

Civil & Environmental Engineering Department

CE 366/ME380: Homework 3
Due Date: 10/08/07

1. The potential energy for a fictitious beam is given by

$$\pi = \int_0^L \left\{ A \frac{du}{dx} \frac{d^2w}{dx^2} + B \frac{d^2u}{dx^2} \frac{dw}{dx} + C (u+w)^2 \right\} dx$$

where $u=u(x)$ is the axial displacement, $w=w(x)$ is the transverse displacement, and A,B, and C are constants. Assuming different interpolation for u and w, that is $u(x)=M_i u_i$ and $w(x)=N_i w_i$, obtain the stiffness matrices by performing the following operations:

$$\frac{\partial \pi}{\partial u_j} = 0 \quad \frac{\partial \pi}{\partial w_j} = 0$$

Your results should be in terms of integrals of the shape functions and their derivatives and will be of the following form:

$$K_{ij}^A u_i + K_{ij}^B w_i = 0$$

$$K_{ij}^C u_i + K_{ij}^D w_i = 0$$

2. Given the following energy integral for one-dimensional heat conduction

$$\pi = \int_x \left[k \left(\frac{dT}{dx} \right)^2 + 2cT \right] dx$$

where $T=T(x)$ is temperature, and k and c are constants (thermal conductivity and internal heat generation, respectively) and assuming the following interpolation for the nodal temperatures

$$T(x) = N_i(x) T_i$$

derive expressions for k and f in the following general element equations

$$k_{ij} T_j = f_i$$