1. The potential energy for a fictitious beam is given by

\[ \pi = \int_0^L \left\{ A \frac{du}{dx} \frac{d^2w}{dx^2} + B \frac{d^2u}{dx^2} \frac{dw}{dx} + C (u + w)^2 \right\} dx \]

where \( u = u(x) \) is the axial displacement, \( w = w(x) \) is the transverse displacement, and \( A, B, \) and \( C \) are constants. Assuming different interpolation for \( u \) and \( w \), that is \( u(x) = M_i u_i \) and \( w(x) = N_i w_i \), obtain the stiffness matrices by performing the following operations:

\[ \frac{\partial \pi}{\partial u_j} = 0 \quad \frac{\partial \pi}{\partial w_j} = 0 \]

Your results should be in terms of integrals of the shape functions and their derivatives and will be of the following form:

\[ K_{ij}^A u_i + K_{ij}^B w_i = 0 \]

\[ K_{ij}^C u_i + K_{ij}^D w_i = 0 \]

2. Given the following energy integral for one-dimensional heat conduction

\[ \pi = \int x \left[ k \left( \frac{dT}{dx} \right)^2 + 2cT \right] dx \]

where \( T = T(x) \) is temperature, and \( k \) and \( c \) are constants (thermal conductivity and internal heat generation, respectively) and assuming the following interpolation for the nodal temperatures

\[ T(x) = N_i(x) T_i \]

derive expressions for \( k \) and \( f \) in the following general element equations

\[ k_{ij} T_j = f_i \]