Lemma 3: Two surface normals at distinct points p_1 and p_2 of F cannot intersect at a point p unless $\max(d(p_1, p), d(p, p_2)) > d(F, MA(F))$.

Proof: Suppose not. Assume that $d(p, p_1) \geq d(p, p_2)$, For $\lambda \in [0, 1]$, let $\overline{B}_\lambda$ be the closed ball centered at $\lambda * p + (1 - \lambda) * p_1$ of radius $d_1 \lambda$ and note that for any $0 \leq \lambda < \lambda^* \leq 1$, we have that $\overline{B}_\lambda \subset \overline{B}^*_\lambda$. Let

$$\Lambda = \{\lambda \mid |\overline{B}_\lambda \cap F| \geq 2\}.$$

Since, $\{p_1, p_2\} \subset \overline{B}_1(p)$, it is clear that $\Lambda \neq \emptyset$. Since $\overline{B}_0$ is a singleton set, it is clear that $0 \notin \Lambda$ and by Proposition 1, there exists some maximal $\lambda_0 \in (0, 1)$ such that for all $\lambda < \lambda_0$, $\overline{B}_\lambda \cap F = \{p_1\}$. However, the maximality of λ_0 and the noted monotonicty of the sets $\overline{B}_\lambda$ imply that $\Lambda = [\lambda_0, 1]$, and the ball $\overline{B}_{\lambda_0}$ contains at least two points of F. On the other hand, any point of $\overline{B}_{\lambda_0} \cap F$ must be in the frontier of $\overline{B}_{\lambda_0}$ (because any interior point of $\overline{B}_{\lambda_0}$ which also appeared in F would also be contained in $\overline{B}_{\lambda_0 - \epsilon}$ for some appropriately small positive ϵ); but this is precluded by our choice of λ_0. Hence, the center point of $\overline{B}_{\lambda_0}$ is in the medial axis of F, and we must have $d(p, p_1) \geq d(F, MA(F))$, as desired.