

## Sample Final Exam

NAME: \_\_\_\_\_

This is a “closed book/closed notes” test  
Make sure to show your work and explain your answers.  
No credit will be given for lucky guesses.

0. [1 pt] Do you understand the instructions above?

Yes	<input type="checkbox"/>
No	<input type="checkbox"/>

Question	Max	Sub-total
<b>0</b>	1	
<b>1</b>	16	
<b>2</b>	30	
<b>3</b>	30	
<b>4</b>	24	
<b>TOTAL:</b>	<b>101</b>	
<b>Earned bonus points:</b>		
<b>GRAND TOTAL:</b>		



## 2 Languages & Automata [30 pts]

Draw a single Venn diagram representing all of the following sets:

1. RL: the set of all regular languages
2. DFSA: the set of all languages recognized by deterministic FSAs
3. NFSA: the set of all languages recognized by non-deterministic FSAs
4. CFL: the set of all context-free languages
5. NPDA: the set of all languages recognized by non-deterministic PDAs
6. NP: the set of all languages decidable in polynomial time by non-deterministic Turing Machines
7. EXPTIME: the set of all languages decidable in exponential time by Turing Machines
8. DL: The set of all Decidable Languages
9. DTM: The set of all languages that have Deterministic Turing Machine deciders
10. TR: The set of all Turing Recognizable languages
11. NTM: The set of all languages that have Non-deterministic Turing Machine (TM) deciders
12.  $TM_{\infty t}$ : The set of all languages that have TM deciders with a single tape unbounded on left and right
13.  $TM_{2t}$ : The set of all languages that have 2-tape TM deciders

### 3 Languages and Language Classes [30 pts]

For each of the following sets, indicate in your diagram from Problem 1 the locations of each set given below. In each case you must reason about the positioning of the sets within the diagram (your answers will be graded on the correctness and relevance of the proofs you give here with respect to each of the sets given below).

1.  $A = \{ a^n b^n \mid n \geq 0 \}$

2.  $B = a^*b^*$

3.  $C = \{ a^n b^n c^n \mid n \geq 0 \}$

4.  $D = \{ a^i b^j c^k \mid i, j, k \geq 0 \}$

5.  $E = \{ a^i b^j c^k \mid i \cdot j = k \text{ and } i, j, k \geq 0 \}$

6.  $F = \{ \langle t, w, c \rangle \mid \text{Turing Machine } t \text{ halts on input } w \text{ within } c \text{ steps} \}$

**4 Closure Properties [24 pts]**

(a) [12 pts] True/False: circle your answer: +2 for each correct answer, -1 for each wrong answer, 0 for blanks.

- 1) True or False: Are Regular Languages closed under *union*?
- 2) True or False: Are Regular Languages closed under *intersection*?
- 3) True or False: Are Regular Languages closed under *concatenation*?
- 4) True or False: Are Context-Free Languages closed under *union*?
- 5) True or False: Are Context-Free Languages closed under *intersection*?
- 6) True or False: Are Context Free Languages closed under *concatenation*?

(b) [6 pts] Show that P is closed under union, concatenation, and complement.

(c) [6 pts] Show that NP is closed under union and concatenation.