical DOA of the } k^{th} \text{ target)} \]  

(17)

\[ \xi(t) = \text{pulse envelope} \]  

(18)

\[ \omega_c = \text{carrier frequency} \]  

(19)

\[ \phi_{kj} = \text{phase angle of the return from the } j^{th} \text{ scattering center of the } k^{th} \text{ target} \]  

(20)

The noises $n_s(t)$, $n_{dh}(t)$ and $n_{dv}(t)$ are zero mean white Gaussian with respective power spectral
densities $N_s$, $N_{dh}$ and $N_{dv}$. The “electronic” monopulse angles $\eta_{hk}$ and $\eta_{vk}$ denote the off-boresight excursion of the $k^{th}$ target, respectively in azimuth and elevation, and are limited to unity in their magnitude: according to standard modeling (e.g., [10][41]) these electronic angles can easily be related to the true off-boresight angles through knowledge of the antenna beam-patterns. Also notice that the scattering centers of the same target have the same electronic angles, as in assumption A3.

The radar returns in (5)–(7) are then processed in detection block as shown in Figure 2, which includes demodulation, matched filtering with impulse response\(^4\) $h(t)$, sampling at rate $1/\Delta$ Hz, and thresholding to decide whether current sample contains signal energy (the reader is referred to [23] for these processes). After the detection block, we have the in-phase (I) and quadrature (Q) parts of the sum, azimuthal- and elevation-difference signals

\[ s_I(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) A_{kj} \cos \phi_{kj} + \nu_{Is}(i) \]  
\[ d_{Ih}(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) \eta_{hk} A_{kj} \cos \phi_{kj} + \nu_{Idh}(i) \]  
\[ d_{Iv}(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) \eta_{vk} A_{kj} \cos \phi_{kj} + \nu_{Idv}(i) \]  
\[ s_Q(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) A_{kj} \sin \phi_{kj} + \nu_{Is}(i) \]  
\[ d_{Qh}(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) \eta_{hk} A_{kj} \sin \phi_{kj} + \nu_{Idh}(i) \]  
\[ d_{Qv}(i) = \sum_{k=1}^{K} \sum_{j=1}^{N_k} q(i\Delta - \tau_{kj}) \eta_{vk} A_{kj} \sin \phi_{kj} + \nu_{Idv}(i) \]

where $q(t) \triangleq \xi(t) \ast h(t)$, and $\ast$ denotes convolution\(^5\). $i$ is the sample sequence index, and we call all the six quantities (in (21)–(26)) with the same index $i$ a sample. The $\nu$’s represent

\(^4\)If we are not interested in Doppler information, we can take the matched filter impulse response $h(t) = \xi(-t)^*$, where the asterisk superscript means complex conjugation. But to allow more freedom in the derivation – specifically to interrogate the effect of a Doppler shift – we keep the filter as $h(t)$.

\(^5\)Note that $q(t)$ is dimensionless (assuming the input and output of the filter are quantities with the same physical dimension, e.g., voltage.)
independent zero mean Gaussian random sequences after the matched filtering and sampling. They are generally no longer white, unless $\Delta$ is longer than or equal to the duration of the pulse.

Both the amplitude $A_{kj}$ and phase $\phi_{kj}$ are random due to target fluctuation [23][30]. In this paper we assume Swerling II model for the RCS fluctuation, which has a pulse-to-pulse fluctuation. Other fluctuation models can also be used with the same framework in this paper, but modifications of end results according to different statistical models are needed.

In Swerling II model, the amplitude $A_{kj}$ has a Rayleigh distribution

$$p(A_{kj}) = \frac{A_{kj}}{\sigma_{kj}^2} \exp\left(-\frac{A_{kj}^2}{2\sigma_{kj}^2}\right)$$

and the phase $\phi_k$ is assumed to be uniformly distributed within $[0, 2\pi)$. $A_{kj}\cos\phi_{kj}$ and $A_{kj}\sin\phi_{kj}$ are independent and identically distributed (i.i.d.) zero mean Gaussian random variables with variance $\sigma_{kj}^2$. Thus the I and Q parts of the sum, azimuthal- and elevation-difference signals in (21)–(26) are zero mean Gaussian random variables, and the I and Q parts are i.i.d. Define $r(t) \triangleq h(t) * h(-t)$, and we have the auto- and cross-correlations of the I and Q parts in (21)–(26) given by

$$\mathcal{E}\{s_I(i_1)s_I(i_2)\} = \mathcal{E}\{s_Q(i_1)s_Q(i_2)\}$$

$$= \sum_{k=1}^{K} \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1\Delta - \tau_{kj})q(i_2\Delta - \tau_{kj}) + \frac{N_s}{2} r((i_1 - i_2)\Delta)$$

$$\mathcal{E}\{s_I(i_1)d_{Ih}(i_2)\} = \mathcal{E}\{s_Q(i_1)d_{Qh}(i_2)\}$$

$$= \sum_{k=1}^{K} \eta_{hk} \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1\Delta - \tau_{kj})q(i_2\Delta - \tau_{kj})$$

$$\mathcal{E}\{s_I(i_1)d_{Iv}(i_2)\} = \mathcal{E}\{s_Q(i_1)d_{Qv}(i_2)\}$$

$$= \sum_{k=1}^{K} \eta_{vk} \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1\Delta - \tau_{kj})q(i_2\Delta - \tau_{kj})$$

$^6$Note that $r(t)$ has dimension 1/time.
\[
\mathbb{E}\{d_{1h}(i_1)d_{1h}(i_2)\} = \mathbb{E}\{d_{Qh}(i_1)d_{Qh}(i_2)\} \\
= \sum_{k=1}^{K} \eta_{hk}^2 \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1 \Delta - \tau_{kj}) q(i_2 \Delta - \tau_{kj}) + \frac{N_{dh}}{2} r((i_1 - i_2) \Delta)
\]

(31)

\[
\mathbb{E}\{d_{1h}(i_1)d_{1v}(i_2)\} = \mathbb{E}\{d_{Qh}(i_1)d_{Qv}(i_2)\} \\
= \sum_{k=1}^{K} \eta_{hk}\eta_{vk}^2 \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1 \Delta - \tau_{kj}) q(i_2 \Delta - \tau_{kj})
\]

(32)

\[
\mathbb{E}\{d_{1v}(i_1)d_{1v}(i_2)\} = \mathbb{E}\{d_{Qv}(i_1)d_{Qv}(i_2)\} \\
= \sum_{k=1}^{K} \eta_{vk}^2 \sum_{j=1}^{N_k} \sigma_{kj}^2 q(i_1 \Delta - \tau_{kj}) q(i_2 \Delta - \tau_{kj}) + \frac{N_{dv}}{2} r((i_1 - i_2) \Delta)
\]

(33)

The functions \(q(\cdot)\) and \(r(\cdot)\) are defined previously. The power of the signal from the \(j^{th}\) scattering center of the \(k^{th}\) target is \(E[A_{kj}q(0)^2] = 2\sigma_{kj}^2 q(0)^2\). The noise power is given by \(N_{s}r(0)\) and the signal to noise ratio in the extended target model is defined as

\[
\text{SNR} \triangleq \frac{2\sigma_{kj}^2 q^2(0)}{N_{s}r(0)}.
\]

(34)

The signals in (21)–(26) are then processed in the segmentation block of the system shown in Figure 2. We term all these signals (sampled at the same time) a \emph{sample}. In the segmentation block, the sample sequence is divided into sets of samples according to certain algorithms. Each sample set consists of finite number of samples containing return signals from a possible group of unresolved extended targets. Assuming as we do that the target size is comparable to the range bin length (good range resolution), and target density in a certain region that is not overwhelmingly high, it is not uncommon to see sets consisting of only two or three samples.

Each of these sets is then processed within the resolution and localization block to find out how many targets are associated with that set and what their DOA’s are. Now, the main goal is to estimate the number of targets \(K\) and the DOA’s \(\{\eta_{hk}, \eta_{vk}\}_{k=1}^{K}\) of each target. Notice that the range resolution of each target is already very good compared with the cross-range resolution, nevertheless, the range\(^7\) \(\{\{\tau_{kj}\}_{j=1}^{N_k}\}_{k=1}^{K}\) of each scattering center of each target will be estimated in sub-range-bin accuracy. The number of scattering centers \(N_k\), and the signal power of each scattering center \(\{\sigma_{kj}^2\}_{j=1}^{N_k}\}_{k=1}^{K}\) will also be estimated.

\(^7\)Here we abuse the notation \(\tau_{kj}\) to represent both the range and the time delay since they can be easily translated into each other.
Note that some of these parameters may be known a priori. For example, we may have prior knowledge or side information of the signal power $\{\sigma^2_{kj}\}_{j=1}^{N_k} \{\text{for each target. This will result in fewer parameters to estimate and can reduce the computation of the estimation algorithm. Here we give the formulation for the general scenario, which can be tailored for specific applications with certain prior information about the targets.}

Define the parameter vector to be estimated as $\Theta \triangleq \{\{\tau_{kj}, \sigma^2_{kj}\}_{j=1}^{N_k}\}_{k=1}^{K}$. In practice, if we have prior knowledge of some of these parameters, we can define $\Theta$ accordingly. For example, if we know the target SNR (equivalent to the signal power $\{\sigma^2_{kj}\}_{j=1}^{N_k} \{\text{since the noise power in the definition of SNR (34) can be easily estimated}), the parameter vector is then defined as $\Theta \triangleq \{\{\tau_{kj}\}_{j=1}^{N_k}, \eta_{hk}, \eta_{vk}\}_{k=1}^{K}$. $K$ and $N_k$ are not included in $\Theta$ because $\Theta$ will be extracted by maximum likelihood estimation (MLE), while $K$ and $N_k$ will be determined with minimum description length (MDL) criterion, as will be shown in Section III.

Normally, radars send more than one pulse in each dwell to get more samples (hence higher overall SNR) of the return signal [23][30]. Recall that in Swerling II model, there is pulse-to-pulse fluctuation, i.e., the returns from different pulses are independent and identically distributed.

We assume the radar send $M$ pulses in each dwell, and consider a set containing $L$ return samples for each pulse. For $M$-pulse radar returns, the segmentation block in Figure 2 forms a set of samples $\{\{y_{im}\}_{i=i_0}^{i_0+L-1}\}_{m=1}^{M}$, where $y_{im}$ is the $i^{th}$ sample from the $m^{th}$ pulse. Notice that the sample sequences from the pulses are time aligned, so the $i^{th}$ samples from all the pulses correspond to the same resolution cell. Note that sample $y_{im}$ contains $\{s_I(i), d_{hI}(i), d_{vI}(i), s_Q(i), d_{hQ}(i), d_{vQ}(i)\}$ from the $m^{th}$ pulse.

Let us define the observation vector $z_m = \{y_{im}\}_{i=i_0}^{i_0+L-1}$, and reset $i_0 = 1$ for convenience. So, we have the observation vector from the $m^{th}$ pulse as $z_m = [z_{mI} \ z_{mQ}]'$, where

$$z_{mI} = [s_I(1) \ldots s_I(L) \ d_{hI}(1) \ldots d_{hI}(L) \ d_{vI}(1) \ldots d_{vI}(L)]'$$

$$z_{mQ} = [s_Q(1) \ldots s_Q(L) \ d_{hQ}(1) \ldots d_{hQ}(L) \ d_{vQ}(1) \ldots d_{vQ}(L)]'$$

The vector $z_m$ is Gaussian, with zero mean and covariance matrix $\mathcal{E} \{z_m z_m'\} = \mathbf{R} = \text{diag}(\mathbf{R}_I, \mathbf{R}_Q)$ having elements as given in (28)–(33). We have the joint probability density function (pdf)

$$p(z_m|\Theta, K, \{N_k\}_{k=1}^{K}) = \frac{1}{\sqrt{|2\pi\mathbf{R}|}} \exp\left(-\frac{1}{2}z_m'\mathbf{R}^{-1}z_m\right)$$

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for the returns from the $m^{th}$ pulse. Note that the dependence of this pdf on $\{\Theta, K, \{N_k\}_{k=1}^K\}$ is implicit through the correlation matrix $R$.

For the case of a Swerling II target (e.g., [23]), we have the likelihood function of all the samples in the set $\{z_m\}_{m=1}^M$ given the parameters $\Theta, K, \{N_k\}_{k=1}^K$

$$p\left(\{z_m\}_{m=1}^M \mid \Theta, K, \{N_k\}_{k=1}^K\right) = \frac{1}{(\sqrt{2\pi R})^M} \exp\left(-\sum_{m=1}^M \frac{1}{2} z_m' R^{-1} z_m\right)$$

Note that in Swerling II model, the observation vectors $\{z_m\}_{m=1}^M$ are independent and identically distributed.

### III. Maximum Likelihood Extraction of the Target Locations and Deciding the Number of Targets by the Minimum Description Length Criterion

With the formulation from the previous section, we can get the maximum likelihood estimates (MLEs) of the parameter vector as

$$\hat{\Theta} = \arg \max_{\Theta} \left\{ p\left(\{z_m\}_{m=1}^M \mid \Theta, K, \{N_k\}_{k=1}^K\right) \right\}$$

subject to the constraints

$$\sigma_{kj}^2 \geq 0$$

$$0 \leq \tau_{kj} \leq (L - 1)\Delta$$

$$-1 \leq \eta_{hk} \leq 1$$

$$-1 \leq \eta_{vk} \leq 1$$

for $k = 1, \ldots, K$ and $j = 1, \ldots, N_k$. These constraints correspond to nonnegative signal power $\sigma_{kj}^2$, the scattering center’s range $\tau_{kj}$ being inside the set, and the targets’ DOA’s $\eta_{hk}$ and $\eta_{vk}$ being within the beam. For this constrained nonlinear maximization, there are a number of methods we can use from the optimization literature: for example, see [5]. We have had good results with the gradient projection method, which reflects the parameter vector back onto the constraint set if an attempt to exit it (outside the beam) occurs. To make the algorithm more efficient, we use the interior-point algorithm from [19], which is designed for affine-scaling optimizations like the above. Their algorithm is proved to have super-linear convergence, and in our experiment, their algorithm converges much faster than the original gradient projection algorithm.
In the Maximum Likelihood estimation above, we can estimate the locations and signal power of the targets (and scattering centers) of a given number of targets and the number of their scattering centers. The number of targets $K$ and the number of their scattering centers $\{N_k\}_{k=1}^K$ are assumed to be known, in order to construct $p\left(\{z_m\}_{m=1}^M | \Theta, K, \{N_k\}_{k=1}^K\right)$. However, $K$ and $\{N_k\}_{k=1}^K$, also must be decided. There are several schemes for this multiple composite hypothesis testing. The vehicle we suggest for this is the minimum description length (MDL) criterion [18] [31].

The general MDL criterion is as follows: Given a data set of interest and a family of competing statistical models, the best model is the one that provides the shortest description length for the data. The description length is defined by:

$$\text{MDL}(K, \{N_k\}_{k=1}^K) = -\ln \left( p\left(\{z_m\}_{m=1}^M | \hat{\Theta}, K, \{N_k\}_{k=1}^K\right) \right) + \frac{1}{2} q \ln(n)$$

(44)

where $q$ is the number of independently adjusted parameters in the model, $n$ is the number of observations and $\log \left( p\left(\{z_m\}_{m=1}^M | \hat{\Theta}, K, \{N_k\}_{k=1}^K\right) \right)$ is the log-likelihood of the maximum-likelihood estimates of the parameters in the model. In general cases, we have $q = \sum_{k=1}^K (2N_k + 2)$ for the estimation of $\{\tau_{kj}, \sigma_{kj}^2\}_{j=1}^{N_k}, \eta_{hk}, \eta_{vk}\}_{k=1}^K$.

If we have prior knowledge of some of these parameters, the prior knowledge can be used and those parameters are not estimated. In this case, we use a different $q$ accordingly. For example, if we know the SNR beforehand, we then have $q = \sum_{k=1}^K (N_k + 2)$ for the estimation of $\{\tau_{kj}\}_{j=1}^{N_k}, \eta_{hk}, \eta_{vk}\}_{k=1}^K$. The number of observations is set to $n = 6LM$, accounting for the facts that we have $L$ samples of the sum, azimuthal- and elevation-difference channels and that we have $M$ complex pulses separated into $2M$ real pulses. We decide the number of targets present and the number of scattering centers for each target by

$$\left\{\hat{K}, \{\hat{N}_k\}_{k=1}^K\right\} = \arg \min_{\{K,\{N_k\}_{k=1}^K\}} \{\text{MDL}(K, \{N_k\}_{k=1}^K)\}$$

(45)

The whole problem can be solved in the following three steps:

- First, we form combinations of possible number of targets and number of scattering centers for each target.
- Next, the parameters for each combination are estimated by maximum likelihood.
- Then all estimated parameters for the combinations are inserted to the MDL test to decide the number of targets.
As will be shown in Section IV, the combinations of possible number of targets and their scattering centers grow exponentially with the size of the set $L$ (determined by the segmentation block). Fortunately, as pointed out in Section II, we are likely to have sets with $L$ equal to two or three in the scenarios of interest. This aspect should also be considered in the segmentation block to avoid long sets.

Another way to reduce computation complexity is to use available prior information on the parameters when possible, even if the prior information is not exact. As will be shown in the simulations, even “guesses” on the scattering center SNRs that are as much +/-5 dB different from the truth can provide reasonably good performance while reducing the computation complexity. This can serve as a design trade-off.

The MDL measure is generally (fortunately) concave in $\{K, \{N_k\}_{k=1}^K\}$, hence a solution is guaranteed. The procedure described above will be referred to as MRSP-E: Monopulse Radar Signal Processing with an Extended target model.

**IV. LIMITS ON THE NUMBER OF TARGETS AND THE NUMBER OF SCATTERING CENTERS**

There are two reasons why we are interested in the limits on the number of targets and their scattering centers. First, it is necessary to know the limits of our method, i.e., how many targets we can correctly extract with our approach. Second, we need to provide candidate numbers of targets and scattering centers to the MDL, and that can be determined by the limits for the the specific segmentation set.

If two sets of parameters yield the same likelihood function, the observations drawn from these two distributions are statistically the same: it is impossible to distinguish between the two sets of parameters from the observations, and we do not have a unique solution to the maximum likelihood estimation in Section III. Thus we call such sets of parameters in the likelihood function *unidentifiable*.

We begin the discussion of identifiability with a few definitions. The ratio of pulse duration $T$ and sampling interval $\Delta$ (we assume the ratio is an integer) is

$$\gamma \triangleq \frac{T}{\Delta}$$

(46)

Let us consider a segmentation set with $L$ samples per radar pulse. We define the set of target
indices $A(l, l_0)$,

$$A(l, l_0) \triangleq \{ k: \text{target } k \text{ has one or more scattering centers located between}$$

$$\text{the } l^{th} \text{ and } (l + l_0)^{th} \text{ samples} \}$$ \hspace{1cm} (47)

and $n_k(l, l_0)$ as the number of scattering centers (from the $k^{th}$ target, $k \in A(l, l_0)$) that are located between the $l^{th}$ and $(l + l_0)^{th}$ samples. We have $l \in \{1, 2, \cdots, L - 1\}$ and $l_0 \in \{1, 2, \cdots, L - l\}$. We also define $g_k(l, l_0)$ as the number of parameters (to be estimated) associated with these $n_k(l, l_0)$ scattering centers. If there is no prior knowledge of any parameters, we have $g_k(l, l_0) = 2(n_k(l, l_0) + 1)$. This is because we have the range and SNR of these $n_k(l, l_0)$ scattering centers to estimate as well as two angle parameters (since we assume different scattering centers from the same target have the same angles). However, if we know any of the parameters, we have to calculate $g_k(l, l_0)$ accordingly. For example, if the SNR of each scattering center is known in advance, we then have $g_k(l, l_0) = n_k(l, l_0) + 2$.

We define $m(l, l_0)$ as the number of nonempty “bins” between the $l^{th}$ and $(l + l_0)^{th}$ samples, where a “bin” is the space between two consecutive matched filter samples and “nonempty” means there is at least one scattering center located in the bin. Obviously we have $m(l, l_0) \leq l_0$.

We have the following necessary condition for the parameters $\Theta$ to be identifiable given a block of $L$ consecutive samples.

**Theorem 1** For any $l \in \{1, 2, \cdots, L - 1\}$ and $l_0 \in \{1, 2, \cdots, L - l\}$, the number of identifiable scattering centers located between the $l^{th}$ and $(l + l_0)^{th}$ samples must satisfy

$$\sum_{k \in A(l, l_0)} g_k(l, l_0) \leq 6\gamma(2\gamma + 1)m(l, l_0)$$ \hspace{1cm} (48)

**Proof:** See the Appendix. \hfill ■

Let us see a few examples.

**Example 1:** All the targets are point targets and we consider unresolved targets within a segmentation set of $L = 2$ samples. We have $l_0 = 1$ and we use $T = \Delta$, that is $\gamma = 1$. The number of scattering centers for each target is $n_k(l, l_0) = 1$ and the number of nonempty bins in this case is $m(l, l_0) = 1$. If the SNR of each target is known, we have $g_k(l, l_0) = n_k(l, l_0) + 2 = 3$. Using equation (48), we have

$$\sum_{k \in A(l, l_0)} g_k(l, l_0) = 3K \leq 6\gamma(2\gamma + 1)m(l, l_0) = 18$$ \hspace{1cm} (49)
Thus the number of targets \( K \leq 6 \). This means with two consecutive samples, we can at most identify six point targets in between them. The same situation was studied in [41], and we use this example to show that the identifiability analysis of point target model\(^8\) is a special case of Theorem 1.

**Example 2:** As depicted in Figure 4, we consider extended targets, each with two scattering centers. Both scattering centers of each target are located in one bin (between two consecutive samples, and \( L = 2 \)), which means there is no bin straddling. We have \( n_k(l, l_0) = 2, \gamma = 1, \) and \( m(l, l_0) = 1 \). If we have no prior knowledge on any of the parameters, \( g_k(l, l_0) = 2n_k(l, l_0) + 2 = 6 \) Using equation (48), we have \( 6K \leq 18 \) and that gives the number of targets \( K \leq 3 \), meaning the maximum number of targets we can identify in this case is three (containing six scattering centers).

**Example 3** Target setup is similar to Example 2, but here each target straddles two consecutive bins, e.g., each target has its first scattering center between the first and second samples and its second scattering center between the second and third samples. Note that in this case, the segmentation set has three samples per pulse. We have \( n_k(l, 1) = n_k(l + 1, 1) = 1, n_k(l, 2) = 2, \gamma = 1, m(l, 1) = m(l + 1, 1) = 1, \) and \( m(l, 2) = 2 \). Here we assume there is no prior knowledge of the parameters and we have \( g_k(l, l_0) = 2n_k(l, l_0) + 2 \). Since we have three consecutive samples to be considered, we have to jointly satisfy the following three inequalities:

\[
\sum_{k \in A(l,1)} g_k(l, 1) \leq 6\gamma(2\gamma + 1)m(l, 1) \tag{50}
\]

\[
\sum_{k \in A(l+1,1)} g_k(l + 1, 1) \leq 6\gamma(2\gamma + 1)m(l + 1, 1) \tag{51}
\]

\[
\sum_{k \in A(l,2)} g_k(l, 2) \leq 6\gamma(2\gamma + 1)m(l, 2) \tag{52}
\]

\(^8\)In [41] we use a rectangular radar waveform and the special shape of the waveform actually allows fewer targets. Here we assume a general waveform \( \xi(t) \) and in the best case six targets can be resolved. This explains why the maximum number of targets that can be identified is five in [41] while it is six here.
We then have

\[ 4K \leq 18 \]  
\[ 4K \leq 18 \]  
\[ 6K \leq 36 \]

In order to satisfy all three inequalities, we have the number of targets \( K \leq 4 \), meaning the maximum number of targets we can identify in this case is four.

Unfortunately, as pointed out in [41], even for a much simpler point target case there is no simple sufficient condition for identifiability because it depends heavily on the locations of the targets and their scattering centers. Further, it is lengthy to list all the possible target location configurations and there can be no generalized rules; nevertheless, with the necessary conditions above, we can have the upper limit on the number of targets and scattering centers, which is essential to the MDL test in Section III.

V. SIMULATION RESULTS

In this section, we present simulation results of the resolution and localization algorithm proposed in this paper. The radar signal processing system is shown in Figure 2, and this paper focuses on the resolution and localization block (detection and segmentation already done). In the simulations, the inputs to the resolution and localization block are segmentation sets that contain samples between which unresolved targets are likely to be located.

We will first present results of resolution and localization when no prior knowledge of the parameters to estimate is available, i.e., we do not know the SNR’s of the scattering centers, the number of scattering centers per target, etc. We will then show resolution and localization results when prior information of some of the parameters is available. We will also show the performance when the prior information is not accurate. Finally, we will present target tracking results from a multiple hypothesis tracker that uses the output of our resolution and localization algorithm as its input.

A. Resolution and Localization without Prior Information of Parameters

We set up the simulation scenario as follows:
• A monopulse radar with 10 GHz carrier frequency. As shown in Section II-A, the range resolution of this radar is 15 m, and cross range resolution is 900 m, at 30 km range.

• The radar transmits 5 pulses in a dwell, and each pulse has a rectangular envelope $\xi(t)$ with pulse duration $T = 1 \times 10^{-7}$ second. The 4-dB bandwidth is 10 MHz. The sampling interval is the same as the pulse width $\Delta = T$ (Nyquist sampling rate).

• Two targets are located 30 km from the radar, and each target has two scattering centers. All the scattering centers are located within the same beam (Figure 4) and are between two sampling points, i.e., the targets are unresolved.

• The length of the target is comparable to the 15 m range resolution. The target length and orientation are not observable because the cross-range resolution is too coarse. As stated previously, the scattering centers from each target have the same DOA’s.

• With the notations defined in Section II-B, we have the target locations represented by $(\tau_{kj}, \eta_{hk}, \eta_{vk})$. In the simulations, the range $\tau_{kj}$ is normalized to the range resolution (15 m), and the off-boresight angles $\eta_{hk}$ and $\eta_{vk}$ are normalized to the beam width. The scattering centers of the first target are located at $(0.4, -0.1, 0.2)$ and $(0.9, -0.1, 0.2)$, while those of the second target are located at $(0.3, 0.7, -0.6)$ and $(0.7, 0.7, -0.6)$.

• The algorithm does not know any prior information about the targets, including the SNR’s of scattering centers or number of scattering centers for each target.

The goal is to determine how many targets are associated with the segmentation set (resolution), and estimate their azimuths and elevations (localization), with sub-beamwidth accuracy. Because of the good range resolution, estimating the ranges of the targets (more precisely the scattering centers for the extended target case) in a sub-range-bin accuracy is not as important. The SNR estimation (of the scattering centers) is not as important as the number of targets and their DOA’s. Nevertheless, the algorithm generates these range and SNR estimates as “by-products”.

Figures 5 shows the percentage of correctly determining the number of present targets as a function of the average SNR of the radar returns from the scattering centers, with the ground truth being two targets. We can see that the algorithm performs reasonably well with 10dB SNR and perfectly identifies the two unresolved target at SNR of 15dB and above.

Figure 6 and Figure 7 show one-sigma error of the estimates of the two targets’ azimuths and elevations. The one-sigma estimation error is calculated from the cases with correctly determined
number of target. This is the reason for the moderate improvement in accuracy with increasing SNR. Recall the azimuth and elevation are in the range of \([-1,1]\), so the accuracy is about 10\% of the beamwidth.

**B. Resolution and Localization with Prior Information of Parameters**

Here we investigate the performance of the algorithm when there is side information on the SNR’s of the scattering centers. We also look at a case for which, even though there is no side

---

**Fig. 5.** The percentage of correct decision on the number of targets present as a function of average SNR of scattering centers from all the targets, without prior knowledge of target SNR.

**Fig. 6.** One-sigma error of azimuth estimation, without prior knowledge of target SNR.

**Fig. 7.** One-sigma error of elevation estimation, without prior knowledge of target SNR.

**Fig. 8.** The percentage of correct decision on the number of targets, for the cases when the SNR’s are unknown, known, and guessed.
information on the SNR’s, assumed SNR’s (equal to 15 dB) are used in the algorithm just to accelerate the computation.

The simulation scenario here is almost the same as that in Section V-A, except that side information is available and the SNR of each scattering center is known exactly in one case.

Another interesting case is when there is no side information on the SNR, but the algorithm assumes 15 dB SNR on each scattering center. One reason we look at this case is to quantify the performance loss if incorrect information on the SNR is used. Because the computation complexity of the algorithm can be reduced if the algorithm does not estimate the SNR’s, we also would like to find out if using guessed SNR’s instead of estimating them can be worthwhile.

In Figure 8, we compare the percentage of correctly estimating the number of targets, for three cases when the SNR is unknown and estimated (as in Section V-A), the SNR is known exactly, and the SNR is guessed to be 15 dB. The knowledge of exact SNR’s does not help increasing the percentage by much. Another finding is an error of +/- 5 dB in the guessed SNR does not incur too much degradation in percentage of correct estimation of the number of targets. However, significant under estimation of the SNR can cause catastrophic results.

In Figure 9 and Figure 10, we compare the accuracy of the azimuth and elevation estimations for the three cases. Here the knowledge of SNR does make a noticeable difference. For example, at 15 dB SNR, the information of exact SNR brings the one-sigma estimation error from 0.3 to 0.15 (from 15% to 7.5% of beamwidth). Another interesting finding is high SNR does not always mean better accuracy. This is because the objective function in the optimization problem in (39) becomes more spiky and ill-conditioned with higher SNR.

We next compare the computation complexity for the case where the SNR’s of the scattering centers are estimated and the case where the SNR’s are either known or guessed. The running time for solving the optimization problem in (39) is recorded. The platform is a PC with Intel processor at 2.8 GHz, 1 GB memory, Windows XP operation system, and MATLAB programming.

The average running time as a function of average ground-truth SNR’s of the scattering centers is plotted in Figure 11. We notice that the cases with ground-truth SNR’s higher than 25 dB can take much more computational time. This is because the objective function in the optimization problem in (39) becomes more spiky and ill-conditioned with higher SNR, same as that stated above in the discussion of Figure 9 and Figure 10.

The best computation time for the case where the SNR’s need to be estimated is at 15 dB
without prior information on SNR
with exact information on SNR
with guessed SNR = 15dB

Fig. 9. One-sigma error of azimuth estimation.

without prior information on SNR
with known or guessed SNR

Fig. 10. One-sigma error of elevation estimation.

without prior information on SNR
with known or guessed SNR

Fig. 11. Computation times for the case where the SNR’s are estimated and the case where the SNR’s are either known or guessed.

Fig. 12. The ratio of the computational time for the case with known or guessed SNR’s over that for the case where the SNR’s need to be estimated.

SNR, and it is 3.16 seconds. For SNR’s from 5 dB to 20 dB, the computational time is below 6 seconds.

In our previous experiences, if the program is in C language, it can run much faster than that in MATLAB (in the order of a few tens times faster). In practice, dedicated digital signal processors are commonly used, and computational time can possibly be reduced even more. If the radar updates the number and locations of targets every second, the proposed algorithm can be implemented with a single processor.

The ratio of the computational time for the case with known or guessed SNR’s over that for
the case where the SNR’s need to be estimated is plotted in Figure 12. In practical designs, one can look at Figure 8 – Figure 10, and Figure 12 to decide if is appropriate to guess the SNR’s instead of estimating them.

C. Tracking Performance with MHT

Perhaps a better performance analysis is by presenting the estimates from the monopulse processor (the resolution and localization block) to a multi-target tracker to compare the tracking performances. The reasons why we are interested in the tracking performance are

- The purpose of the monopulse radar signal processing is to provide measurements to track these targets. Thus it is meaningful to test how the monopulse signal processing affects the tracking performance.
- Tracking algorithms use both the current measurements and previous target states to update current target states, declaring birth of new targets and death of old targets, and decide if one measurement is a false alarm or from a certain target. Thus the number of present targets can be determined more intelligently with tracking algorithms.

Here we use a hypothesis oriented multiple hypothesis tracking (MHT) algorithm [3][4][29], which has track management (declaration of track birth and death) incorporated. References [3] and [7] provide general background on target tracking and a large collection of target tracking algorithms.

We compare the proposed Monopulse Radar Signal Processing with an Extended target model (MRSP-E) with the point target approach in [40] and [41] which seems to be the only comparable technique available in the literature. Since the point target approach is not for extended targets, in order to be “fair” to the point target approach, we have to use an (admittedly) ad-hoc modification on the point target oriented approach to decide the number of extended targets. We use the following criterion for the Monopulse Radar Signal Processing with a Modified Point target model (MRSP-MP)

**Criterion for MRSP-MP:** After the point-target oriented approach, we perform a centroiding which groups (with weighted average) the estimated point scattering centers into a single point target if the distance between these estimated point scattering centers is less than or equal to \( l_{\max} \), the maximum possible target length and it is a design factor.
In the simulations here, we choose $l_{\text{max}}$ to be the range resolution, i.e., the longest target we are interested in is the range resolution (15 m), which is larger than the true length of the target (10 m) in our simulations.

The simulation scenario is set up as follows. There is one 10-meter-long target (with two scattering centers as before) entering the scene at 100km from the radar and coming right toward the radar. The velocity of the target is 1000 m/s. The radar generates estimations of the number of targets and their locations every second. The parameters of the radar are the same as those in Section V-A, unless stated otherwise. At the 8th second, the target is separated into two targets, each with the same length as the original target and each with two scattering centers. Their relative ranges remain the same and they are separating in both azimuth and elevation with constant speed. Note that the two targets remain unresolved (in the same resolution cell) throughout the simulation (from the 8th second to the 20th second). We use the same SNR of all the scattering centers before and after the separation. We apply both the extended and point target monopulse processors in the radar to estimate the number of targets as well as their locations. The results are then presented to an MHT to track the targets.

For all the simulations here (Section V-C), the monopulse signal processors do not have any prior information about the targets’ SNR, the number of scattering centers for each target, the target length, etc.

First, the results for a single run are plotted in Figure 13 and Figure 14, where the ground-truth SNR of each scattering center is 15 dB. At the beginning, the MHT with the proposed MRSP-E clearly recognizes that there is only one extended target while the MHT with MRSP-MP thinks there are two extended targets. After the target splits at the 8th second, the MHT with the proposed MRSP-E detects the target split 2 seconds later and generates a new track for the new target. The MHT with MRSP-MP also detects the split 2 seconds later, but it generates a third track. As the targets move on, the MHT with MRSP-E maintains the two established tracks, but the MHT with MRSP-MP generates a fourth track and the tracks experience several switches. As we can see, the MHT with the proposed MRSP-E provides much more stable tracks.

To clearly illustrate the tracking performance of MHT with these two monopulse radar signal processing scheme, we look at the average number of confirmed targets of the most likely hypothesis in the MHT algorithm. In Figure 15–18, the number of targets in the MHT with both MRSP-E and MRSP-MP are plotted versus time. We also include the ground truth in the figure.
to clearly illustrate the ground-truth. The SNR of each scattering center varies from 5 dB in Figure 15 to 20 dB in Figure 18, and 100 Monte Carlo runs are used to generate this figure.

MRSP-MP has a tendency of recognizing more targets than what are actually present. This tendency is more obvious at higher SNR (Figure 17 and 18). For example, in Figure 17, the MHT with MRSP-MP gets an average number of two confirmed targets, while the MHT with MRSP-E has one confirmed target, which is correct according to the ground truth. After the target split, the average number of confirmed targets from the MHT with MRSP-MP rises to three and eventually to four. On the other hand, the average number of targets from the MHT with MRSP-E follows the ground truth and reaches two four seconds later and stays there. This is consistent with the discussion above based on a single run.

In all the cases with different SNR’s, MRSP-E makes the correct decision on the number of unresolved targets. There is a delay for MRSP-E to recognize that the target has split into two. In cases with higher SNR’s, MRSP-E recognize the target split quicker than MRSP-E in cases with lower SNR’s. For the case with 5 dB SNR, it takes 13 seconds for MRSP-E (with MHT) to detect the target split in all the Monte Carlo runs. But for the case with 20 dB SNR, it only takes 3 seconds for MRSP-E in all the Monte Carlo runs to recognize the target split.
VI. SUMMARY

We have presented a model for detection and localization of multiple unresolved extended targets via joint multiple-bin monopulse processing. A maximum likelihood estimator (MLE) was developed to extract estimates of the parameters (sub-bin range, azimuth, elevation and possibly the SNR) of multiple unresolved targets. We suggest the use of the minimum description length (MDL) criterion to decide the number of targets present. We also derived the upper limit on
the number of targets and their scattering centers that can be detected by the proposed method. Simulations show that the extended target monopulse processor can successfully resolve extended targets, and it is superior to that of the point target monopulse processor in tracking performance when followed by a multitarget tracker.

APPENDIX

PROOF OF THEOREM 1

Proof: As can be seen from (37) the parameters are represented in the likelihood only in the form of the covariance matrix \( R \). From (33) we know that \( R_I = R_Q \), so \( R = \text{diag}(R_I, R_Q) \) is determined by \( R_I \). To test whether the parameters are identifiable, it is equivalent to see whether there exist two (or more) sets of parameters that yield the same covariance matrix \( R_I \).

With a segmentation set of \( L \) consecutive samples, \( R_I \) is a \( 3L \times 3L \) matrix (since in each sample we have the sum, horizontal and vertical difference channels). Write these as a vector \( g \):

\[
g = [R_{1,1}, R_{2,1}, \ldots, R_{3L,3L}]^T
\]

(56)

Note that \( g \) has \((3L)^2\) elements. Define the Jacobian matrix

\[
J = \frac{\partial g}{\partial \Theta} = \begin{bmatrix}
\frac{\partial g_1}{\partial \eta_{h1}} & \frac{\partial g_1}{\partial \eta_{v1}} & \frac{\partial g_1}{\partial \tau_{11}} & \cdots & \frac{\partial g_1}{\partial \sigma_{211}} & \frac{\partial g_1}{\partial \sigma_{21N_1}} & \frac{\partial g_1}{\partial \eta_{h2}} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\
\frac{\partial g_{(3L)^2}}{\partial \eta_{h1}} & \frac{\partial g_{(3L)^2}}{\partial \eta_{v1}} & \frac{\partial g_{(3L)^2}}{\partial \tau_{11}} & \cdots & \frac{\partial g_{(3L)^2}}{\partial \sigma_{21N_1}} & \frac{\partial g_{(3L)^2}}{\partial \sigma_{21N_1}} & \frac{\partial g_{(3L)^2}}{\partial \eta_{h2}} & \cdots
\end{bmatrix}
\]

(57)

We have the following observations of the Jacobian matrix \( J \),

1. Each column of \( J \) consists of the derivatives of all the elements in \( R_I \) with respect to one parameter to be estimated.
2. The total number of columns of \( J \) is exactly the number of parameters of the targets and it is equal to \( \sum_{k=1}^{K} 2(N_k + 1) \).
3. If the energy from a scattering center is not picked up at a sample (because of the finite duration of the pulse), the corresponding element in \( R_I \) is not related to the parameters of this scattering center, and the corresponding elements in \( J \) is zero.
4. There are \((3L)^2\) rows in \( J \), but only \( 3L(L + 1) \) of them are unique. They are located in the shaded area in Figure 19. This is because the covariance matrix \( R_I \) is symmetric, and
from equations (28) – (33), we also have

\[
\mathcal{E}\{s_I(i_1)d_{Ih}(i_2)\} = \mathcal{E}\{s_I(i_2)d_{Ih}(i_1)\}
\]

(58)

\[
\mathcal{E}\{s_I(i_1)d_{Iv}(i_2)\} = \mathcal{E}\{s_I(i_2)d_{Iv}(i_1)\}
\]

(59)

\[
\mathcal{E}\{d_{Ih}(i_1)d_{Iv}(i_2)\} = \mathcal{E}\{d_{Ih}(i_2)d_{Iv}(i_1)\}
\]

(60)

Thus only the shaded items in Figure 19 are unique, and there are \(3L(L+1)\) of them.

A necessary condition for identifiability is that there exists no \(h\) such that

\[
Jh = 0
\]

(61)

Otherwise, we will have (for a small real value \(\epsilon\)),

\[
g(\theta) = g(\theta + \epsilon h)
\]

(62)

which contradicts identifiability. Thus the necessary condition for the parameters \(\Theta\) to be identifiable is the Jacobian \(J\) has full column rank.

If \(J\) has full column rank, any matrix constructed with a subset of the columns of \(J\) has full column rank. Now let us construct a matrix \(J(l, l_0)\) with the columns from \(J\) which are related to the parameters of all the scattering centers located between the \(l^{th}\) and \((l + l_0)^{th}\) samples. Then \(J(l, l_0)\) has full column rank for any \(l \in \{1, 2, \ldots, L-1\}\) and \(l_0 \in \{1, 2, \ldots, L-l\}\). That
means the number of columns of $\mathbf{J}(l, l_0)$ has to be less than or equal to the number of unique and nonzero (not all the items in the row are zero) rows of $\mathbf{J}(l, l_0)$.

From the construction above, we know that the number of columns of $\mathbf{J}(l, l_0)$ is the number of parameters associated with all the scattering centers located between the $l^{th}$ and $(l + l_0)^{th}$ samples. These scattering centers are from the targets with indices in the set $A(l, l_0)$, as we have defined before. We also have $n_k(l, l_0)$ as the number of scattering centers (from the $k^{th}$ target, $k \in A(l, l_0)$) that are located between the $l^{th}$ and $(l + l_0)^{th}$ samples. Recall that we define $g_k(l, l_0)$ as the number of parameters (to be estimated) associated with these $n_k(l, l_0)$ scattering centers. So we have the number of columns of $\mathbf{J}(l, l_0)$ given by $\sum_{k \in A(l, l_0)} g_k(l, l_0)$.

Now let us take a look at the number of unique and nonzero rows. Recall that each row of $\mathbf{J}(l, l_0)$ is the derivative of one item of the covariance matrix $\mathbf{R}_I$ with respect to the parameters of the scattering centers located between the $l^{th}$ and $(l + l_0)^{th}$ samples. The elements of $\mathbf{R}_I$ can be separated into 6 groups as different combinations of the sum channel, and the horizontal and vertical channels, as listed in (28) – (33). From these equations, it is obvious that the numbers of unique and nonzero elements in each group are the same. So we now concentrate on the group of $\mathcal{E}\{s_I(i_1)s_I(i_2)\}$ and later just multiply the number of unique rows (in $\mathbf{J}(l, l_0)$) and related to $\mathcal{E}\{s_I(i_1)s_I(i_2)\}$ by 6 to get the total number of unique and nonzero rows.

From equation (28) we can see, if there is at least one scattering center located between sample $l'$ and $l' + 1$, $\mathcal{E}\{s_I(i_1)s_I(i_2)\}$ will be nonzero, $\forall i_1, i_2 \in \{l' - \gamma + 1, \ldots, l' + \gamma\}$, where $\gamma$ is the integer ratio of $T$ and $\Delta$. Simple algebra shows that the number of unique and nonzero elements is $\gamma(2\gamma + 1)$, because of the scattering centers located between sample $l'$ and $l' + 1$. For example, if there is one scattering center located between sample 1 and sample 2, and $\gamma = 1$ (the sampling interval is the same as the pulse width), we have $\gamma(2\gamma + 1) = 3$ unique and nonzero elements $\mathcal{E}\{s_I(1)s_I(1)\}$, $\mathcal{E}\{s_I(1)s_I(2)\}$ and $\mathcal{E}\{s_I(2)s_I(2)\}$.

There are $m(l, l_0)$ nonempty bins between the $l^{th}$ and $(l + l_0)^{th}$ samples. Thus the total number of unique and nonzero elements is less than or equal to $\gamma(2\gamma + 1)m(l, l_0)$. Note that the total number can be less than $\gamma(2\gamma + 1)m(l, l_0)$, because there may be overlaps in the counting of unique items. In the last example, if we have another scattering center between sample 2 and sample 3, we will have another 3 unique and nonzero elements $\mathcal{E}\{s_I(2)s_I(2)\}$, $\mathcal{E}\{s_I(2)s_I(3)\}$ and $\mathcal{E}\{s_I(3)s_I(3)\}$. So that brings the total number of unique items to 6, but we notice that the item $\mathcal{E}\{s_I(2)s_I(2)\}$ is counted twice and the number of unique items is 5.
From the analysis above, we know that the total number of unique and nonzero elements, related to the scattering centers located between the \( l \)th and \( (l + l_0) \)th samples, is less than or equal to \( 6\gamma(2\gamma + 1)m(l, l_0) \). Generally the unique elements in \( R(l) \) (related to the scattering centers located between the \( l \)th and \( (l + l_0) \)th samples) can be translated into unique and nonzero rows in \( J(l, l_0) \). Thus we have the number of unique and nonzero rows in \( J(l, l_0) \) less than or equal to \( 6\gamma(2\gamma + 1)m(l, l_0) \). From the previous analysis, we conclude that

\[
\sum_{k \in A(l, l_0)} g_k(l, l_0) \leq 6\gamma(2\gamma + 1)m(l, l_0) \tag{63}
\]

REFERENCES


