Civil & Environmental Engineering Department

CE 366/ME380: Homework 2  Due Date: 9/24/07

Problem 1: The element equations for a rod element with a distributed axial force \( n \) are

\[
K_{ij} = \int \frac{1}{2} EA \frac{dN_i}{d\xi} \frac{dN_j}{d\xi} d\xi \\
F_i = \int L n N_i d\xi
\]

Derive the terms of the stiffness matrix and consistent force vector (for \( EA=\text{constant} \) & \( n=\text{constant} \)) for the three-node element shown. The shape functions in the natural coordinate system are

\[
N_1(\xi) = \frac{\xi(\xi - 1)}{2} \quad N_2(\xi) = (1 - \xi^2) \quad N_3(\xi) = \frac{\xi(\xi + 1)}{2}
\]

Problem 2: Using only one element, calculate the displacement at the midpoint and end for a rod that is fixed at one end and subjected to a constant distributed axial load \( n \), and compare to the following exact solution.

\[
u(x) = \frac{nL^2}{EA} \left( \frac{x}{L} \right) \left( 1 - \frac{x}{2L} \right)
\]