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CSE 254, F08, Dr. T. J. Peters

1. Draw two polygons, denoted as  $A$ ,  $B$ , such that  $A \cap B$  is non-empty and is a subset of some shared edge between  $A$  and  $B$ .
  - a) Draw their set subtraction. This corresponds to infinite precision arithmetic.
  - b) Draw their regularized set subtraction.
  - c) Draw what pathology could occur for 1a under floating point arithmetic.
2. If one considers the unit interval  $(0, 1)$  with the usual operations of addition and multiplication, is this algebraically closed? Either prove or give a counterexample.
3. For your answer to 1c, show what problems could happen with a graphics algorithm that tried to fill the resultant object as if it were a polygon. Specifically, show some good scan lines and one that goes awry. What happens with the one that goes awry – namely, how would this error become apparent?
4. In each of the indicated sets, specify whether the indicated point is in the interior or is a limit point. Give a brief justification of your answer.
  - a.  $(0, 1)$  and the point 0.
  - b.  $(0, 1)$  and the point  $1/2$ .
  - c.  $(0, 1)$  and the point  $1/200$ .
5. Given a polygon and the definition of an interior point, is it possible to write an algorithm that *exhaustively* checks whether any particular point is an element of the interior of a set? Your answer should make very careful use of quantifiers.
6. Given a polygon and the definition of a limit point, is it possible to write an algorithm that *exhaustively* checks whether any particular point is a limit point? Your answer should make very careful use of quantifiers.

**Definition 1:** For a set  $A$  and a point  $x \in A$ , the point  $x$  is in the interior of  $A$  (denoted  $\text{int}(A)$ ) iff there exists some open ball,  $B$  about  $x$  such that  $B \subset A$ .

**Definition 2:** For a set  $A \subset X$  and a point  $x \in X$ , the point  $x$  is a limit point of  $A$  iff for every open ball,  $B$  about  $x$ , it is true that  $B \cap A \neq \emptyset$ .